Imperfect competition

Industrial Organization

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Welcome to this Industrial Organization course

This is a core course in the Microeconomics field

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How markets work when perfect competition is not an acceptable assumption for firms, i.e. when firms have market power?

The course is about the analysis of markets under imperfect competition and of various pricing strategies, marketing strategies and other strategic manipulations by firms in an attempt to gain or maintain market power (capacity to influence prices).

Mostly about the theory of industrial organization: the course relies on Standard microeconomics (Production theory) and Game theory

Gain for economic analysis

More realistic account of how markets, fundamental institution of economics, actually work and better prediction of market outcomes

Fundamental extension of the micro-economic paradigm, relaxing the perfect competition assumption

Often at the cost of a partial equilibrium approach: "toy models", surplus analysis,...

Useful applications

Practical implications for managers : how to decide on market strategy (prices, investments, R&D, product variety / niches, advertising, supply chain,...)

Practical implications for public authorities: competition policy / competition law, mergers guideline, industrial policy, regulation, innovation policy

0.2. What will we study and why ?

1. Introduction and market power
2. Product differentiation and marketing strategies
3. Entry, exit and the dynamics of market structure
4. Sophisticated pricing strategies
5. Quality and experience goods
6. Consumers' inertia and **Midterm exam**
7. Behavioral model of non-fully rational consumers
8. Horizontal agreements, cartels and mergers
9. Vertically agreements
10. Vertically related markets: foreclosure, mergers
11. Innovation, R& D and intellectual property
12. Networks and platforms

0.2. What will we study and why ?

Readings:

- Tirole, 1988, *The Theory of Industrial Organization*, MIT Press
- Belleflamme - Peitz, 2010, *Industrial Organization: Markets and Strategies*, Cambridge University Press
- Shy, 1996, *Industrial Organization: Theory and Applications*, MIT Press
- Additional required readings and recommended readings provided at each session

No recitation: yet ... ! Practice by yourselves (Tirole, Shy, online material)

Final mark = Mid-term exam: 25% + Final exam: 75%

0.3. Topics for dissertation

Informal presentation of research topics by JPT and BC

0.4. A brief history of IO

The "Harvard tradition" (Bain, Mason, 50s): It developed the **Structure-Conduct-Performance** paradigm:

- The market structure (# of sellers, degree of product differentiation, cost structure, etc.) determines conduct (price, investment, R&D, etc.), and conduct determines market performance (efficiency, ratio of price to marginal cost, innovation rate, profits, etc.)
- No theories, only empirical studies which explain, e.g., profit as a function of concentration, entry barriers, etc.

Empirical analyses relied on measures of profitability ...

- Accounting profits / sales, margin ratio, stock value, rate of return on capital ...
- Poor quality; measures highly correlated with size; low variation with demand shocks

... on measures of concentration ...

- Mostly CR2, Herfindhal index, Entropy index, ...
- All ad-hoc; correlation across countries; stability over time

... on measures of "entry barriers"

- Efficient scale, advertising ratios, R& D ratios
- Poor quality, correlated with size

Econometrics: Profitability (Concentration, Entry barriers)

Weak correlation with accounting profits - concentration, weakly signif., unstable across countries and over time

Positive and signif. correlation margin ratio - concentration and pro-cyclicity of margin ratio increases in concentration
Interpretation: Market power, cartel, role of differentiation, differences in cost-efficiency ?

Strong positive correlation profitability - entry barriers, positive correlation rate of entry - entry barriers
Interpretation: Technology leads to "natural concentration" or strategic manipulation / deterrence?

0.4. A brief history of IO

The "Chicago tradition" (Director, Stigler, 60s):

- Problem in Harvard regressions: these variables are determined simultaneously; the links must be interpreted as correlation, not causal relationship.
- It emphasized the need for rigorous theoretical analysis and empirical identification of competing theories. Important impact of the development of the field.
- Very permissive view of market behavior and relative distrust of government intervention.

The second wave of interest, with unified paradigm, starts in the late 70s. Due to the growing dissatisfaction with empirical studies and game theory imposing itself as the standard tool for the analysis of strategic conflict.

Today's topic: **Basic models of imperfect competition / market power with an homogeneous good.**

A necessary preliminary step because the models we will use rely on the building blocks that I will present today and in the first-half of session 2.

Mostly a reminder for most of you. Make sure you are perfectly familiar with the material before session 3 ! Practice with exercises.

I. Today's session

We are interested in modeling a market in which one, or a few strategic firms interact.

- Monopoly, in particular
  - Standard behavior
  - Multi-product monopoly
  - Durable good monopoly
- Oligopoly
  - Bertrand competition
  - Bertrand-Edgeworth competition
  - Cournot competition
  - Supply functions
  - Applications

Monopoly

Monopolistic market = market where there is only one producer of the good / service, aware of its influence on the price

Why are there monopolies ?

- Informational reasons: secrecy
- Technological reasons: economies of scale, increasing returns
- More generally (multi-dimension) sub-additive cost function (economies of scope):

$$\forall(y_1, ..., y_n), C(\sum_1^n y_i) \leq \sum_1^n C(y_i)$$

- Legal reasons: patents, public franchises
- Yet, examples of pure monopolies are fairly rare

Partial equilibrium framework

- Market demand $D(.)$ twice differentiable, decreasing on $[0, \bar{p}]$, null for $p \geq \bar{p}$, with inverse demand $P(.)$
- One single producer with cost function $C(.)$ twice differentiable, with $C'(0) < \bar{p}$
- Assume: $\exists$ (unique) social optimum (competitive equilibrium) q^o such that: $C'(q^o) = P(q^o)$

Monopoly chooses price p to maximize profits, internalizing the impact on demand:

$$\max_{p \geq 0} \{pD(p) - C(D(p))\}$$

Solution exists within $[p^o, \bar{p}]$

Lerner formula:

- FOC yields implicit equation in monopoly price p^m:

$$\frac{p^m - C'(q^m)}{p^m} = \frac{1}{\varepsilon(p^m)}$$

where $q^m = D(p^m)$ is the monopoly production and $\varepsilon(p^m)$ is the price elasticity of demand at price p^m

$$\varepsilon(p^m) = -\frac{p^m D'(p^m)}{D(p^m)} = -\frac{dLn(D(p))}{dLn(p)}|_{p=p^m}$$

- Trade-off: increase in margin vs demand contraction
- Operates always in region where elasticity is larger than 1

Alternative (dual) formulation, using $R(q) = P(q)q$, the revenue function:

$$\max_{q \geq 0} \{R(q) - C(q)\}$$

- Solution exists within $[0, q^o]$
- FOC yields "Marginal revenue = marginal cost" formula:

$$R'(q^m) = P(q^m) + q^m P'(q^m) = C'(q^m)$$

- Picture in class

A word on SOC (either program): decreasing returns and decreasing demand do not guarantee concavity ! Requires concavity of revenue function (SC: concave demand)

Efficiency properties

- $p^m > C'(q^m)$; moreover, $q^m < q^o$
- Monopoly optimum is inefficient: excessive pricing or restrictive supply
- Yet, productive efficiency (cost minimization)
- The deadweight loss can be measured by change in Marshallian surplus:

$$\int_{q^m}^{q^o} (P(x) - C'(x))dx > 0$$

- Harberger's measure (1954): .1% of GNP...! More thorough analysis lead to 7% of GNP

Additional allocative inefficiency and rent-seeking:

- Posner (1975) argues that the social cost of a monopoly should include the overall monopoly profit a firm obtains: firms would waste resources in activities which do not have any social value in the attempt to maintain or acquire monopoly power (lobbying, bribing, etc.).
- Argument hinges on: (1) perfect competition among agents who engage in rent-seeking; (2) the rent-seeking technology exhibits constant returns-to-scale (so that the whole monopoly profit is dissipated in this process); (3) these activities have no social value (but adequacy of a partial equilibrium framework?).

Possible productive inefficiency:

- Leibenstein (1966) and the concept of "X-inefficiency": monopoly power, and the quiet life which comes with it, brings about managerial inefficiency, hence higher costs.
- Link between competition on the product market and the power of managerial incentives, with shareholders - manager relationships under asymmetric informational (Schmidt, 1996, and Aghion-Dewatripont-Rey, 1998 & 1999).
- Some empirical support (Nickell, 1996), but strong stakeholder's monitoring can be a substitute, e.g. financial pressure or external shareholder control.
- The monopoly has low incentives to innovate, because of the replacement effect (Arrow, 1962): more about this in the R&D - innovation chapter

Remedy to monopoly market power:

- Can taxation of monopoly's output at rate t restore efficiency?

$$\max_{p \geq 0} \{D(p)(p-t) - C(D(p))\}$$

- To restore efficiency, t must be such that the optimum is reached at p^o; hence: $t/p^o = -1/\varepsilon(p^o) < 0$. Output must be subsidized
- Is this a convincing answer ?

Optimal regulation of monopolies: see Martimort - Gagnepain's course

II.4. Multi-product monopoly

Microsoft is a (quasi-)monopolist on its OS but also on many softwares. Monsanto on GMO seeds and anti-insect / anti-parasites spray. How does it affect (multi-product) monopoly pricing ?

- Consider n-good monopoly, with demand $D_i(\mathbf{p})$ on good i for $\mathbf{p} = (p_1, ...p_i, ...p_n)$ and cost function $C(q_1, ...q_n)$
- Monopolist's program:

$$\max_{\mathbf{p}}[\sum_{i=1}^{n} p_i D_i(\mathbf{p}) - C(D_1(\mathbf{p}), ...D_n(\mathbf{p}))]$$

- FOC: for all $i = 1, 2, ..., n$

$$[D_i + p_i \frac{\partial D_i}{\partial p_i}] + \sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i} = \sum_{j=1}^{n} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i}$$

Independent demands $D_i(p_i)$, **separable costs** $\sum_{i=1}^n C_i(q_i)$: equivalent to n mono-product monopolies

Demand substitutability, separable costs:

$$\frac{p_i - C_i'}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j \neq i} \frac{p_j - C_j'}{p_j} \cdot \frac{p_j q_j}{p_i q_i} \cdot \frac{\varepsilon_{ij}}{\varepsilon_{ii}}$$

with $\varepsilon_{ij} = -\frac{p_i}{q_j} \cdot \frac{\partial D_j}{\partial p_i}$, cross-elasticity of demand for j w.r.t. p_i

- If substitute, $\varepsilon_{ij} < 0$, higher margin ratio than with independent monopolies: cross-participation of profit centers
- If complements: low margin on platform and high margin on complement softwares

Independent demands, dependent costs:

Example of learning by doing in which $i = 1, 2$ denotes the period, δ the discount factor, and the cost at $i = 2$ depends on the production in period $i = 1$: $C(q_1, q_2) = C_1(q_1) + C_2(q_1, q_2)$

$$\frac{p_1 - C_1'}{p_1} = \frac{1}{\varepsilon_1} + \frac{\delta}{p_1}\frac{\partial C_2}{\partial q_1}$$

Positive learning effect: $\frac{\partial C_2}{\partial q_1} < 0$, reduces further the margin ratio in the first period

II.5. Monopoly and Durable Good

Durable goods

Assume a partial equilibrium setting: a good is said to be a perfectly durable if, starting from a given date, consumers get the same inter-temporal utility from the consumption of the good whether they buy it at this date or if they bought it before.

Durable goods are often imperfect: the flow utility they yield may decrease over time as they deteriorate or get obsolete. Durable goods with non-trivial durability represent roughly 60% of production.

- Dynamic setting to analyze pricing over time.
- Inter-temporal link between pricing and marketing strategies: commitment, choice of durability, planned obsolescence, leasing

II.5. Monopoly and Durable Good

A two-period model of a monopoly selling a durable good:

- Two periods, $t = 1, 2$; discount factor δ
- Cost of production is null;
- Willingness to pay per period for representative consumer when q units for consumption overall: $P(q) = 1 - q$

If the monopoly sells q_1 at $t = 1$ and q_2 at $t = 2$, with consumers anticipating at $t = 1$ that q_2^e will be sold at $t = 2$, the prices for both periods are:

$$\begin{aligned} P(q_1 + q_2) &\quad \text{at date } 2, \\ P(q_1) + \delta P(q_1 + q_2^e) &\quad \text{at date } 1. \end{aligned}$$

Full commitment model:

Suppose that the monopoly can commit at $t = 1$ to a policy for both periods: q_1 and q_2 decided at $t = 1$, which fixes expectations at $q_2^e = q_2$

$$\max_{q_1, q_2} [P(q_1) + \delta P(q_1 + q_2)]q_1 + \delta P(q_1 + q_2)q_2,$$

or:

$$\max_{q_1, Q_2} P(q_1)q_1 + \delta P(Q_2)Q_2 \quad \text{where } Q_2 = q_1 + q_2,$$

which leads to:

$$q_1 = Q_2 = q^m \equiv \arg\max_q P(q)q \Leftrightarrow q^m = \frac{1}{2} \text{ and } \pi^m = \frac{1}{4}.$$

Optimal for the monopoly to sell the monopoly quantity at $t = 1$ and then to sell nothing at $t = 2$: buy now or never !

For the commitment solution, monopoly faces a residual demand at $t = 2$: the consumers' willingness to pay for q_2 additional units would be: $P(q^m + q_2)$. Problem of the credibility of the commitment !

No commitment model:

The monopoly cannot credibly commit to q_2 at $t = 1$.

- Instead, q_2 is chosen at $t = 2$ once q_1 has been purchased
- Consumers have to form expectations at $t = 1$ about q_2, denoted q_2^e
- Subgame-perfect equilibrium: expectations are correct.

II.5. Monopoly and Durable Good

At $t = 2$, given q_1 the monopoly solves:

$$\max_{q_2} P(q_1 + q_2)q_2 \Leftrightarrow \hat{q}_2(q_1) = \frac{1 - q_1}{2}$$

At $t = 1$, buyers rationally anticipate that the monopoly will put an additional quantity $\hat{q}_2(q_1)$ at $t = 2$ whenever it offers q_1 at period $t = 1$, so that their willingness to pay:

$$\hat{P}(q_1) \equiv P(q_1) + \delta P(q_1 + \hat{q}_2(q_1)).$$

The first-period quantity maximizes the monopolist profit, given its behavior in the consecutive period:

$$\max_{q_1} \quad \underbrace{\hat{P}(q_1)q_1}_{\text{1st period profit}} \quad + \delta \underbrace{P(q_1 + \hat{q}_2(q_1))\hat{q}_2(q_1)}_{\text{2nd period profit}}.$$

Durable good: equilibrium without commitment

The monopoly creates, and is hurt by, its own competition:

$$q_1^* < q^m, Q_2^* > q^m, p_1^* < (1+\delta)p^m, p_2^* < p^m, \pi^* < (1+\delta)\pi^m.$$

- At $t = 1$, monopoly would like to convince buyers to pay the high price $(1+\delta)P(q_1)$. But, anticipating $\hat{q}_2(q_1)$ at $t = 2$, buyers' willingness to pay is:

  $$\hat{P}(q_1) = P(q_1) + \delta P(q_1 + \hat{q}_2(q_1)) < (1+\delta)P(q_1).$$

  Demand at $t = 1$ is lower, the monopoly sells less: $q_1^* < q_1^m$.
- At $t = 2$, monopoly does not account for the negative externality q_2 imposes on the $t = 1$ demand, so: $Q_2^* > q^m$.

The Coase Conjecture:
(Bulow, 1982, Stokey,1981, Gul-Sonnenschein-Wilson, 1986)

The monopolist suffers from the consumers' rational belief that he will flood the market. Even worse, assume:

- Infinitely-lived consumers and monopoly, infinitely durable good, discount factor δ
- Consumers have unit demand, with valuation (i.e. present discounted value of services from the date of purchase on) distributed on $[c, +\infty)$, with c monopolist's unit cost.

Coase conjecture

When $\delta \to 1$, the monopolist's inter-temporal profit tends to zero (its price at any date tends to c in any stationary sequential equilibrium).

How to Evade the Coase Problem:

- Non-stationary equilibria (Ausubel-Deneckere, 1987): weird...
- Leasing instead of selling: the good is implicitly returned to the firm, hence repeated static monopoly pricing. But moral hazard and costly monitoring.
- Price-guarantees and return policy: de facto reimbursement of the difference b/w the prices at $t = 1$ and at $t = 2$.
- Destruction of production facilities
- Planned obsolescence
- Reputation to maintain prices

II.6. Non-price distortions

Besides prices, other corporate decisions matter and may also generate inefficiencies (Examples: choice of quality, of variety, advertising, ...)

For all these decisions:

- Monopoly decision: marginal revenue = marginal cost
- Social optimum: marginal surplus = marginal cost

Hence, in general, imperfect appropriation of surplus associated with a decision $\Rightarrow$ inefficient decision (for any given quantity traded), and inefficiency may go either way

Moreover, since there is inefficient contraction of supply, final decision may be distorted upwards or downwards

What kind of bias does the monopolist introduce in its choice of quality or durability of its product(s)?

Inverse demand depends both on quantity q and quality s:

$$p = P(q, s) \quad \text{with } \frac{\partial P}{\partial s} > 0.$$

Cost $C(q, s)$ with $\frac{\partial C}{\partial s} > 0$.

Remark: s can be interpreted as the durability of the good.

Social Optimum:

Welfare:

$$W(q,s) = \int_0^q P(x,s)dx - C(q,s),$$

FOC for surplus-maximizing decisions:

$$P(q,s) = C_q(q,s), \tag{1}$$

$$\int_0^q P_x(x,q)dx = C_s(q,s). \tag{2}$$

(1): Usual marginal cost pricing rule.
(2): $P_s(x,s)$ is the marginal valuation for quality for the marginal buyer when the price is $P(x,s)$; the social planner is interested by the average marginal valuation for quality.

Monopoly Optimum:

Profit:

$$\pi(q, s) = P(q, s)q - C(q, s),$$

FOC for profit-maximizing decisions:

$$P(q, s) + qP_q(q, s) = C_q(q, s), \quad (3)$$
$$qP_s(s, q) = C_s(q, s). \quad (4)$$

(3): Usual monopoly pricing rule.
(4): The marginal revenue revenue associated with a unit increase in quality is equal to the marginal cost of producing this quality.

II.6. Non-price distortions

The social planner looks at the effect of an increase in quality on all buyers; by contrast, the monopolist considers the effect of an increase in quality on the marginal buyer, that determines the price.

$$P_s(q,s) \quad \text{vs} \quad \left(\int_0^q P_s(x,s)dx\right)/q$$

indicates the way the monopoly biases its choice of quality.

No general result can be obtained ! E.g. consumers indexed by θ distributed according to $F(.)$ on $[0,\alpha]$ demand 0/1 unit with either:

- $u(1,p) = \theta s - p$
- $u(1,p) = \theta + (\alpha - \theta)s - p$

Most common situation is that of imperfect competition among a finite, and often small number of firms = an oligopoly. Then, a firm no longer encounters a passive environment, strategic interactions among firms modeled via non-cooperative game theory: which game to consider ?

Each firm takes into account the influence of its behavior on the market: firms are price-makers.

Firms use different "instruments" to compete in a market: price, capacities, product characteristics, R&D, etc. Some instruments can be changed more quickly than others: difference b/w short-run and long-run.

Duopoly facing an homogeneous good demand $D(.)$ with a symmetric technology exhibiting CRS: $C_i(q_i) = cq_i$

Bertrand duopoly game: Firms choose simultaneously their prices p_i. Demand goes to the lowest price firm (equal split if tie) and firms have to supply all the forthcoming demand they get:

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j, \\ \frac{1}{2}D(p_i) & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j. \end{cases}$$

Profit of firm i: $\pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j)$.

Symmetric Bertrand equilibrium

There exists a unique Nash equilibrium; it corresponds to $p_i = c$ and implies full efficiency.

Bertrand paradox: apart from monopoly, everything is fine...!?

- What if richer models of Bertrand competition maintaining CRTS? Robustness apart from some pathological case
- What if increasing returns ? Problems of existence
- What if decreasing returns ? Problems of multiplicity
- Decreasing returns and voluntary trading: Edgeworth model
- What if competition in quantities ? Cournot model
- What if differentiated products ? models of horizontal differentiation, vertical differentiation, diversity
- What is various forms of cooperative behavior, e.g. cartel ? Models of collusive pricing

More than 2 firms:

- $p_i \geq c$ for all i and at least two firms set $p_i = c$.
- The only equilibrium in which all firms are active is the symmetric one, $\forall i, p_i = c$
- With bounded monopoly profits, the unique Bertrand equilibrium outcome is the zero-profit outcome
- Curiosity: with unbounded monopoly profit, there is a Folk theorem, any positive profit can be achieved as the expected per firm profit in a symmetric mixed strategy equilibrium (Baye-Morgan, 1999): e.g. with $D(p) = p^{-1/2}$.

Increasing returns:

- Assume constant unit cost c and avoidable fixed cost F if $q > 0$, such that monopoly is viable
- If firms decide at the same time about entry and price, no (p.s.) equilibrium with 2 active firms nor with only one active firm, with the equal splitting assumption if tie
- If random selection if tie, unique Bertrand equilibrium at $\hat{p} = \inf\{p, (p-c)D(p) - F = 0\}$, i.e. average cost pricing
- Then, zero-profits, industry cost-efficiency (one only produces), *constrained* efficiency (s.t. firms make no loss) (Cf sustainability - contestability, Baumol-Panzar-Willig, 1982)
- If, however, firms enter first (and pay F) and then decide prices: unique equilibrium outcome is monopoly ! Harsh post-entry competition implies low entry

Decreasing returns:

- Assume increasing smooth convex $C(.)$, with $C(0) = 0$; $p = C'(q)$ is the firm's supply curve
- Demand strictly decreasing, with choke price $\overline{p}$

$$\pi_n(p) = \frac{pD(p)}{n} - C\left(\frac{D(p)}{n}\right)$$

$$\overline{p}_n \equiv \{p \in [0, \overline{p}); \pi_1(p) = \pi_n(p)\} > \underline{p}_n \equiv \{p \in [0, \overline{p}); \pi_n(p) = 0\}$$

- All firms charging common price within $[\underline{p}_n, \overline{p}_n]$ is a Bertrand equilibrium (Dastidar, 1995 & 1997)
- Among these, the Walrasian price, $p^w = C'\left(\frac{D(p^w)}{n}\right)$.
- Deviating slightly below p^w yields profit:

$$\pi_1(p^w) < max_q \left[p^w q - C(q)\right] = \pi_n(p^w)$$

Intuition with decreasing returns:

When one firm undercuts, it gets the whole demand which implies a sharp increase in marginal cost, possibly above price, hence lower profits after undercutting.

Undercutting deviations are deterred, hence the existence of many possible equilibrium prices

Critical assumption: the firm has to satisfy all the ensuing demand, i.e. produces more than its competitive supply.

What if **voluntary trading** ? Bertrand-Edgeworth model

III.3. Edgeworth model

Assume: firms can decide how much they supply given the demand that they get

A firm never wants to supply more than its competitive supply: $S_i(p) = \arg\max_{q\in[0,D(0)]}\{(pq - C_i(q)\}$ at price p.

If $p_1 < p_2$, firm 1 supplies $S_1(p_1)$ and so may not serve all demand if $D(p_1) > S_1(p_1)$. Some consumers turn to firm 2, this is firm 2's residual demand curve. Firm 2 can now exert market power on its residual demand curve, and make profit.

A force towards higher prices: the Walrasian price ceases to be an equilibrium price. Precise results depend on how residual demand is constructed.

Rationing rules: what is the residual demand for firm 2, when $p_1 < p_2$? Lottery, auction, queuing, random,...

"Efficient" rule: $D_2(p_1, p_2) = \sup\{D(p_2) - S_1(p_1); 0\}$

- It yields a parallel shift of demand curve to the left by $S_1(p_1)$
- Highest-priced portion of the demand curve served first, hence surplus maximizing rule.

Proportional rule: $D_2(p_1, p_2) = \sup\{\left(1 - \frac{S_1(p_1)}{D(p_1)}\right) D(p_2); 0\}$

- Corresponds to queuing system, residaul consumers are a random sample of all consumers
- Rules out resale possibilities

A Bertrand-Edgeworth equilibrium is an Nash equilibrium of the Edgeworth game

Walrasian prices vs Edgeworth equilibrium

With smooth strictly decreasing returns (smooth strictly convex costs), the Walrasian price p^w, given by $\sum_i S_i(p^w) = D(p^w)$, is not a Bertrand-Edgeworth equilibrium price vector.

Intuition with 2 firms and efficient rule: once firm 1 has supplied $S_1(p^w)$, firm 2 has monopoly power on the residual demand $D_2(p^w, p_2)$. The right-derivative of firm 2's profit at $p_2 = p^w$ is equal to its Walrasian supply, hence positive. Firm 2 has an incentive to charge above p^w.

Duopoly with capacity constraints

Firm $i = 1, 2$ has installed capacity k_i with zero marginal cost below capacity.

- Only candidate pure strategy equilibrium is the competitive price: $p^w = 0$ if $k_1 + k_2 > D(0)$ and $p^w = P(k_1 + k_2)$ if $k_1 + k_2 \leq D(0)$, where firms sell at capacity whenever they do not flood the market
- If $k_i \geq D(0)$ for $i = 1, 2$, standard Bertrand as capacities are irrelevant: existence is OK.
- If $0 < \min\{k_1, k_2\} < D(0)$, firm 2 pricing slightly above $p_2 > p^w$ gets: $p_2[D(p_2) - k_1]$ (efficient rationing) or $p_2 D(p_2)[1 - \frac{k_1}{D(p^w)}]$ (proportional rationing), this may be profitable if demand is sufficiently inelastic at p^w: problem of existence of p.s. equilibrium

Example with linear demand, small capacities, efficient rationing

Suppose moreover demand is $D(p) = 1 - p \Leftrightarrow p = P(Q) = 1 - Q$

- Assume installed capacities are: $k_i \leq \frac{1}{3}$
- There is a unique equilibrium: both firms charge the competitive price $p^w = P(k_1 + k_2)$ and both firms dump their full capacities on the market
- Deviating below makes the firm sell full capacity at a lower price
- Deviating above yields $p[D(p) - k_1]$, marginal profit at p^w:

$$D(p^w) - k_1 + p^w D'(p^w) = k_2 - [1 - k_1 - k_2] = -[1 - k_1 - 2k_2] < 0$$

- Ultimately, firms' profits in equilibrium are: $k_i P(k_1 + k_2)$, as in **a Cournot setting**

III.3. Edgeworth model

In general (demand functions, large enough capacities), pure strategy equilibria do not exist; mixed strategy equilibria do (Dasgupta-Maskin, 1986, Maskin, 1986)

Nature of mixed strategy equilibria (concave demand; efficient rationing, $k_1 > k_2$)

- Common randomization support: interval $[\underline{p}, \overline{p}]$ with $\overline{p} = \arg\max_p \{p(D(p) - k_2)\}$ and $\underline{p} > p^w$: all prices above the competitive price
- Firm 1's cdf FOS-dominates firm 2's cdf (firm 1 may even have a mass at $, \overline{p}$): the penalty for firm 1 of being undercut is limited because firm 2 cannot sell that much (e.g. if k_2 close to 0, incentive to monopoly pricing for firm 1)
- High capacities lead to low equilibrium prices (FOSD)

Assuming capacities are costly to build, consider a 2-stage duopoly game: first capacities, then prices.

Idea: building capacities involves a longer-term strategy that setting prices

If capacity cost is high enough, only small capacities make sense and the price subgame after $(k_1 + k_2)$ has $p_i = P(k_1 + k_2)$ as the unique p.s. equilibrium, as in former linear example. Firm i's payoff in first stage:

$$k_i P(k_1 + k_2) - C_i(k_i)$$

as if Cournot competition in capacities.

Foundations of Cournot equilibrium (Kreps-Scheinkman, 1983)

Under efficient rationing, with a smooth concave demand and a smooth convex cost for capacities $C(.)$ such that $C'(0) > 0$, the unique (SP) equilibrium outcome of the two-stage game involves Cournot productions (assuming unique Cournot equilibrium).

- The equilibrium tends to be more competitive than Cournot with proportional rationing (Davidson-Deneckere, 1986)
- With uniformly elastic demand and no production costs for given capacities, the Cournot outcome obtains as an equilibrium whatever the rationing rule

If firms make irreversible production decisions first, a quantity-setting reduced form model may be appropriate, Cournot corresponding to the least competitive situation

III.4. Cournot model

Dual approach: competition through quantities, price adjusts to clear supply and demand.

- "Ventes à la criée" on daily markets in which prices adjust so that the quantities of product are sold.
- Tour operators: Approximately 18 months before the season, tour operators decide their offers and book hotel rooms and flights. Once catalogs are determined, prices adjust in the short run.
- Theoretical justification: as hinted previously, the Cournot reduced form can *sometimes* be rationalized with a two stage game in which firms first decide capacities and then compete in prices.

Cournot game form:

Firms $i = 1, ..., N$ choose simultaneously their quantities q_i of an homogeneous good with cost function $C_i(.)$, increasing.

Inverse demand function: $P(.)$, continuous decreasing when positive, null for high enough quantity. Once quantities are decided, the market price adjusts 'unspecified mechanism) so that $p = P\left(\sum_i q_i\right)$.

Denoting $Q_{-i} \equiv \sum_{j \neq i} q_j$, firm i's profit is:

$$\pi_i(q_1, ...q_N) = P(q_i + Q_{-i})q_i - C_i(q_i)$$

Existence of a pure strategy Cournot equilibrium

Standard smooth (C^2) setting:

- Decreasing marginal revenue: $P' + q_i P'' \leq 0$ (restrictive)
- $C_i'' - P' > 0$ for all i (mild)
- Then profit is strictly concave in q_i, use Debreu's existence theorem

Other existence results (Vives, 2000, Amir 1996, Amir-Lambson 1996)

- With strictly decreasing marginal revenue
- With super-modularity
- With general demand and identical convex costs

Assume $P' + q_i P'' \leq 0$ and $C_i'' - P' > 0$, $\forall i$. Best reply of firm i to Q_{-i}, $R_i(Q_{-i})$ is unique solution (if positive) to the FOC:

$$\frac{\partial \pi_i}{\partial q_i} = P(q_i + Q_{-i}) + q_i P'(q_i + Q_{-i}) - C_i'(q_i) = 0$$

$R_i(.)$ is smooth when positive and has slope within $(-1, 0]$:

$$R_i'(Q_{-i}) = -\frac{\partial^2 \pi_i / \partial q_i \partial Q_{-i}}{\partial^2 \pi_i / (\partial q_i)^2} = -\frac{P'(Q) + q_i P''(Q)}{2P'(Q) + q_i P''(Q) - C_i''(q_i)}$$

Decreasing best replies: common wisdom of Cournot, but not necessary in general. For $P(Q) = (1+Q)^{-\alpha}$ with $\alpha > 2$ and zero costs, best replies are increasing with unique intersection...! Costs bias best replies into being decreasing.

Uniqueness:

Under $P' + q_i P'' \leq 0$ and $C_i'' - P' > 0$ for all i, unique Cournot equilibrium. Proof:

- Let $\phi_i(Q)$ the unique solution to: $q_i = R_i(Q - q_i)$; when positive, it is smooth and non-increasing.
- So $\sum_i \phi_i(.)$ non-increasing and has unique interaction with 45^o-line: unique aggregate equilibrium Cournot output

For uniqueness, one can relax $P' + q_i P'' \leq 0$ to $P(.)$ being log-concave; or even more ... (see index theorem, technical)

III.4. Cournot model

Characterization of interior equilibrium with $Q^c = \sum_{i=1}^{N} q_i^c$:

$$P(Q^c) + q_i^c P'(Q^c) = C_i'(q_i^c) \text{ or } \frac{p^c - C_i'}{p^c} = \frac{q_i^c}{Q^c} \cdot \frac{1}{\varepsilon(p^c)}$$

- Productive inefficiency: marginal costs not equalized
- Allocative inefficiency: price > marginal costs of active firms
- Monopoly pricing (inverse elasticity rule) on residual demand $P_R(q_i) = P(q_i + Q_{-i})$
- With constant or decreasing returns, $Q^o > Q^c > Q^m$: inefficient supply in equilibrium ...
- ... but less inefficient than under monopoly: not profit maximization for the industry as externality of q_i on π_j

III.4. Cournot model

Linear Cournot oligopoly

- $P(Q) = a - bQ$
- N symmetric firms: $C_i(q_i) = cq_i$ with $c < a$
- Benchmarks: $Q^o = \frac{a-c}{b}$ and $Q^m = \frac{a-c}{2b}$
- Cournot FOC yields

$$Q_N^c = \frac{a-c}{b} \cdot \frac{N}{1+N} \text{ and } p_N^c = \frac{a+cN}{1+N}$$

- The model covers the whole range from monopoly ($N = 1$) to perfect competition asymptotically when $N \to \infty$
- It also yields natural comparative statics results (e.g. increase in cost c leads to increase in price and decrease in individual and total output)

Comparative statics

Conform to intuition provided regularity conditions are met:

- (Local) Stability of best-reply dynamics, i.e. best replies cross in the correct order
- Decreasing best replies

Symmetric cases

- $E = -\frac{QP''(Q)}{P'(Q)}$ (decreasing marginal revenue implies $E < 1$)
- If $E + \frac{C''}{P'} < 1 + n$ and $\frac{C''}{P'} < 1$, there exists a unique locally stable Cournot equilibrium
- Unit tax increase induces a decrease in productions and an increase in price, possibly with over-shifting (if $E + \frac{C''}{P'} > 1$)
- Even an increase in profits if $E + \frac{C''}{P'} > 2$ (not if $E < 1$)

Define the average Lerner index:

$$L = \sum_{i=1}^{n} \left(\frac{q_i}{Q}\right) \frac{(p - C_i')}{p} = \sum_{i=1}^{n} \left(\frac{q_i}{Q}\right)^2 \frac{1}{\varepsilon} = \frac{H}{\varepsilon}.$$

H: Herfindhal-Hirschman Index of concentration: used in merger analysis. Note that H increases when either n decreases or firms have more unequal market shares

Concentration and welfare:

- For identical firms and identical unit costs, an increase in H induces a decrease in welfare
- Not true in general: e.g. with asymmetric costs, welfare increases when low-cost firms gain market share at expenses of high-cost firms, which also increases H.

Large markets

Existence under mild conditions: the decreasing marginal revenue assumption boils down to decreasing demand

Asymptotic results when the number of firms $n \to \infty$

- For constant or decreasing returns, individual outputs converge to 0 as $1/n$, margin ratio vanish and Cournot equilibrium tends to a price-taking behavior
- For U-shaped cost curve or everywhere-decreasing average costs, approximate efficiency also prevails, although individual output does not converge to 0 (Novshek, 1980)

Adapted from Sullivan (1985)

- US Data. Per state and per year: common price p, number of packs q_i, tax rate t
- Starting from Cournot with N (effective number of firms): $\frac{p-t-C_i'}{p} = \frac{q_i}{Q} \cdot \frac{1}{\varepsilon}$
- Summing: $N(p-t) - \sum_i C_i' = \frac{p}{\varepsilon}$
- Using a lower bound c on the cost of a pack $C_i'(q_i) \geq c$ and viewing p and Q as function of tax t:

$$N \geq N(t,c) = -\frac{p'(t)Q(t)}{Q'(t)(p(t)-t-c)}$$

- Strategy: estimate $Q(t)$ and $p(t)$ and deduce $N(.)$ for all possible values of c

	Const	t	$(t - t_{mean})^2$
$Ln(Q)$	5.111	-.0245	-.00013
	(.016)	(.0012)	(.00018)
p	14.24	1.089	.0090
	(.34)	(.026)	(.0035)

- $p'(t) > 0$ and $Q'(t) < 0$: intuitive
- $p'(t) > 1$ statistically significant: reject an hypothesis that this market is perfectly competitive !

c	0	3	6	9	12
estim N	2.88	3.57	4.70	6.89	12.88
95% interval lower bd	2.57	3.19	4.21	6.16	11.52
95% interval upper bd	3.18	3.95	5.20	7.62	14.24

$N > 2.5$ at 95% confidence level for t_{mean}: reject hypothesis that the market is monopolized !

For reasonable cost between 6 and 10 cents / pack, market behaves as a 4- or 5-firm Cournot oligopoly

III.6. Supply functions

Why not generalize and consider that firms choose supply functions. That is, a firm chooses $S_i(.)$, thereby committing to select a price-quantity pair (p_i, q_i) satisfying $q_i = S_i(p_i)$.

Relevance: electricity markets work that way (Green-Newberry, 1992)

Game in supply functions:

- n firms, smooth increasing convex costs $C_i(.)$, smooth decreasing concave demand $D(.)$.
- If firms $i = 1, 2, ..., n$ choose $S_i(.)$ and if there exists a unique price p such that $D(p) = \sum_i S_i(p)$, then firm i's payoff is:

$$\pi_i = pS_i(p) - C_i(S_i(p))$$

- Otherwise, say $\pi_i = 0$.

Multiplicity of equilbria in supply functions

Given $S_j(.)$ for $j \neq i$, firm i faces a residual demand

$$D_i^r(p) = D(p) - \sum_{j \neq i} S_j(p)$$

Firm i's optimal response is any $S_i(.)$ such that $q_i^* = S_i(p^*)$, with:

$$p^* \in \arg\max_p \{pD_i^r(p) - C_i(D_i^r(p))\} \quad \text{and} \quad q_i^* = D_i^r(p^*)$$

There are many such supply function through (p^*, q_i^*).

Folk theorems: any market-clearing allocation that give firms their min-max payoffs can be supported as a supply function equilibrium !

Introducing uncertainty (Klemperer-Meyer, 1989)

Suppose $D(p)+\theta$, where θ is random on $[0,+\infty)$, unknown when firms decide their C^2 supply functions .

Given the supply functions of rivals, firm i has now a residual demand that depends on θ and therefore an optimal choice $(p_i(\theta), q_i(\theta))$ that depends on θ. If $p_i(/)$ is invertible, this defines entirely a supply function that is best response to the others.

The supply function is not simply pinned down at one point but for a whole set of points, as θ varies: this reduces multiplicity.

Analysis (assuming identical $C(.)$):

- Residual demand: $D_i^r(p,\theta) = D(p) + \theta - \sum_{j \neq i} S_j(p)$
- FOC, replacing $D_i^r(p,\theta)$ by $S_i(p)$ at optimum:

$$p - C'(S_i) = -\frac{S_i}{\partial D_i^r / \partial p}$$

- If θ has full support $[0, +\infty)$, a symmetric equilibrium in supply functions exists, characterized by DE:

$$(n-1)S'(p) = \frac{S(p)}{p - C'(S(p))} + D'(p)$$

  with $0 < S'(p) < \infty$ for all $p > 0$.
- No asymmetric equilibrium and equilibrium set is either a singleton or connected.

Properties of the supply function equilibrium

It does not depend on the distribution of θ ! So, if it is unique, natural candidate for selection under certainty.

Using the DE written as:

$$p - C'(q) = \frac{q}{(n-1)S'(p) - D'(p)}$$

supply function equilibrium generates an outcome (price, quantities, profits) for a given θ intermediate between Cournot (corresponding to $S' = 0$) and Bertrand (corresponding to $S' = \infty$ and $p - C' = 0$) since $0 < S'(p) < \infty$.

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