Product differentiation

Industrial Organization

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September 22, 2016

Motivation

The Bertrand paradox relies on the fact buyers choose the cheapest firm, even for very small price differences.

In practice, some buyers may continue to buy from the most expensive firms because they have an intrinsic preference for the product sold by that firm: Notion of differentiation.

Indeed, assuming an homogeneous product is not realistic: rarely exist two identical goods in this sense

- For objective reasons: products differ in their physical characteristics, in their design, ...
- For subjective reasons: even when physical differences are hard to see for consumers, branding may well make two products appear differently in the consumers' eyes

Differentiation among products is above all a property of consumers' preferences:

- Taste for diversity
- Heterogeneity of consumers' taste

But it has major consequences in terms of imperfectly competitive behavior: so, the analysis of differentiation allows for a richer discussion and comparison of price competition models vs quantity competition models.

Also related to the practical question (for competition authorities) of market definition: set of goods highly substitutable among themselves and poorly substitutable with goods outside this set

Motivation

Firms have in general an incentive to affect the degree of differentiation of their products compared to rivals'. Hence, differentiation is related to other aspects of firms' strategies.

Choice of products: firms choose how to differentiate from rivals, this impacts the type of products that they choose to offer and the diversity of products that consumers face.

Entry: is there enough room for differentiation so that a firm could enter profitably

Advertising: advertising is a powerful marketing strategy to create differentiation in the consumers' perception about products

Today's session

Product differentiation:

- Horizontal differentiation
- Vertical differentiation and natural oligopolies
- Differentiated products oligopoly: general results
- Nature of competition: prices vs quantities
- Market definition
- Empirical strategy to analyze diffferentiation
- Monopolistic competition and the Chamberlin model

Advertising:

- Imperfect competition with "reach" advertising
- Dispersion of consumers and demand rotation
- Advertising: the content matters

Views on product differentiation

Differentiation can be incorporated in representative consumer's preferences that rely on consumption of the various products

- Preferences usually on existing products
- Problematic when considering entry by a new product

Or as a consequence of consumers' heterogeneity

- Heterogeneity in valuations of consumers with respect to the set of characteristics of products: characteristics approach (Lancaster, 1971)
- Often discrete approach: consumers buy 1/0 units, focus on one characteristic, others treated as random variables

Views on product differentiation

Representative consumer approach used to compare price vs quantity competition

Discrete choice models widely used: easily tractable for theoretical analysis, nice interpretation, used in empirical work.

Horizontal vs vertical differentiation models:

- Horizontal: a product always preferred by some consumers
- Vertical: consumers agree which one they prefer (quality)
- But for which prices ? What if very different costs of production: production side also matters (think about quality)
- Vertical differentiation: when all consumers prefer one product to the other when both priced at marginal cost; otherwise, some horizontal differentiation

Horizontal Differentiation: Hotelling model

Each version of the good attached to an "address" in space of possible configurations. Each consumer's utility depends on quantity and "distance" between ideal version and purchased version

Initially, a spatial competition (discrete choice model); where to buy? Disutility interpreted as transportation costs.

Hotelling's linear city:

- Buyers uniformly distributed on $[0,1]$. They have unit demand, with gross utility v (assumed large) for the product
- Firms 1 and 2, located at extremities, produce same good at unit cost c.
- Consumers incur quadratic transportation cost: buyer at x,

$$\max\{v - p_1 - tx^2; v - p_2 - t(1-x)^2\}$$

Hotelling model

Marginal or indifferent buyer $\tilde{x}$:

$$v - p_1 - t\tilde{x}^2 = v - p_2 - t(1 - \tilde{x})^2.$$

Demands faced by the firms:

$$D_1(p_1, p_2) = \tilde{x} = \frac{1}{2} - \frac{p_1 - p_2}{2t},$$
$$D_2(p_1, p_2) = 1 - \tilde{x} = \frac{1}{2} - \frac{p_2 - p_1}{2t}.$$

Profit functions:

$$\pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j).$$

Hotelling model

FOC conditions yields increasing best replies:

$$P_i(p_j) = \frac{p_j + t + c}{2}$$

Price equilibrium is unique and symmetric: $p^* = c + t$ and equilibrium profits are equal to $t/2$.

When t increases, products are increasingly differentiated (for consumers). Firms compete less fiercely for the same clients, their neighboring consumers become somehow captive and market power increases

But the model reduces to standard Bertrand when:

- $t = 0$, i.e. no differentiation
- If firms were located at same address (minimal differentiation), instead of at extremities (maximal differentiation)

Hotelling with endogenous locations

Assume firm 1 located at a, and firm 2 at $1-b$ with $0 \leq a \leq 1-b \leq 1$. Now, consumer at x:

$$\max\{v - p_1 - t(x-a)^2; v - p_2 - t(x+b-1)^2\}$$

Demands are given by:

$$D_1(p_1, p_2; a, b) = a + \frac{1-a-b}{2} - \frac{p_1 - p_2}{2t(1-a-b)} = 1 - D_2(p_1, p_2; a, b)$$

Characterization of equilibrium prices for given locations:

$$\begin{aligned} p_1^*(a,b) &= c + t(1-a-b)\left(1 + \frac{a-b}{3}\right) \\ p_2^*(a,b) &= c + t(1-a-b)\left(1 + \frac{b-a}{3}\right) \end{aligned}$$

Hotelling with endogenous locations

Consider a two-stage game to endogenize the choice of location, i.e. the choice of firms about how to differentiate from rivals.

- Firms choose the characteristic of their products/locations;
- Firms choose their prices.

At stage 1, problem of firm 1:

$$\begin{aligned}\max_a \Pi_1(a,b) &\equiv \pi_1\left(p_1^*(a,b), p_2^*(a,b); a, b\right) \\ &= \left(p_1^*(a,b) - c\right) D_1\left(p_1^*(a,b), p_2^*(a,b); a, b\right)\end{aligned}$$

Hotelling with endogenous locations

At interior equilibrium (in locations) for firm 1:

$$\frac{d\Pi_1}{da}(a,b) = \underbrace{\frac{\partial \pi_1}{\partial a}}_{\text{Direct effect}} + \underbrace{\frac{\partial \pi_1}{\partial p_1}}_{=0 \text{ (Enveloppe Thm)}} \frac{\partial p_1^*}{\partial a} + \underbrace{\frac{\partial \pi_1}{\partial p_2}\frac{\partial p_2^*}{\partial a}}_{\text{Strategic effect}} = 0.$$

Direct effect:

$$\frac{\partial \pi_1}{\partial a} = (p_1^* - c)\frac{\partial D_1}{\partial a} > 0$$

Given p_2, firm 1 wants to get closer (lower differentiation) to the center to increase its own demand if $a < 1/2$.

Strategic effect:

$$\frac{\partial \pi_1}{\partial p_2}\frac{\partial p_2^*}{\partial a} < 0$$

Moving away from the center (higher differentiation) allows to soften the competition in prices: firm 2 is less aggressive

Hotelling with endogenous locations

The maximum differentiation principle

In the Hotelling model described above: $\frac{d\Pi_1}{da} < 0$, so that firms want to differentiate as much as possible! The strategic effect dominates the demand effect.

- Critical: uniform distribution, v large (fully covered market)
- With linear transportation costs, non existence of p.s. price equilibrium is many subgames...
- If prices are fixed, equal to $\overline{p}$, say because of regulation, a minimum differentiation principle prevails (Hotelling's original point !), as only demand effect exists
- Socially optimal differentiation minimizes total transportation costs, hence $x_1^0 = \frac{1}{4} = 1 - x_2^0$.

Horizontal differentiation and entry

Force towards excessive differentiation compared to what's socially optimal. But when firms differentiate, they create an opportunity for entry in between ! Is there excessive entry, i.e. too many products in equilibrium ?

Hotelling's linear city not well fitted to analyze this question because of boundary effects. Salop (1979) proposes a version without boundaries to address the question of excessive entry

- A circular space: unit-length circle
- Consumers are uniformly distributed
- N firms are symmetrically located

Horizontal differentiation and entry

Look for a symmetric price equilibrium p

- By deviating at p_i slightly below p, firm i gets market share $2S$ given by: $p_i + tS^2 = p + t(\frac{1}{N} - S)^2$ and profits:

$$\frac{N}{t}(p_i - c)(\frac{t}{N^2} + p - p_i)$$

- NC for p to be a symmetric equilibrium: $p = c + \frac{t}{N^2}$ with equilibrium profits $\frac{t}{N^3}$

Free entry with setup cost f leads to an equilibrium number of firms $N* = (\frac{t}{f})^{1/3}$ and $p = c + (tf^2)^{\frac{1}{3}}$

Horizontal differentiation and entry

Price above marginal cost, yet no profits: market power is about pricing above marginal cost, NOT above average cost !

- As f decreases, more firms with smaller price; in the limit market becomes approximately competitive and consumers purchase a product close to their preferred one
- As t increases, more firms with higher price (increase possibility of differentiation)

There is too much entry compared to social optimum

- Price doesn't matter, all consumers are served one unit
- Social cost = duplication of entry costs + total transportation costs paid by consumers
- Socially optimal diversity: $N^0 = (\frac{t}{6f})^{1/3} < N^*$
- Business - stealing effect

Model of vertical differentiation

A model of competition with different qualities.

Two firms, $1, 2$, with qualities s_1 and s_2, producing each at unit cost c; let $\Delta s = s_2 - s_1 > 0$.

Mass 1 of buyers:

- Heterogeneity indexed by θ uniformly distributed on $[\underline{\theta}, \overline{\theta}]$, with $\overline{\theta} = \underline{\theta} + 1$
- Unit demand, valuation for quality given by: $u(\theta, s) = \theta s$.
- Assume that the market is covered at equilibrium.

The marginal buyer:

$$\tilde{\theta} s_2 - p_2 = \tilde{\theta} s_1 - p_1 \Leftrightarrow \tilde{\theta} = \frac{p_2 - p_1}{\Delta s}.$$

Model of vertical differentiation

Demand faced by firm 1 and 2, assuming $\underline{\theta} \leq \tilde{\theta} \leq \overline{\theta}$

$$D_1(p_1, p_2) = Prob(\theta \leq \tilde{\theta}) = \tilde{\theta} - \underline{\theta} = \frac{p_2 - p_1}{\Delta s} - \underline{\theta},$$
$$D_2(p_1, p_2) = Prob(\theta \geq \tilde{\theta}) = \overline{\theta} - \tilde{\theta} = \overline{\theta} - \frac{p_2 - p_1}{\Delta s}.$$

Price equilibrium, assuming that $\overline{\theta} > 2\underline{\theta}$ and $c + \frac{\overline{\theta} - 2\underline{\theta}}{3}\Delta s \leq \underline{\theta} s_1$:

$$p_2^* = c + \frac{2\overline{\theta} - \underline{\theta}}{3}\Delta s > p_1^* = c + \frac{\overline{\theta} - 2\underline{\theta}}{3}\Delta s.$$

Profits:

$$\Pi_2(s_1, s_2) = (2\overline{\theta} - \underline{\theta})^2 \Delta s / 9 > \Pi_1(s_1, s_2) = (\overline{\theta} - 2\underline{\theta})^2 \Delta s / 9$$

Model of vertical differentiation

The high-quality firm charges a higher price than the low-quality firm and makes higher profits

Both profits increase in the difference of quality, i.e. in the extent of vertical differentiation; they vanish for homogenous product, when Δs goes to 0

Endogenizing the choice of quality ?

- Say, at first stage, simultaneous choice of $s_i \in [\underline{s}, \overline{s}]$
- 2 asymmetric ps equilibria with maximal differentiation

$\overline{\theta} > 2\underline{\theta}$ means that there is high degree of heterogeneity among consumers: it guarantees that $p_1^* > c$.

For low heterogeneity among consumers, ie $\overline{\theta} < 2\underline{\theta} \Leftrightarrow \underline{\theta} > 1$, the equilibrium is $p_1^* = c$ and $\pi_1^* = 0$ while $\pi_2^* > 0$. Even though entry is costless, only one firm makes a strictly positive profit at equilibrium

Intuition: When low-quality is indeed low compared to high quality, they do not really compete, while if both quality levels are close, this triggers intense price competition that swamps the increase in demand associated to the increase in quality

Note that if there were an infinitesimal cost of entry, a entry-the-price game would yield an asymmetric equilibrium with only one firm entering and then behaving as a unrestricted monopoly !

We could easily extend the result with N different qualities (and zero cost): if $\overline{\theta} < 2^{n-1}\underline{\theta}$, not all firms can capture a positive market share: finiteness property

Vertical Differentiation and natural oligopoly

This result can be generalized (Shaked-Sutton, 1983):

- $c(s)$ unit cost of producing quality s
- Assume: if all products were priced at their respective marginal costs, all consumers would buy the highest quality product
- Finiteness Property / Natural oligopoly : then, there can be at most a finite number of firms with positive market share.
- Price competition among high quality firms drives prices down to a level at which there is no room for low-quality products.

Contrast with the horizontal differentiation case with no entry cost: there is unbounded entry !

Result may be more relevant when quality improvements come from R&D fixed costs rather then from higher quality material and more skilled labor

Differentiated products oligopoly

Basic assumption: goods / products are differentiated. Each good is unique (although it is substitutable with others at least on the aggregate level) and:

one product = one producing firm (almost always) = one price

- N normal and differentiated products / firms
- $D_i(\mathbf{p}) = D_i(p_1, ..., p_N)$ demand for product i, assumed smooth decreasing in p_i.
- Assume Jacobian is neg. def.. Demand system is invertible and inverse demands: $p_i = P_i(\mathbf{q})$, smooth decreasing in q_i.
- **Game form**: firms simultaneously choose their prices p_i with profits:

$$p_i D_i(p_i; p_{-i}) - C_i(D_i(p_i; p_{-i}))$$

Properties of demand system:

- The system can be deduced from a representative consumer's maximization problem: $\max_{\mathbf{q}}\{U(\mathbf{q}) - \mathbf{pq}\}$. Assume $U(.)$ smooth strictly concave: $P_i(\mathbf{q}) = \partial_i U(\mathbf{q})$ for $\mathbf{q} > 0$
- Symmetry of cross effects: $\frac{\partial P_i}{\partial q_j} = \frac{\partial P_j}{\partial q_i}$ for all $j \neq i$, and downward-sloping, $\frac{\partial P_i}{\partial q_i} < 0$.
- **Substitutes (resp. Complements)**: $\frac{\partial^2 U}{\partial q_i \partial q_j} = \frac{\partial P_i}{\partial q_j} \leq 0$, for $j \neq i$ (resp. $\frac{\partial^2 U}{\partial q_i \partial q_j} = \frac{\partial P_i}{\partial q_j} \geq 0$
- **Gross substitutes**: $\frac{\partial D_i}{\partial p_j} \geq 0$ for all $j \neq i$, implies that they are substitutes (reverse does not hold)
- If the goods are complements, they are necessarily **gross complements**: $\frac{\partial D_i}{\partial p_j} \leq 0$ for all $j \neq i$

Differentiated products oligopoly

Quadratic utility / linear demands

$$U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}\left(\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2\right)$$

$\gamma > 0$ (resp. < 0) for substitutes (resp. complements). Duopoly:

$$p_i = \alpha_i - \beta_i q_i - \gamma q_j \quad \text{or} \quad q_i = a_i - b_i p_i + c p_j$$

whenever positive. Demand for i has a kink at critical price at which it becomes monopoly:

$$D_i(p_1, p_j) = \max\left\{0, \min\{a_i - b_i p_i + c p_j; \frac{\alpha_i - p_i}{\beta_i}\}\right\}$$

Symmetric oligopoly: with $\beta > \gamma > 0$ and $\alpha > 0$,

$$P_i(\mathbf{q}) = \alpha - \beta q_i - \gamma \sum_{j \neq i} q_j$$

Differentiated products oligopoly

Other examples

- Constant elasticity $\sigma = \frac{1}{1-\beta}$, with $0 < \theta < 1$ and $0 < \beta \leq 1$:

$$U(\mathbf{q}) = [\sum_i q_i^\beta]^\theta \quad \text{and} \quad P_i(\mathbf{q}) = \beta\theta[\sum_j q_j^\beta]^{\theta-1} q_i^{\beta-1}$$

- Logit demands

$$D_i(\mathbf{p}) = \frac{k_i \exp\{\lambda p_i\}}{\sum_j k_j exp\{\lambda p_j\}} \quad \text{with } \lambda < 0 \text{ and } k_j > 0$$

- Constant expenditure

$$D_i(\mathbf{p}) = \frac{1}{p_i} \frac{g(p_i)}{\sum_j g(p_j)} \quad \text{with } g(.) \text{ positive decreasing e.g.}$$

$g(p) = p^r$ with $r < 0$ or $g(p) = e^{-\beta p}$ with $\beta > 0$

Existence of an equilibrium in pure strategies in prices

- With constant marginal costs c_i, existence if $1/D_i$ convex in p_i (weaker than D_i concave in p_i)
- With convex costs and D_i log-concave in p_i
- Natural existence results using supermodularity, e.g. if goods are gross substitutes, cost are convex and demand shows increasing differences $\frac{\partial^2 D_i}{\partial p_i \partial p_j} \geq 0$ (linear demand), or with constant marginal costs and the log of demand showing increasing differences (CE, logit or constant expenditure cases)
- But price game need not be supermodular in general (Hotelling)!

Uniqueness:

Sufficient conditions (for contraction mapping condition)

$$
\begin{aligned}
1 - \frac{\partial D_i}{\partial p_i} C_i'' &> 0 \\
\sum_j \frac{\partial D_i}{\partial p_j} &< 0 \\
\frac{\partial^2 D_i}{(\partial p_i)^2} + \sum_{j \neq i} \left| \frac{\partial^2 D_i}{(\partial p_i \partial p_j)} \right| &< 0
\end{aligned}
$$

Own price effects dominate the cross price effects

Differentiated products price competition

Characterization of an equilibrium:

$$\frac{p_i - C_i'}{p_i} = \frac{1}{\eta_i}$$

where the own-price elasticity $\eta_i = -\frac{p_i}{q_i}\frac{\partial D_i}{\partial p_i}$ depends upon p_j

Allocative inefficiency: price above marginal cost

Productive efficiency: here almost tautological since one good = one firm !

Quasi-monopoly on interdependent markets with inefficiency: as substitutability $\to \infty$, price $\to$ marginal cost

Existence of a Cournot equilibrium in differentiated products:

- If $P_i(\mathbf{q})$ is log-concave and downward sloping in q_i (when positive), and $\frac{\partial P_i}{\partial q_i} - C_i'' < 0$
- For duopoly, using supermodularity if the revenue $R_i(q_i, q_j)$ displays decreasing differences (cross derivative non-positive), i.e. goods are strategic substitutes (decreasing best reply)

Note: Uniqueness is particularly difficult to ensure with close substitutes (contraction condition unnatural). A sufficient condition when $P_i(q_i, \sum_{j \neq i} q_j)$ (!) is that the slope of the pseudo-best reply is within $(-1, 0)$.

Differentiated products quantity competition

Characterization of Cournot equilibrium in differentiated products:

$$p_i + q_i \frac{\partial P_i}{\partial q_i} - C_i' = 0 \Leftrightarrow \frac{p_i - C_i'}{p_i} = \varepsilon_i$$

with $\varepsilon_i \equiv -\frac{q_i}{p_i}\frac{\partial P_i}{\partial q_i}$ stands for the elasticity of inverse demand.

With an homogeneous good, the elasticity of inverse demand equals the inverse of the elasticity of demand.

But with differentiated products, if products are gross substitutes or if they are complements, $\varepsilon_i \geq \frac{1}{\eta_i}$.

Price vs quantity competition

Under perfect competition or monopoly, the equilibrium depends only on the fundamentals of the economy (supply and available technologies, demand and the buyers' preferences)

Under oligopolistic competition, the equilibrium depends *also* on the assumptions made on the anticipations of each firm with respect to the reaction of its rivals following a modification of its behavior.

This highlights the need to study carefully the industry under consideration.

Price vs quantity competition

Consequence of $\varepsilon_i \geq \frac{1}{\eta_i}$:

- $\frac{\partial \pi_i}{\partial p_i}(p^{Cournot}) \leq 0$: firms have an incentive to cut price at the Cournot equilibrium
- $\frac{\partial \pi_i}{\partial q_i}(q^{Bertrand}) \leq 0$: firms have an incentive to reduce output at the Bertrand equilibrium

Intuition: in Cournot competition, i expects j to cut price in response to a decrease in p_i, corresponding to an increase in q_i, in order to maintain q_j as fixed; while in Bertrand competition, it expects j to maintain its price.

Cournot competition penalizes price cuts, hence leads to less competitive outcome.

Price vs quantity competition

With linear demands / inverse demands, it is rather immediate to find that the equilibrium outcome under quantity competition is less competitive (higher price) than the equilibrium outcome under price competition

In general, it is more complicated but the general idea is robust:

- If the price-game is supermodular and quasi-concave (e.g. gross substitutes, increasing convex costs, demand log-concave and log-demand has increasing differences), any interior equilibrium in the quantity-game is characterized by a price strictly larger than the smallest Bertrand equilibrium price
- If gross substitutes with symmetric market and contraction property holds, the unique Bertrand equilibrium displays lower prices and profits, and higher total surplus than any symmetric quantity-game equilibrium.

Price vs quantity competition

- Cournot in homogenous goods: with decreasing marginal revenue and convext costs, best replies are decreasing
- In Hotelling or in canonical vertical differentiation model, best replies are increasing

In general, the slope of best replies is related to the cross derivative of profit for firm i w.r.t. its own decision variable and a rival's decision variable (whether profit exhibits increasing / decreasing differences in own decision and cross-decision)

Common wisdom: the quantity-game exhibits strategic substitutability (decreasing best replies) while the price-game exhibits strategic complementarity (increasing best replies)

Price vs quantity competition

Bertrand competition w/ differentiated products:

$$\pi_i(p_i, p_j) = p_i D_i(p_i, p_j) - C(D_i(p_i, p_j))$$
$$\Rightarrow \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = \left[1 - \frac{\partial D_i}{\partial p_i} C_i''\right] \frac{\partial D_i}{\partial p_j} + (p_i - C_i') \frac{\partial^2 D_i}{\partial p_i \partial p_j}.$$

Strategic complementarity if supermodularity; other cases ...

Cournot competition w/ differentiated products:

$$\pi_i(q_i, q_j) = P_i(q_i, q_j) q_i - C_i(q_i) \Rightarrow \frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = \frac{\partial P_i}{\partial q_j} + q_j \frac{\partial^i P_i}{\partial q_i \partial q_j}.$$

Strategic substitutability not natural even with substitute products

Price vs quantity competition

Implications:

- To encourage its rivals to be less aggressive on the market, a firm would like to convince them that it will be less (resp. more) aggressive if choices variables are strategic complements (resp. substitutes).
- The incentives to become more aggressive (investment to reduce cost, increase quality, etc.) depends on the anticipation about the reaction of the rivals.
- Same spirit: the incentives to disclose information about its cost.
- Prediction on mergers: A merger between two firms makes them internalize the negative externalities there create on each other; hence they become less aggressive; positive reaction from the rivals under Bertrand competition, but negative one under Cournot competition.

Market Definition

Market definition is the first step to establish the presence or absence of market power.

The relevant set of products: Not the products which "resemble" each others, but rather the set of products (and geographical areas) that exercise some competitive constraints on each others

The SSNIP (Small but Significant Non-transitory Increase in Prices) or "hypothetical monopoly test":

- Consider a product and suppose that there exists a hypothetical monopolist that is the only seller.
- Would this hypothetical monopolist find it profitable to increase the price of this product above the current level by $5-10\%$?

If yes, the product under scrutiny does not face significant competitive constraints from other products: separate market.

If no, there exist other products that are substitutes (demand substitutability) enough to exercise competitive constraints: so enlarge the set of products that are considered close substitutes and apply the SSNIP test to the broader set.

There could also be supply substitutability, if producers that are currently producing a different product can switch production if a price rise occurs. For supply substitutability to be a competitive constraint, switching production must be easy, rapid and feasible.

However, the “cellophane fallacy” when applying the SSNIP test to non-merger case (cf a monopolist would NOT find an increase in price profitable starting from the monopoly price !): the test would induce to consider too large a market

In the case of a firm alleged of abuse of dominant position, the relevant price level is not the *current* level, but the *competitive* one.

When defining the relevant market, many considerations have to be taken into account: the presence of secondary markets, the change of market over time, the geographic dimension, etc.

Market Definition

Once the relevant market is defined, antitrust agencies measure the market power with market shares (and their persistence over time). In practice: below 40% a firm is unlikely to be considered as dominant, above 50%, dominance can be presumed.

Many other aspects must be taken into account: Ease and likelihood of entry, buyers' power, etc.

Empirical analysis of product differentiation

To get useful restrictions on aggregate demand for differentiated products, start from individual choice.

Most useful approach corresponds to probabilistic choice theory

- Consumers are ex ante homogeneous but random psychological stimuli shocks affect their choice: truly random choice
- Consumers look ex ante homogeneous to the observer but there is unobserved heterogeneity that leads to different choice: lack of information

Approach has proved extremely appropriate in applied studies in many industries, most notably the car industry (Nevo, 2000, for a survey).

Empirical analysis of product differentiation

- $i = 1, 2, ...n$ products and an outside option $i = 0$ (no participation)
- Random utility for product i: $v_i = \overline{v}_i + \tilde{\varepsilon}_i$, where $\overline{v}_i$ is the observable / measurable utility, common to all consumers in the sub-population ($v_0 = 0$, normalization), and $\tilde{\varepsilon}_i$, with zero-mean, captures the shocks that affect consumers
- Choosing i over j depends on the probability that $v_i > v_j$.
- If $\{\tilde{\varepsilon}_i\}_i$ i.i.d. following double exponential distribution

$$F(\varepsilon) = \exp\left\{ - \exp\{ - \left(\frac{\varepsilon}{\mu} + \gamma \right) \} \right\} \text{ with } \mathbb{E}\varepsilon = 0 \text{ and } \mathbb{V}\varepsilon = \frac{\pi^2}{6}\mu^2$$

- Multinomial logit choice proba: $Q_i = \frac{\exp\{\overline{v}_i/\mu\}}{1+\sum_j \exp\{\overline{v}_j/\mu\}}$

Empirical analysis of product differentiation

- All consumers share same mean utility that depends upon product characteristics x_i and unobserved characteristics ξ_i:

$$\overline{v}_i = \beta.x_i + \xi_i - \gamma p_i$$

- Taking $\mu = 1$ for simplicity, the market share equation:

$$\log \alpha_i - \log \alpha_0 = \beta.x_i + \xi_i - \gamma p_i$$

- On supply side, assume constant marginal cost c_i depends on product characteristics w_i and on unobserved characteristics yielding a mean cost ω_i: $c_i = \kappa w_i + \omega_i$
- Assuming FOC uniquely determine price equilibrium:

$$p_i = \kappa w_i + \frac{1}{\gamma}\frac{1}{1-\alpha_i} + \omega_i$$

- Joint estimation demand side - supply side

Empirical analysis of product differentiation

- Multinomial logit implies a restrictive substitution pattern e.g. in car industry: introducing a new "family van" has same effect on market share of another family van or of a SUV, say !
- Group different products together in a single nest (e.g. family van) and use nested logit model, in which consumers first choose among nests, and then within nests

$$\log \alpha_i - \log \alpha_0 = \beta . x_i + \xi_i - \gamma p_i + \sigma \log \overline{\alpha}_{i|g}$$

- Endogenous extra term depends on substitution parameter σ and market share of product i within group g

$$\overline{\alpha}_{i|g} = \exp \left[\frac{(\beta x_i + \xi_i - \gamma p_i)/(1-\sigma)}{\sum_{j \in g} \exp[(\beta x_j + \xi_j - \gamma p_j)/(1-\sigma)]} \right]$$

- Different substitution patterns within and across nests
- Pricing has also to be adapted to the nested structure

Monopolistic competition

Canonical horizontal and vertical differentiation models involve a small number of firms with competing "neighbors". Instead, we focus on market structure exhibiting monopolistic competition :

- A large number of firms, each producing a differentiated good
- Firms do not interact strategically, but indirectly through the aggregate demand effect
- FIrms make no profit
- Each firm faces a downward-sloping demand curve, hence enjoys market power

The model (Dixit-Stiglitz, 1977) does not assume heterogeneity among consumers but rather a representative consumer with a taste for variety

Monopolistic competition

Consumer's utility: $U = q_0^{1-\gamma} Q^\gamma$ with q_0 numeraire and Q is a composite differentiated good defined as the aggregation of n varieties:

$$Q = \left(\sum_{i=1}^{n} q_i^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}}$$

σ is the elasticity of substitution across any 2 varieties.

With revenue y and given a price index P for the composite good, two step utility maximization problem:

1. Maximize utility w.r.t. q_0 and Q: constant budget share for the composite good $PQ = \gamma y$
2. Miximize Q w.r.t. all q_i given prices p_i and budget:

$$\sum_i p_i q_i \leq \gamma y$$

Demand function for variety i: $q_i = \left(P^{\sigma-1}\gamma y\right) p_i^{-\sigma}$

Demand for variety i depends upon the prices of all varieties, through the price index P. An increase in p_i compared to other prices implies a reduction of the demand for variey i, but still a positive demand (taste for variety)

Computing Q from individual demands and using $PQ = \gamma y$, one gets:

$$P = \left(\sum_j p_j^{-(\sigma-1)} \right)^{-1/(\sigma-1)}$$

From this, the demand elasticity for variety i is: $\eta_i = \sigma$

Monopolistic competition

In equilibrium, each firm uses the Lerner pricing formula:

$$\frac{p_i - c}{p_i} = \frac{1}{\sigma} \Leftrightarrow p_i = \frac{\sigma}{\sigma - 1}c$$

Equilibrium profits: $\pi_i(n) = \frac{\gamma y}{n\sigma} - e$ with e entry cost.

Free entry equilibrium would then result in: $n^* = \frac{\gamma y}{\sigma e}$. Immediate comparative statics.

Here, insufficient entry compared social optimal entry (entry controlled, not pricing), but in general it is ambiguous as there are 2 effects:

- Imperfect appropriate of the created surplus
- business-stealing effects

References

- *Belleflamme-Peitz, Ch.5
- D'Aspremont, C., J. Gabszewicz and J. Thisse (1979), EMA, 17, 1145-1151
- Dixit, A. and J. Stiglitz (1979), AER, 67, 297-308
- Nevo, A. (2000), JEMS, 9, 513-548
- Salop, S. (1979), Bell J. of Econ., 10, 141-156
- Shaked, A. and J. Sutton (1983), EMA, 51, 1469-1484
- Tirole, Ch.7
- Vives, Ch.6