# Markets with Asymetric Information

## Microeconomics 2

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The economy is fundamentally uncertain / risky, not all information is available to the agents when they make their decisions

Moreover, agents receive information about the state of nature that may differ from one agent to the other: asymmetric information

- Firms have better information about their own costs than their rivals or than consumers
- Citizens have better information about their willingness to pay for a public good than the central planner

# I.1. Introduction

Prominent importance of asymmetric information framework in modern economic theory (IO, Macro, Labor econ., Finance, Public econ., ...) as well as in related social sciences (e.g. Political sciences)

- **Labor market and unemployment**: workers know what they are capable of doing, their talent,... employers do not
- **Credit market and credit rationing**: entrepreneurs know how risky their projects are, the investors do not
- **Product markets and barriers to entry**: firms have better information on their costs than their rivals

# I.2. Road map for today

Up to now, we saw that:

- Classical market failure easy to fix under perfect information, but much harder when planner has imperfect information, or more precisely when information is asymmetrically distributed between economic agents and the central planner
- Imperfect (but symmetric) information *per se* does not affect the efficiency properties of perfectly competitive equilibria provided financial / asset markets are rich enough (complete markets)

**Today's main message**: Asymmetric information in itself is a source of inefficiency, it is the fundamental maket failure.

# I.2. Road map for today

**Roadmap:**

- (RE) Equilibrium under asymmetric information may fail to be efficient: Akerlof's (simplified) example

- Solving the inefficiency problem by transmitting information to the markets before they operate ?
  - Hard information transmission (Milgrom): when non-manipulable proofs are available
  - Signaling (Spence): when costly actions can be taken (see Game Theory course)
  - Communication: (Crawford - Sobel): when costless messages can be exchanged (If time permits....)

# II. Akerlof's market for lemons

Example of a market under asymmetric information

- $N$ sellers, selling one unit of the good each
- Units of good can be of quality $q \in \{L, H\}$
- There are $N_H = \lambda N$ sellers of H-quality and $N_L = (1-\lambda)N$ sellers of L-quality
- A seller of quality $q$ gets utility

$$\begin{array}{ll} r_q & \text{if she keeps her good} \\ m & \text{if she sells her good at price } m \end{array}$$

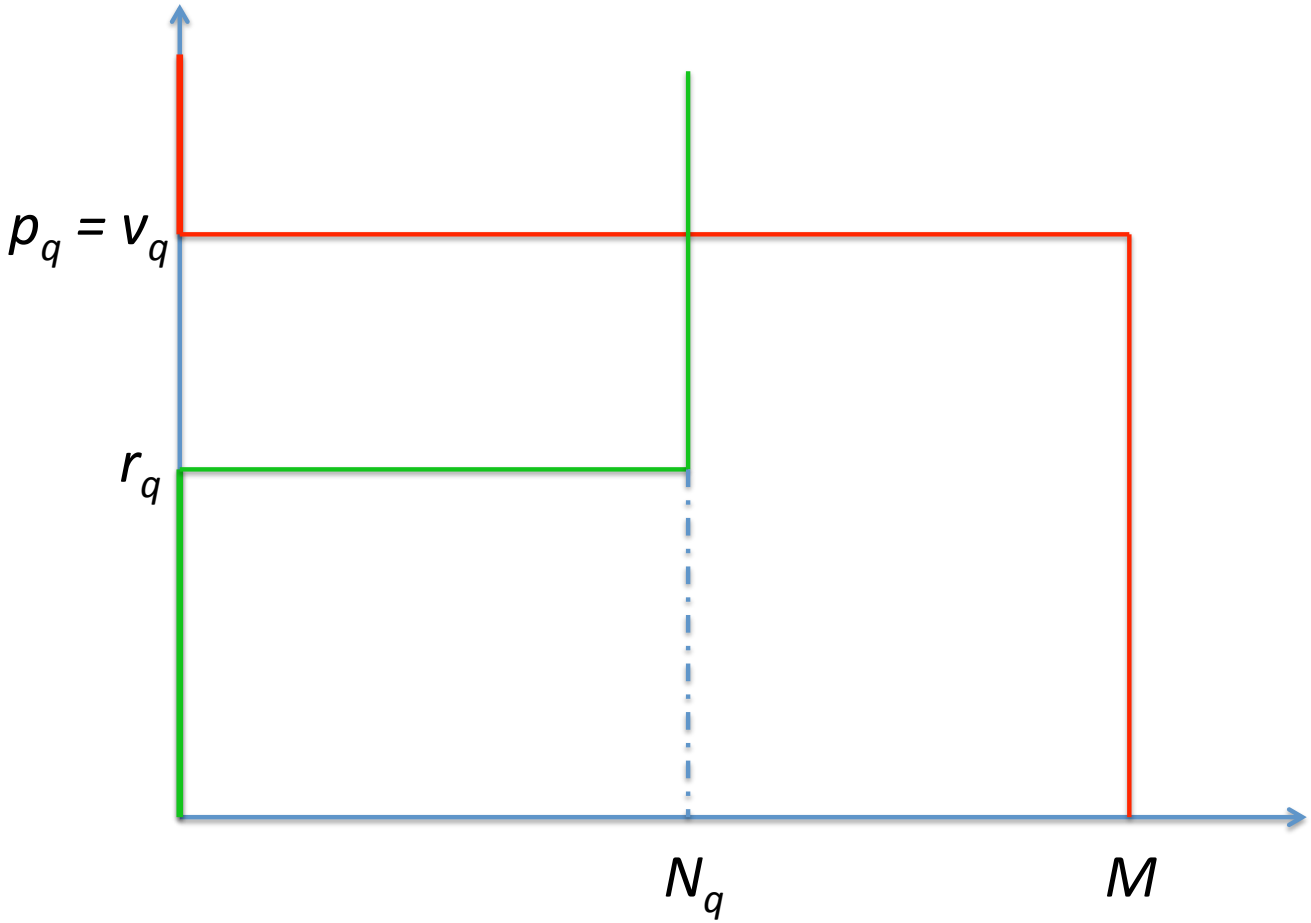
- $M \gg N$ buyers, interested in buying one unit each
- A buyer buying a good of quality $q$ at price $m$ gets utility $v_q - m$ and he gets 0 if he does not acquire the good.

# II. Akerlof's market for lemons

| quality ↓ | % | buyer's value | seller's value |
|---|---|---|---|
| High | $\lambda$ | $v_H = 50$ | $r_H = 40$ |
| Low | $1 - \lambda$ | $v_L = 30$ | $r_L = 25$ |

- **Efficient allocation**: all goods of all qualities transferred from sellers to buyers
- **Perfect information**: $q$ observable, 2 markets prices $p_q$
- Competitive prices: $(p_L = v_L, p_H = v_H)$ since $v_q > r_q$; as expected, it is efficient
- **Imperfect but symmetric information**: $q$ unobservable (by sellers and by buyers): one market with price $p$
- Competitive price: $p = v^e = \lambda v_H + (1 - \lambda) v_L$ since $p = v^e \geq r^e = \lambda r_H + (1 - \lambda) r_L$; equilibrium is efficient

# II. Akerlof's market for lemons



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- **Efficient allocation**: only H-quality should be traded
- **Imperfect symmetric information**: $p = v^e$ is a competitive equilibrium iff $v^e \geq r^e$, and then all goods are traded: ex-post inefficient equilibrium ...!
- But, it is constrained efficient: i.e. optimal given the information publicly available in the economy
- Nobody knows $q$; as if good is of homogenous (average) quality, with buyers' value $v^e$ and sellers' value $r^e$.

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**Under asymmetric information**: only sellers know the quality of their good $q$.

Sellers' competitive supply curve (they're under full information):

- 0, if $p < r_L$
- $N_L = (1 - \lambda)N$, i.e. L-quality goods, if $r_L \leq p < r_H$
- $N$, i.e. all goods, if $r_H \leq p$.

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REE: buyers should extract information from the price, assuming sellers are rational, to formulate their demand

Buyers willing to buy if $\mathbb{E}[v_q \mid p] \geq p$. Hence their demand:

- For $r_L \leq p < r_H$, $\mathbb{E}[v_q \mid p] = v_L$, $D(p) = M$ if $p \leq v_L$
- For $r_H \leq p$, $\mathbb{E}[v_q \mid p] = v^e$, $D(p) = M$ if $p \leq v^e$
- For $p < r_L$, no supply; assume e.g. beliefs $\mathbb{E}[v_q \mid p] = v^e$

Demand is not monotonic (if $v^e > r_H$): we can expect problems!

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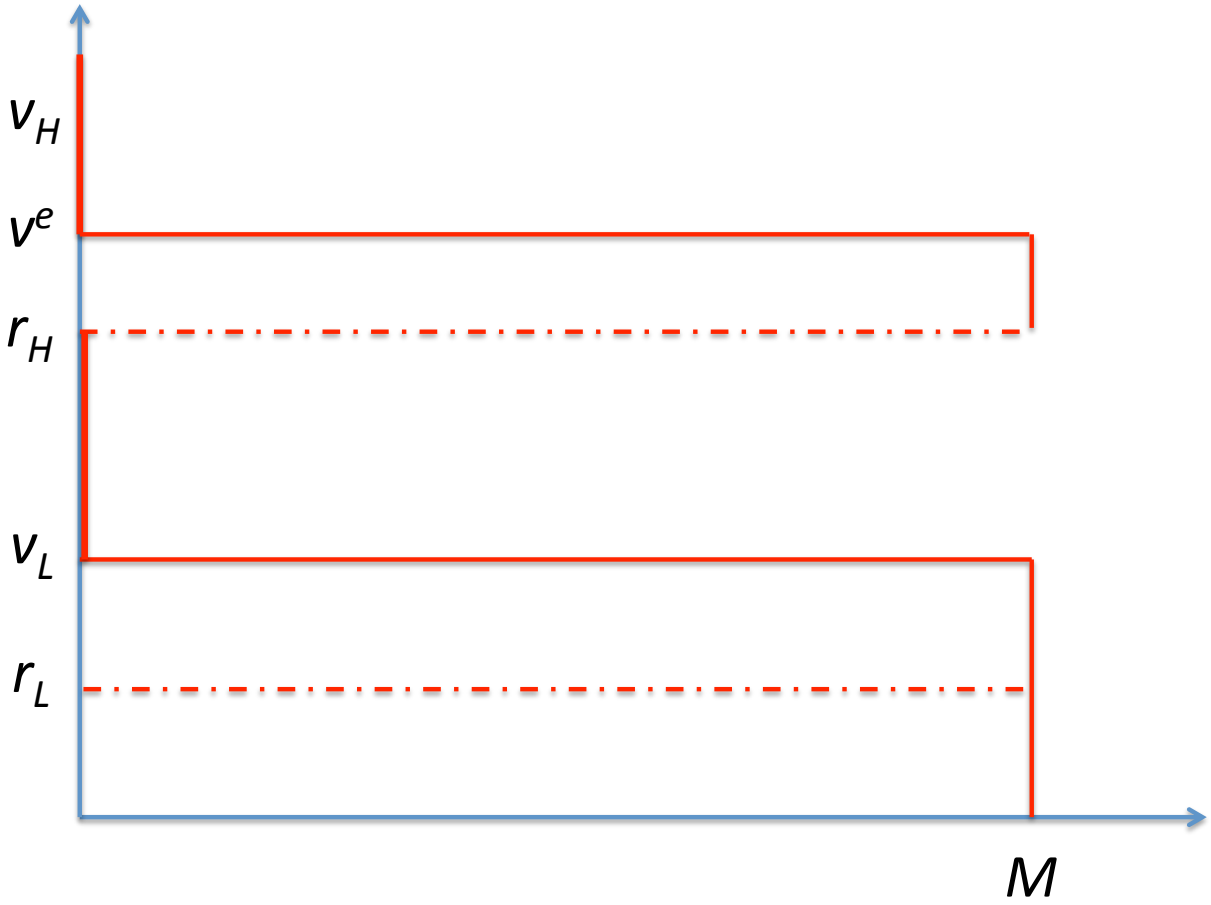

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Suppose $\lambda \geq 1/2$: there are many H-quality sellers.

- Price $p = v^e \geq r_H$ clears the market: all goods traded
- This RE Equilibrium is efficient
- But price $p$ with: $r_L \leq p = v_L \leq r_H$ also clears the market: only L-quality is traded.
- This L-quality RE Equilibrium is inefficient !
- Two competitive equilibria that are Pareto ranked.

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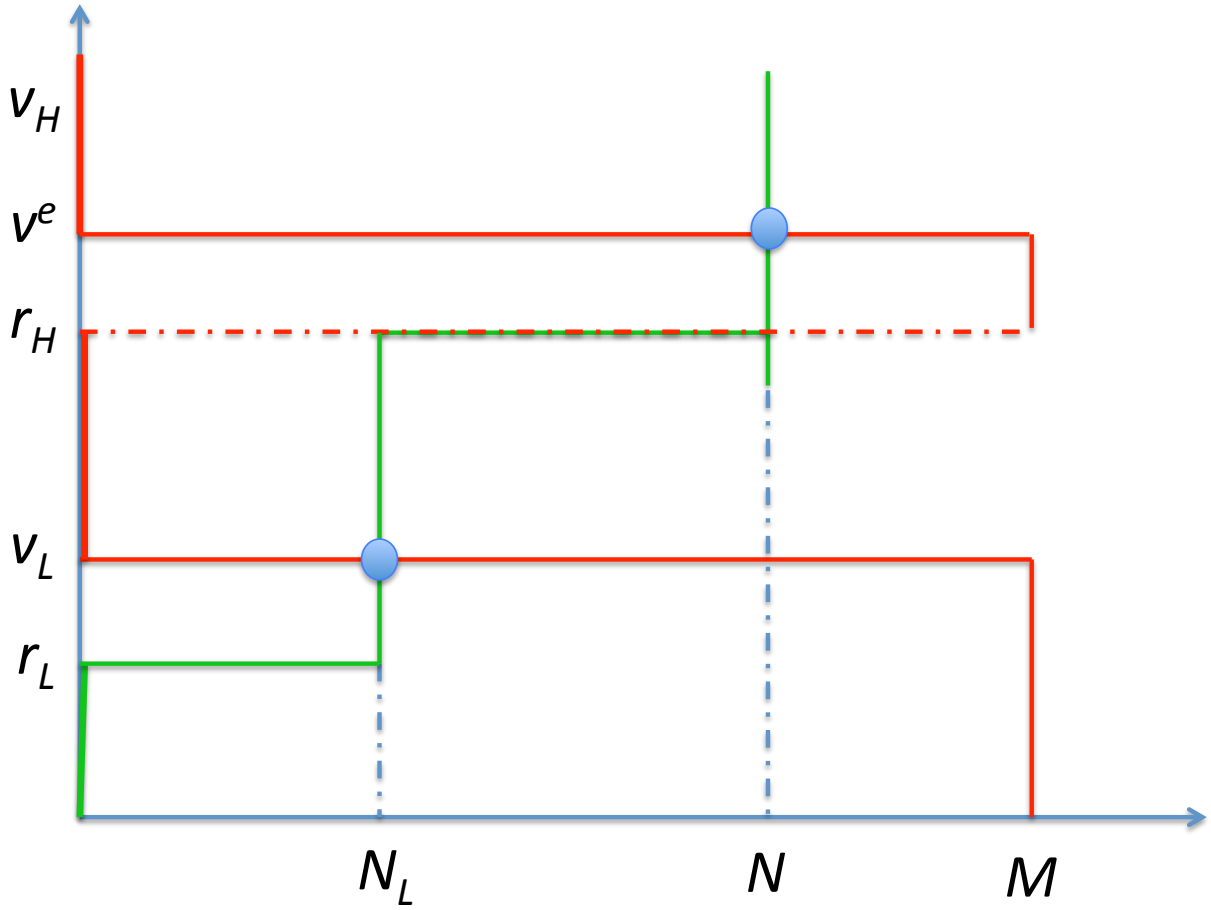

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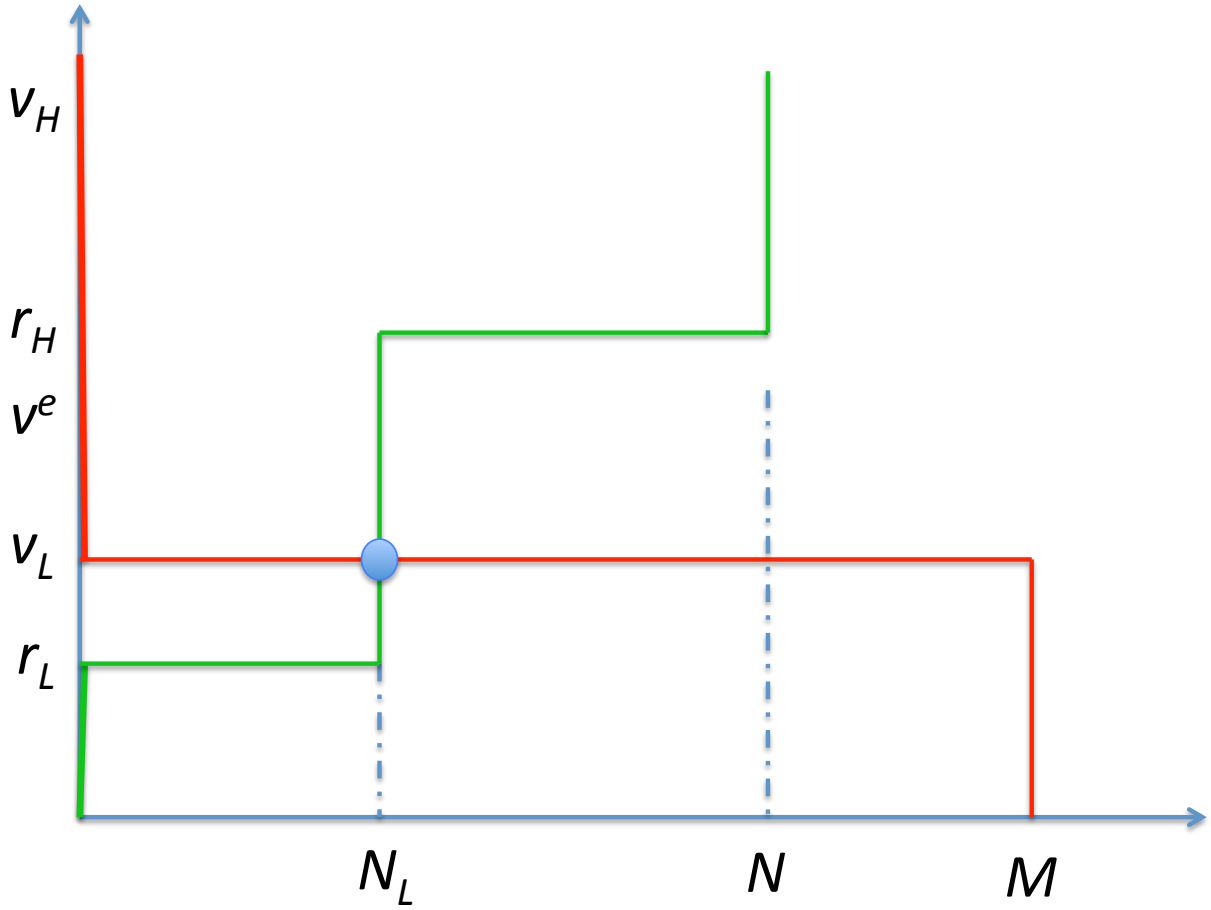
Suppose now $\lambda < 1/2$: there are few H-quality sellers.

- There cannot exist an equilibrium with all goods traded since $r_H = 40 > v^e$: no price is high enough for H-quality sellers to sell and low enough for buyers to buy the average quality
- There is adverse selection: L-quality goods kick H-quality goods out of the market. At moderate price, only L-quality sellers are all willing to sell, they drive expected quality down, hence also the price, which prevents H-quality sellers to sell...
- $p = v_L$ is now the unique equilibrium and it is inefficient

# II. Akerlof's market for lemons



# II. Akerlof's market for lemons

**Adverse selection in the labor market:**

- Heterogeneity of workers with respect to their talent, related to their productivites; private information
- Higher talent workers also have better alternative occupations (i.e. work as an independent)
- For wage $\omega$ equal to average productivity in population, labor supply only by workers with talent *below* a threshold corresponding to the alternative that is equivalent to $\omega$
- Hence, for $\omega$, the average productivity of workers participating in the labor market smaller than average productivity, there is no demand for labor at $\omega$
- Lowering $\omega$ kicks medium productivity workers out of the market, hence reduces average productivity further ...

# II. Akerlof's market for lemons

**Conclusion**:

- Asymmetric information may lead to competitive (RE) equilibria that are inefficient: failure of the st Welfare Theorem.
- The market may even completely collapse: non-existence of non-degenerate (active trade) equilibrium

The strength of the competitive equilibrium paradigm rests on existence and efficiency results: less appeal under asymmetric information !

Two routes from here:

1. Circumvent the asymmetry of information by having information transmitted to the market (rest of today)
2. Taking asymmetry as given, provide a more appropriate model of elementary transactions under asymmetric information (Principal - Agent models)

In the Akerlof's example, sellers of high-quality goods would like to convince buyers that they sell high quality

Yet, sellers of low-quality would be willing to fool the market and pretend they sell high quality as well !

Hence, information transmission has to be made convincingly, in a credible way:

- By disclosing of hard information (i.e. non-manipulable proof of information)
- By undertaking observable and costly actions that signal information
- By communicating information (costless actions)

# III.1. Transmission of information

Formally, this means introducing a pre-market stage, where informed agents make decisions (disclose proofs, emit signals, communicate)

Two stages: pre-market actions and then (competitive) market mechanism (hence, not strictly-speaking a game).

Information may be revealed:

- directly: hard proof, message
- indirectly, as a (bayesian) strategy maps an agent's information into the set of his possible actions: "inverting" the mapping yields information

Typical questions in such a setting: full / partial / no revelation of information ? Efficiency ?

# III.2. Disclosure of certifiable / hard information

Introduce in Akerlof's example a technology that allows a seller to certify / prove the quality of his good at a cost $K$ before the market opens

- If seller certifies his quality $q$, he obtains price equal to $v_q$
- So high-quality sellers have an incentive to certify provided $K$ is not too large, as otherwise they do not sell ($\lambda$ small) or they sell at price $v^e < v_h$ ($\lambda$ large)
- Low-quality sellers don't
- Immediate equilibrium: high-quality sellers show proofs, hence absence of proof induces beliefs that quality is low
- Full equilibrium disclosure of information to the market

**Disclosure of hard information (Milgrom)**

In models with richer information (more than 2 types), there is full disclosure of hard information if the cost of disclosure $K$ is small enough.

**About efficiency**:

- Certification may improve efficiency when $K$ negligeable and $\lambda < 1/2$ and then the right amount of information is disclosed to the market
- It may be wasteful when there exists an efficient equilibrium without certification, as the cost $K$ is wasted.
- In general, too much disclosure of information: disclosure is decided on the basis of the private value of information for informed agents, not on the basis of the social value of information

# III.3. Signaling in Akerlof's model

Coming back to the example of the labor market: it is probably difficult for a worker to provide a "proof" of his/her analytical skills (as determining productivity in some type of job)

But people do different things: e.g. they invest in education !

Education may of course increase your analytical skills, hence your productivity (APE certainly does...!)

But education is also a way to send information about how "intrinsically" good you are ! Passing micro 2 speaks about your intrinsic analytical skills

It is harder for one who has low analytical skills to achieve a good grade than for one who has high analytical skills: observing a good grade may be a signal of high intrinsic analytical skills.

| quality ↓ | % | buyer's value | seller's value |
|---|---|---|---|
| high | $\lambda$ | $v_H = 50$ | $r_H = 40$ |
| low | $1-\lambda$ | $v_L = 30$ | $r_L = 25$ |

**Specific Akerlof example with a performance test**:
Each seller can run a test and achieve a publicly observable level of performance $g \geq 0$; the test has no impact on the intrinsic quality $q$ but the cost of achieving performance level $g$ varries with quality:

$$\begin{aligned} C(g,q) &= g \quad \text{if } q = L \\ &= \alpha g \quad \text{if } q = H \text{ with } \alpha < \frac{1}{2}. \end{aligned}$$

Goods differentiated by disclosed performance: $p(g)$ and buyers draw inferences when observing a performance level

# III.3. Signaling in Akerlof's model

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An equilibrium is: $(g_L, g_H, p(.), \mu(.))$, where $g_q$ is the level of performance chosen by sellers of quality $q$, $\mu(g)$ are buyers' (common) beliefs about quality when observing performance level $g \in \mathbf{R}_+$ and $p(g)$ is the equilibrium price for goods with label $g$, such that:

- $p(g) = \mu(g)v_H + (1 - \mu(g))v_L$ (competition among buyers given beliefs)
- $q$-type sellers are better off choosing $g_q$ rather than deviating and choosing another $g$
- Buyers' beliefs are consistent with Bayes law, whenever this has some bite

# III.3.a. Separating equilibrium in Akerlof's model

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Separating equilibrium $g_H \neq g_L$, $p(g)$, $\mu(g)$ beliefs after any $g$

- Buyers' beliefs: $\mu(g_H) = 1$, $\mu(g_L) = 0$ and so competition induces $p(g_H) = v_H = 50, p(g_L) = v_L = 30$
- Take worst possible beliefs for all other $g$: $\forall g \neq g_h, \mu(g) = 0$ and $p(g) = v_L = 30$
- $q_L$-sellers: $g_L$ rather than $g_H$, another $g$ or not selling:

$$p(g_L) - g_L \geq \sup\{p(g_H) - g_H, p(g) - g, r_L\}$$

- NC: $g_L = 0$ and $30 \geq 50 - g_H$
- $q_H$-sellers: $g_H$ rather than $g_L$, another $g$ or not selling:

$$p(g_H) - \alpha g_H \geq \sup\{p(g_L) - \alpha g_L, p(g) - \alpha g, r_H\}$$

- NC: $50 - \alpha g_H \geq 40$

# III.3.a. Separating equilibrium in Akerlof's model

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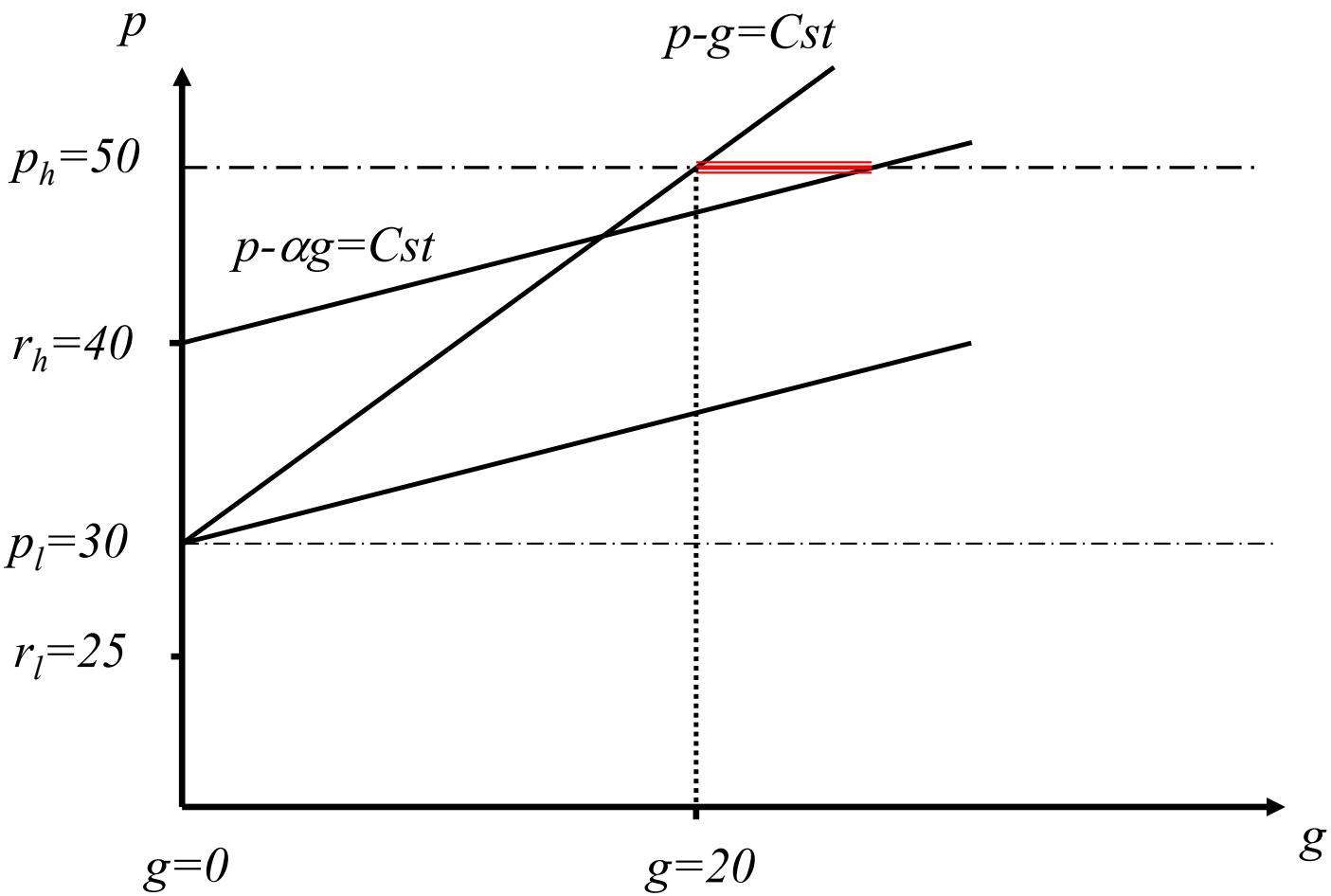
**Continuum of separating equilibria:** Fix any $y$ with $20 \leq y \leq \frac{10}{\alpha}$, then $g_H = y, g_L = 0, \mu(y) = 1, p(y) = 50, \forall g \neq y, \mu(g) = 0, p(g) = 30$ is a separating equilibrium.

More convincing (monotonic) beliefs: $\mu(g) = 0, p(g) = 30$ for $0 \leq g < y$, and $\mu(g) = 1, p(g) = 50$ for $y \leq g$

Least-cost separating equilibrium: $g_H = 20$, minimal level of performance that enables $q_H$-sellers to signal themselves

# III.3.a. Separating equilibrium in Akerlof's model



# III.3.b. Non-separating equilibrium in Akerlof's model

| quality ↓ | % | buyer's value | seller's value |
|---|---|---|---|
| high | $\lambda$ | $v_H = 50$ | $r_H = 40$ |
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Non-revealing equilibrium $g_H = g_L = g^*$, $p(g)$, $\mu(g)$ after any $g$

- Buyers don't get any information: $\mu(g^*) = \lambda$, so competition leads to $p(g^*) = v^e$
- For all other $g \neq g^*$, take worst beliefs $\mu(g) = 0$ and so $p(g) = 30$
- $q_L$-sellers: $g^*$ rather than any other $g$ or not selling:

$$p(g^*) - g^* \geq \sup\{p(g) - g, r_L\}$$

- So, $v^e - g^* \geq 30 > 25 = r_L$
- $q_H$-sellers: $g^*$ rather than any other $g$ or not selling:

$$p(g^*) - \alpha g^* \geq \sup\{p(g) - \alpha g, r_H\}$$

- So, $v^e - \alpha g^* \geq 40$

# III.3.b. Non-separating equilibrium in Akerlof's model

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| high | $\lambda$ | $v_H = 50$ | $r_H = 40$ |
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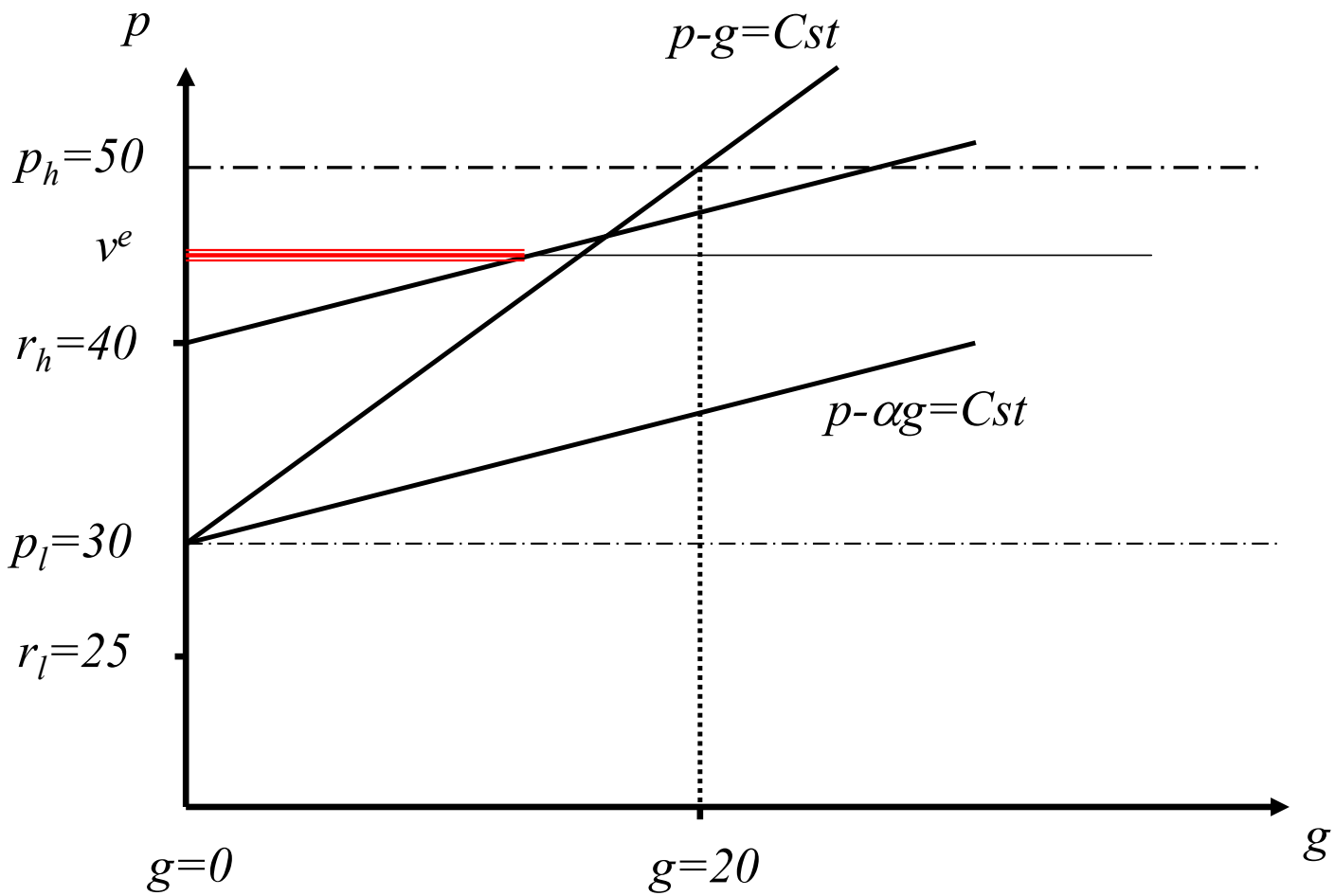
**Continuum of pooling equilibria**: Fix any $z$ such that $0 \leq z \leq \inf\{v^e - 30, \frac{v^e - 40}{\alpha}\}$, then $g_H = g_L = z$, $\mu(z) = \lambda, p(z) = v^e$ and $\forall g \neq z, \mu(g) = 0$ and $p(g) = 30$ is a pooling equilibrium (Remark: As $v^e > r_h = 40$, $\inf\{v^e - 30, \frac{v^e - 40}{\alpha}\} > 0$)

More reasonable beliefs such as: $\mu(g) = 0, p(g) = 30$ for $0 \leq g < z$, and $\mu(g) = \lambda, p(g) = v^e$ for $z \leq g$

No information at all is revealed in equilibrium

# III.3.b. Non-separating equilibrium in Akerlof's model



# III.3.c. Efficiency of signaling in Akerlof's model

Natural idea: in separating signaling equilibrium, decentralized information passed onto the market, which restores efficiency

Not so simple: the use of signals is driven by the private value of the signal for sellers, not by its social value

- If $\lambda < 1/2$:
  - trade is inefficient (only $q_L$ traded) without test technology but efficient under signaling: great !
  - but the cost of test $g_H > 0$ is a pure social waste ...
  - here, overall more efficient with than without test technology
- If $\lambda > 1/2$:
  - trade may be efficient (in one equilibrium) with (in separating and pooling equilibria) and without test technology;
  - but social cost of signals.

# IV. Cheap talk models, costless signals

To transmit information about their talent, people also talk: labor market interviews ...

They can send very positive messages about their productivity: in contrast to education, which involves a cost in acquiring diploma and obtaining good grades that depends upon intrisinc talent, it is not less costly to say that one is extremely good when one actually is rather than when one is not: no differential cost of messages, *talk is cheap*

So, can information be actually transmitted through costless communication? And if so, how much?

# IV. Cheap talk models, costless signals

Elementary Sender-Receiver game that is now the basis of all papers dealing with communication

- Information: S informed about state of nature $\theta \in [0,1]$, while R only has a prior $F(.)$
- Moves: S sends a costless (cheap talk) message $m \in [0,1]$ to R, then R takes a decision $y \in \mathbf{R}$

Utility:

$$U_R(y,\theta) = -(y-\theta)^2, \text{ maximum for } y_R(\theta) = \theta$$
$$U_S(y,\theta) = -(y-\theta-b)^2, \text{ maximum for } y_S(\theta) = \theta + b$$

$b$ measures the lack of congruence between S and R's objectives

# IV. Cheap talk models, costless signals

Concentrate on (m.s.) Perfect Bayesian Equilibria:

- $y(m)$, decision rule after any message $m$,
- $q(. \mid \theta)$, distribution over messages conditional on any $\theta$
- $r(. \mid m)$, R's beliefs on the state of nature after any message $m$,

Say that $\theta_0$ induces action $y_0$ if:

$$\int_{\{m;y(m)=y_0\}} q(m \mid \theta_0) dm > 0$$

# IV. Cheap talk models, costless signals

If R does not listen to S, S has no reason to send meaningful messages, and if S sends meaningless messages, R has no reason to listen to S

**Babbling equilibria**

There always exists a babbling equilibrium in cheap talk games; here $q$ uniform over $[0,1]$ for all $\theta$, $y(\theta) = \mathbf{E}[\theta]$ and $r = f$.

**Question:** Are there other PBE where some information is revealed, and if so, how much information is revealed ?

**Central result:** Full information transmission is impossible !

**Central lemma: a finite number of actions induced**

If $y'$ and $y''$ are induced actions in equilibrium, then: $\mid y'-y'' \mid \geq b$; hence, the set of induced actions is finite.

**Proof:** take $y' < y''$

- $U_S(y', \theta') \geq U_S(y'', \theta')$ and $U_S(y'', \theta'') \geq U_S(y', \theta'')$
- So, there exists $\theta^*$ with $U_S(y', \theta^*) = U_S(y'', \theta^*)$ and $y' < \theta^* + b = \frac{y'+y''}{2} < y''$
- $y'$ cannot be induced by $\theta > \theta^*$, so the support of beliefs leading to $y'$ must be included in $[0, \theta^*]$ and so, $y' \leq \theta^*$.
- Therefore: $y'' \geq \theta^* + b > \theta^* \geq y'$, i.e. $y'' - y' > b$

# IV. Cheap talk models, costless signals

Important result: even locally, perfect revelation of information is impossible as soon as objectives are not perfectly aligned ($b > 0$)

One can characterize partition PB equilibria, where $[0, 1]$ is split into $p$ intervals, and in each interval, S sends the same message: i.e. there is partial / coarse information revelation

Moreover, there exists an upper bound $P(b)$ such that, only partition equilibria with $p \leq P(b)$ exist

$P(b)$ decreasing in $b$: more congruent objectives may lead to more information being communicated / conflict limits communication

# V. Conclusion

Asymmetric information is a major impediment to markets functioning properly

- It makes classical market failures more difficult to solve
- And it is the source of inefficiency by itself
- There are no easy way to fill the informational gap by sending information convincingly to the market, without generating residual inefficiencies

Need for a re-formalization of transactions under asymmetric information to provide better foundations to the analysis of markets under asymmetric information

# Required Reading


- * Akerlof, G. (1970), *Quarterly Journal of Economics*, 89, 488-500
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