Principal - Agent model under moral hazard

Microeconomics 2

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Principal - Agent model as the elementary block to build up models of transactions under asymmetric information

- Principal, who lacks information, proposes a setting for the transaction
- Agent, who is informed, accepts or refuses the transaction setting
- If agreement, the transaction is implemented

Previously: incomplete information or screening, i.e. missing information on some exogenous parameters

Today: imperfect information or moral hazard, i.e. missing information about some endogenous variables (Agent's decisions)

I.1. Introduction

Principal - Agent relationship: Agent takes payoff-relevant action in exchange of a reward / compensation.

When there is no issue:

- Principal does not care about the action: let Agent do his job and compensate him for the opportunity cost
- Agent does not care about the action: Agent takes action prefered by Principal if compensated for his opportunity cost
- No observable variable available: the Agent takes his prefered action and is compensated for his opportunity cost
- Perfect information about the action: Principal imposes her prefered action taking into account the Agent's compensation for his opportunity cost

So assume actions impact both players' utilities and there is an imperfect signal about the actions undertaken.

I.1. Introduction

Wide applicability of moral hazard model:

- Insurance company / insured agent
- Employer / employee: provide incentives to the employee so that he takes profit-enhancing actions that are costly to him
- Shareholders / CEO: induce the manager to implement projects that enhance the firm value and not his own private benefits
- Plaintiff / attorney: induce attorney to expend costly effort to increase plaintiff's chances of prevailing at trial (also all expertise relationships)
- Homeowner / contractor: induce contractor to complete work on time by expending appropriate but costly effort
- Landowner / farmer: induce farmer to grow crops and preserve soil quality, sharecropping...

I.2. Road map for today

Canonical two-action model:

- General presentation
- Cost-minimizing contract implementing a given action
- Optimal contract and inefficiency result

General (discrete) framework:

- Existence and general inefficiency theorem
- About monotonicity
- Sufficient statistics theorem and information structures
- Discussion: first-order approach, asymptotic efficiency
- Dynamic issues: memory, savings

Applications:

- Linear schemes in Holmström-Milgron: Multitask schemes and organizational design
- Limited Liability models: Corporate finance

Transaction between Principal and Agent:

- Agent takes a transaction-relevant action $a \in A$ compact in $\mathbf{R}_+$, unobservable by any other party
- Observable signal, i.e. random variable $x \in X \subset \mathbf{R}$
- a affects the signal: conditional cdf or proba (if finite) of x given a: $F_a(x)$.
- The other part of the transaction is an observable and contractable action by Principal: payment $w \in \mathbf{R}$
- Principal often assumed risk-neutral: $V = x - w$
- Agent's risk-averse preferences: $U = u(w) - C(a)$, $u(.)$ concave increasing unbounded, $C(.)$ convex increasing (separability is a strong and important assumption)

I.2. Canonical setting

Principal proposes a compensation mode, called a contract: specifies how w is determined based on variables that can be observed without ambiguity by both parties and a lawyer who would enforce the contract

These variables are called verifiable, or contractible variables: a contract can be based on their specification

- If Agent refuses, he obtains a reservation utility U_R and Principal a reservation utility normalized to 0
- If Agent accepts, then he decides a, outcome x arises and contractual transfers are implemented

I.3. Perfect information benchmark

Benchmark case: (a, x) are observable and verifiable, i.e. they can be the basis of a contract $w = w(x, a)$.

- Ex ante Pareto program:

$$
\begin{aligned}
(w^0(.), a^0) \in \quad & \arg\max_{w(.),a} \mathbf{E}[x - w(x)] \\
& \mathbf{E}[u(w(x)) - C(a)] \geq U_R
\end{aligned}
$$

- Two steps: **First**, perfect information optimum for given a:

$$
\begin{aligned}
\max_{w(.)} \quad & \int (x - w(x)) f_a(x) dx \\
U_R \leq \quad & \int u(w(x)) f_a(x) dx - C(a).
\end{aligned}
$$

- Participation constraint = individual rationality constraint will obviously be binding:

$$
\int u(w_a(x)) f_a(x) dx - C(a) = U_R
$$

I.3. Perfect information benchmark

- Optimal risk-sharing in Pareto optimum: equalized MRS across states (called Borch rule): $u'(w_a(x))$ must be constant across all x
- That is: $w_a(x) = u^{-1}(U_R + C(a))$, i.e. perfect insurance
- In general (risk averse Principal), optimal risk sharing: $0 \leq w'_a(x) \leq 1$
- **Second step** is easy: maximize w.r.t. a

$$a^0 = \arg\max_a \int \left(x - u^{-1}(U_R + C(a))\right) f_a(x)dx$$

- **Optimum**: Principal proposes a forcing contract: *you take action a^0 and you'll be paid $w^0(x) = u^{-1}(U_R + C(a^0))$ irrespective of the outcome x*: Full efficiency.

I.4. First hint on imperfect information

Suppose now a not observable by Principal (or anybody else)

- Perfect information optimum is action a^0 and a constant transfer $w^0 = u^{-1}(U_R + C(a^0))$ (risk-netrual Principal)
- Faced with perfect information contract, Agent chooses his action

$$\max_a \big(u(w^0) - C(a)\big) = U_R + C(a^0) - \min_a C(a) > U_R$$

- $\underline{a} \equiv \min A$: he chooses the minimal-cost effort, since he is perfectly insured !

Tension between optimal risk sharing (full insurance) and incentives to expend effort

I.4. First hint on imperfect information

- Suppose the Agent is **risk-neutral**: $u(w) = w$
- Full information optimum yields profit for P:

$$\Pi = \mathbf{E}[x \mid a^0] - C(a^0) - U_R = \max_a \left(\mathbf{E}[x \mid a] - C(a)\right) - U_R$$

- **Sell-out contract**: $w(x) = x - \Pi \to$ Agent residual claimant of profits for purchase price of Π and chooses

$$\begin{aligned} \max_a \left(\int w(x) f_a(x) dx - C(a) \right) &= \max_a \left(\mathbf{E}[x \mid a] - C(a) - \Pi\right) \\ &= U_R \end{aligned}$$

for $a = a^0$! He takes efficient action, full efficiency !

- BUT with risk-aversion, this violates optimal risk-sharing

Fundamental conflict: Pareto efficiency vs incentives

I.4. First hint on imperfect information

- How much risk is necessary ? What is the optimal contract?

$$\max_{w(.),a} \int (x - w(x))\, f_a(x) dx$$

$$\text{s.t.} \quad : \quad \int u(w(x)) f_a(x) dx - C(a) \geq U_R$$

$$\text{and} \quad : \quad a \in \arg\max_{a'} \left(\int u(w(x)) f_{a'}(x) dx - C(a') \right)$$

- Agent accepts the contract: participation / IR constraint
- New constraint = incentive constraint: the action induced is the one preferred by the Agent to any other action a' wihtin the framework of the contract
- Simplify this (too general) setting to get intuition

Solving this problem may be tricky in general → Start with the binary version with 2 actions: much of the intuition.

- Effort can take 2 values: $A = \{0, 1\}$, $C(1) = C > 0 = C(0)$
- Principal is risk neutral
- Principal's program under moral hazard solved in 2 stages
  - For a given a, what is the best contract that induces the Agent to take action a ? That is, what is the cost-minimizing contract that implements a
  - Compare the cost-minimizing contract that implements $a = 1$ and the cost-minimizing contract that implements $a = 0$
- NB: cost-minimizing contract that implements $a = 0$ is the perfect insurance contract $w_0 = u^{-1}(U_R)$

II.2. Cost-minimizing contract that implements $a = 1$

$$\min_{w(.)} \int w(x) f_1(x) dx$$

$$\text{s.t.} \quad : \quad \int u(w(x)) f_1(x) dx - C \geq U_R$$

$$\text{and} \quad : \quad \int u(w(x)) f_1(x) dx - C \geq \int u(w(x)) f_0(x) dx$$

- $\lambda \geq 0$ multiplier associated to IR constraint
- $\mu \geq 0$ multiplier associated to IC constraint
- If $\mu = 0$, solving without IC leads to $w_1 = u^{-1}(U_R + C)$, which induces Agent to choose $a = 0$! So, $\mu > 0$
- IR binding; otherwise, consider $dw(x)$ such that, for all x

$$u(w(x)) - u(w(x) + dw(x)) = \varepsilon > 0$$

II.2. Cost-minimizing contract that implements $a = 1$

Optimizing the Lagrangean, FOC:

$$\frac{1}{u'(w_1^*(x))} = \lambda + \mu\left(1 - \frac{f_0(x)}{f_1(x)}\right) = \lambda + \mu\left(1 - r(x)\right)$$

$$\int u(w_1^*(x))f_1(x)dx - C = U_R$$

$$\int u(w_1^*(x))f_1(x)dx - C = \int u(w_1^*(x))f_0(x)dx$$

where $r(x)$ is the likelihood ratio $\frac{f_0(x)}{f_1(x)}$

- $r(x)$ measures how likely it is that x comes from a draw from $(x \mid a = 0)$ compared to a draw from $(x \mid a = 1)$
- Compensation $w_1^*(x)$ is higher (lower) when the likelihood ratio is lower (higher), i.e. when it is relatively likely (unlikely) that Agent has chosen $a = 1$ ($a = 0$)

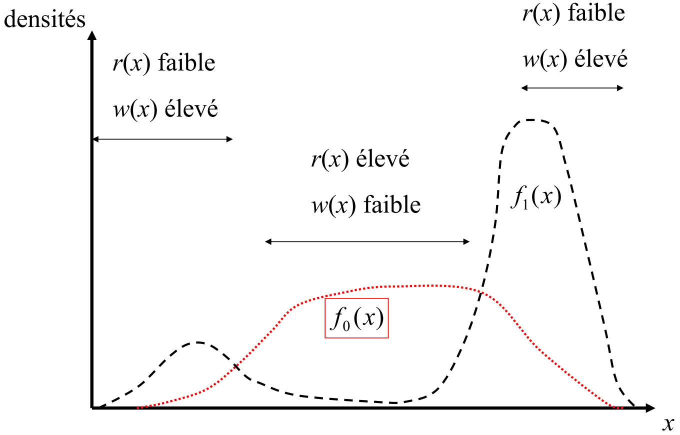
II.2. Cost-minimizing contract that implements $a = 1$

Does compensation increase in performance x ($w_1^*(.)$ increasing)?

$$\frac{1}{u'(w_1^*(x))} = \lambda + \mu\,(1 - r(x))$$

- YES iff $r(x)$ is decreasing in x, i.e. if higher performance gives more confidence that $a = 1$ has been chosen ! (MLRP, *Monotone likelihood ratio property*)
- NO in general !
- See the counter-example on the picture below

II.2. Cost-minimizing contract that implements $a = 1$



w(.) non-monotone

II.2. Cost-minimizing contract that implements $a = 1$

The nature of the problem is stochastic: stochastic link from a to x matters, not physical link

In fact, there is nothing special about x except that it is an observable signal about the action; could be different from Principal's gross profit.

Note also that MLRP implies FOSD, i.e. $F_1(w) \leq F_0(x)$ for all x, but the reverse is not true.

- Necessary that effort stochastically increases output for the optimal compensation to increase in performance
- But not sufficient !

If several signals about Agent's action, which one to use ?

- Two verifiable signals: $(x, y) \sim f_a(x, y)$
- FOC are similar:

$$\frac{1}{u'(w_1^*(x,y))} = \lambda + \mu \left(1 - \frac{f_0(x,y)}{f_1(x,y)}\right).$$

- $w_1^*(x, y)$ iff $r(.)$ depends on x and on y
- A contrario, suppose $f_a(x, y) = k(x, y)g_a(x)$, then $r(.)$ depends only on x and $w_1^*(.)$ should not depend on y optimally.
- $f_a(x, y) = k(x, y)g_a(x) \Leftrightarrow x$ is a sufficient statistics on a for the pair (x, y), i.e. y does not convey any additional information on a that is not already contained in x
- This is the sufficient statistics theorem (Holmström)

II.3. Optimal contract

- Cost-minimizing contract that implements $a = 1$ is more costly due to imperfect information on a:

$$\int w_1^*(x) f_1(x) dx > u^{-1}(U_R + C)$$

- Principal's net profit of inducing $a = 1$ is smaller than perfect information profits:

$$\int (x - w_1^*(x)) f_1(x) dx < \int x f_1(x) dx - u^{-1}(U_R + C)$$

- Even if $a = 1$ is optimal under perfect information, $a = 0$ may become optimal under imperfect information on a

Cost of moral hazard

Moral hazard implies a strict loss for the Principal: either induce $a = 1$ at a larger cost, or induce $a = 0$.

How general / robust are previous conclusions ? Established model of Grossman - Hart with finite signals and finite actions.

- Verifiable signal takes a finite number of values: $x \in X = \{x_1, x_2, ..., x_n\}$, ranked increasingly in i.
- Effort: $a \in A$ finite, and $f_i(a) = \Pr\{x = x_i \mid a\}$
- Cost-minimizing contract that implements a: $\{w_i\}_{i=1}^n$, viewed using utilities $\{v_i\}_{i=1}^n$ with $v_i = u(w_i)$:

$$\min_{(v_i)_i} \sum_i f_i(a) u^{-1}(v_i)$$

s.t.: $$\sum_i f_i(a) v_i - C(a) \geq U_R$$

and: $$\sum_i f_i(a) v_i - C(a) \geq \sum_i f_i(a') v_i - C(a'), \forall a'$$

III.2. General results

All constraints are linear constraints, $u^{-1}(.)$ is convex; standard polygonal program where the only problem is whether the set of constraints is empty.

Definition of an implementable action

a is implementable if the set of constraints is not empty

Proposition: Binding participation constraint

If a implementable, participation constraint binds at cost-minimizing contract that implements a

- Proof: If not, consider a uniform decrease in v_i, $\forall i$.
- This result depends on Agent's utility being additively separable in money / action
- OK with multiplicatively separable: $u(w)\gamma(a)$ (check it!)

Existence of cost-minimizing contract iff set of constraints is not empty, i.e. a implementable

Proposition: Condition of implementability / existence

a implementable iff there does not exists a distribution $\nu(a')$ over $a' \in A \setminus \{a\}$, such that: for any i

$$\sum_{a' \neq a} \nu(a') f_i(a') = f_i(a) \text{ and } \sum_{a' \neq a} \nu(a') C(a') < C(a)$$

- Proof: existence of a solution to set of IC inequalities, related to Farkas' Lemma (Theorem 22.1, Rockafellar 1970)
- Intuition: no way to achieve the same distribution over outcomes at smaller (expected) cost.

III.2. General results

Cases where moral hazard does not matter:

- $\underline{a}$ can be implemented at the same cost as under perfect information (with perfect insurance contract)
- The case of shifting support:
  - Distribution $f_i(.)_i$ has shifting support relative to a if there exists i_0 such that $f_{i_0}(a) = 0 < f_{i_0}(a')$ for all a' such that $C(a') < C(a)$
  - If the distribution has shifting support relative to a, a can be implemented at the same cost as under perfect information
  - Intuition: take $v_{i_0} \to -\infty$ (assumption $u(.)$ unbounded from below) and $v_i = U_R + C(a)$ otherwise
- If Agent is risk neutral

General loss for the Principal due to moral hazard

Assume a implementable, $u(.)$ stricly concave, $f_i(.)_i$ has full support and $C(a) > \min_{a' \in A} C(a')$, cost of implementing a under moral hazard is strictly higher than under perfect information.

Proof:

- Cannot be smaller: more constraints
- Under the assumptions, there exists i, j such that $v_i \neq v_j$
- So, Jensen inequality + concavity + full support yield:

$$\begin{aligned}\sum_k f_k(a) u^{-1}(v_k) &> u^{-1}\left(\sum_k f_k(a) v_k\right) \\ &\geq u^{-1}\left(U_R + C(a)\right)\end{aligned}$$

Monotonic properties of cost-minimizing contract

Assume previous assumptions:

- there exists i such that $w_i < w_{i+1}$
- there exists j such that $x_j - w_j < x_{j+1} - w_{j+1}$

Very weak properties ! Going further with Kuhn-Tücker?

$$\frac{1}{u'(u^{-1}(v_i))} = \left\{ \lambda + \sum_{a' \neq a} \mu(a') \right\} - \sum_{a' \neq a} \mu(a') \frac{f_i(a')}{f_i(a)}$$

with $\mu(a') > 0$ means Agent is indifferent between a and a'

- At optimum, there exists (at least) one such less costly action a' for which $\mu(a') > 0$
- If there exists just one such a', as in the two-action model.
  - MLRP hypothesis: For all $(a, a') \in A^2$, if $C(a') \leq C(a)$ then $\frac{f_i(a')}{f_i(a)}$ is decreasing in i
  - Then, the cost-minimizing contract is increasing in i under MLRP
- But if there exists a' and a" with $C(a') < C(a) < C(a")$, $\mu(a') > 0$ and $\mu(a") > 0$, MLRP does NOT imply that:

$$\mu(a')\frac{f_i(a')}{f_i(a)} + \mu(a")\frac{f_i(a")}{f_i(a)} \text{ decreases in } i$$

III.3. Further results about monotonicity

- Spanning condition: There exists two distributions $\underline{f}_i$ and $\bar{f}_i$ over X, with $\frac{\bar{f}_i}{\underline{f}_i}$ decreasing in i and non-decreasing mapping $\lambda(.)$ from A to $[0,1]$ such that:

$$\forall a, f_i(a) = \lambda(a)\bar{f}_i + (1-\lambda(a))\underline{f}_i$$

- Spanning condition is sufficient for monotonicity, but strong
- CDFC assumption: For all $(a, a', a")$ such that $C(a) = \lambda C(a') + (1-\lambda)C(a")$, the following holds:

$$F(\cdot \mid a) \succeq^{FOSD} \lambda F(\cdot \mid a') + (1-\lambda)F(\cdot \mid a")$$

- MLRP + CDFC are sufficient conditions for monotonicity, but again quite strong
- Conclusion: monotonicity is not a natural property in the moral hazard framework

In two-effort setting: a glimpse on when one should make the compensation contingent on an additional signal and when not.

General link between information structures (signal technologies) and Principal's cost of implementing a given action a under moral hazard ?

A moral hazard environment is characterized by the information structure, summarized by $f(.)$ from A into a simplex

Question: compare the expected compensation to implement action a (i.e. moral hazard cost of implementing a) under information structure $f(.)$ (vector of dim n) and information structure $g(.)$ (of dim m) ?

Let $\Gamma(a; f)$ denote the value of the cost-minimizing program that implements action a when the structure of verifiable signals is given by $f(.)$

Compare $\Gamma(.; f)$ and $\Gamma(.; g)$ for two signal technology $f(.)$ and $g(.)$.

Recall a basic definition:

Blackwell sufficient information structures

$f(.)$ (of dim n) is sufficient for $g(.)$ (of dim m) in the sense of Blackwell if there exists a transition matrix P (of dim $m \times n$) such that: $g(.) = P.f(.)$ (i.e. $g_j(.) = \sum_i p_{ji} f_i(.)$)

Comparison of information structures

If $f(.)$ (of dim n) is sufficient for $g(.)$ (of dim m) in the sense of Blackwell, then $\Gamma(.;f) \leq \Gamma(.;g)$.

Proof:

- v_j: cost-min contract under $g(.)$ and let define a contract u_i based on signal $f(.)$ as: $u_i = \sum_j p_{ji} v_j$ (or $u = P^t.v$)
- Sufficiency implies for any action α, $g(\alpha)^t.v = f(\alpha)^t.P^t.v = f(\alpha)^t.u$, so that u satisfies also (IC) and (IR)
- New contract is cheaper than original one (Jensen again):

$$\begin{aligned} \sum_i f_i(a) u^{-1}(u_i) &= \sum_i f_i(a) u^{-1}\left(\sum_j p_{ji} v_j\right) \\ &\leq \sum_{ij} f_i(a) p_{ji} u^{-1}(v_j) = \sum_j g_j(a) u^{-1}(v_j) \end{aligned}$$

Getting back: suppose there are 2 signals, $x \in \{x_1, x_2, ..., x_i, ...x_n\}$, characterized by (marginal) $f_i(.)$, and $y \in \{y_1, y_2, ..., y_j, ...y_m\}$, characterized by (joint) $h_{ij}(.)$ for $i = 1, 2, .., n$ and $j = 1, 2, ..., m$.

In general, Principal will incur a smaller moral hazard cost of implementing any action (except lowest cost one) if he makes the compensation contingent on both x and y.

Proof:

- Let $g_k(.) = h_{ij}(.)$ for $k = (i-1)m + j$, $k = 1, ..., nm$.
- With P such that $p_{ik} = 1$ if and only if $k \in \{(i-1)m+1, (i-1)m+2, ..., im\}$ and 0 otherwise, $g(.)$ sufficient for $f(.)$:

$$f_i(a) = \sum_j h_{ij}(a) = \sum_{k=(i-1)m+1}^{im} g_k(a) = \sum_k p_{ik} g_k(a)$$

However, suppose there exists a transition matrix K such that $h_{ij}(.) = k_{ij} f_i(.)$:

- Let define the matrix D of dimension $nm \times n$ such that $d_{ki} = p_{ji}$ if $k = (i-1)m + j$ and 0 otherwise. D is a transition matrix since K is one.
- We have: $g(.) = D.f(.)$ so that x is sufficient for (x, y)

It follows that: $\Gamma(.; f) = \Gamma(.; g)$ that is, the cost-minimizing contract that implements a given action a needs not depend on y (Holmström)

Application:

- A salesman's effort aims at convincing buyers to buy the firm's product. When he visits a buyer, the buyer may end up signing up for a pre-order
- Then macroeconomic shocks impact buyers' budget and a buyer cancel his order before actual delivery (and payment)
- (nb orders, nb sales) jointly distributed depending on effort, but the distribution of sales conditional on orders only depend on macroeconomic shocks
- Observed performance: number of units ordered and number of units actually sold
- Salesman's compensation should only depend on the observed number of orders, not on the observed sales as macro shocks are not informative about the salesman's effort

III.5. Pitfall of the "first-order approach"

- Natural way to formalize moral hazard: the continuous model with $A = [0, \bar{a}]$ and X real interval (X could be discrete as before, this is not the important point)
- Use calculus to replace IC constraint by its local FOC:

$$\int u(w(x))\partial_a f_a(x)dx - C'(a) = 0$$

- FOC uses the differential version of likelihood ratio:

$$\frac{1}{u'(w(x))} = \lambda + \mu \frac{\partial_a f_a(x)}{f_a(x)}$$

- Under differential version of MLRP, i.e. $\frac{\partial_a f_a(x)}{f_a(x)}$ increasing in x, FOC implies $w(.)$ increasing provided $\mu > 0$!
- BUT ! a might be only a local extremum

III.5. Pitfall of the "first-order approach"

- Weak local IC: $\int u(w(x))\partial_a f_a(x)dx - C'(a) \geq 0$
- Then $\mu \geq 0$ and, as before, FOC imply $w(.)$ increasing under differential version of MLRP
- Differential CDFC: $F_a(x)$ is convex in a for all x.

$$\begin{aligned} \partial^2_{aa} & \left(\int u(w(x)) f_a(x)dx - C(a) \right) \\ &= \partial^2_{aa} \left(- \int u'(w(x))w'(x)F_a(x)dx - C(a) \right) \\ &= - \int u'(w(x))w'(x)\partial^2_{aa}F_a(x)dx - C''(a) \leq 0 \end{aligned}$$

- Agent's objectives are concave, first-order approach OK; but restrictive assumptions (MLRP + CDFC) as in discrete case

III.6. Asymptotic perfect information optimum

We've considered X an interval or even $\mathbf{R}$; when the density vanishes, situation looks like a shifting support

- Suppose $x = a + \epsilon$, with ϵ distributed according to $F(.)$ cont. diff. unimodal with zero mean.
- Take $a = [0, 1]$, $C(a) = a^2/2$, $u(0) = 0 = U_R$
- Suppose full information optimum is: $a = 1$, $v^0 = 1/2$
- Consider the following threshold contract: for a given b, $v(x) = v_-$ if $x < b$ and $v(x) = v_+$ if $x \geq b$
- Agent's objectives:

$$\max_{0 \leq a \leq 1} \left(F(b-a)v_- + (1 - F(b-a))v_+ - \frac{a^2}{2} \right)$$

- If $(v_+ - v_-)f(b-1) = 1$ and $f'(b-1) > 0$, Agent chooses $a = 1$

III.6. Asymptotic perfect information optimum

- To make IR binding:

$$F(b-1)v_- + (1-F(b-a))v_+ - \frac{1}{2} = 0$$

- Solving: $v_+ = \frac{1}{2} + \frac{F(b-1)}{f(b-1)}$ and $v_- = \frac{1}{2} - \frac{1-F(b-1)}{f(b-1)}$
- Suppose $\lim_{y\to-\infty} \frac{F(y)}{f(y)} = 0$; e.g. normal distribution
- Then, when $b \to -\infty$, $v_+ \to 1/2$, $v_- \to -\infty$ but $(1 - F(b-1))u^{-1}(v_-) \to 0$ (infinite punishment but with vanishing probability)
- The cost of implementing $a = 1$ using this contract:

$$F(b-1)u^{-1}(v_-) + (1-F(b-1))u^{-1}(v_+) \to u^{-1}(1/2)$$

- Non-existence; but approximate full information optimum

IV. Dynamic issues – IV.1. Why study dynamics and how?

Moral hazard models are widely used to model organizations, firms,... and these are long-lasting institutions: nexus of contracts for repeated interactions.

Similarly, contractual arrangements in markets (distribution and retailing, insurance, credit...) span over long time periods during which several transactions take place.

Sources of dynamics:

- Agent takes actions today, tomorrow,...
- Information changes over time
- The contractual setting is modified: related to commitment issues

IV.1. Why study dynamics and how?

Commitment = capacity to tie one's own hands

- Long term contract proposed once and for all;
- Principal does not play after contract signature: no non-contractible action
- Principal commits not to use information if it was not explicitly stated in original contract
- Abide by (possibly) ex-post (.i.e once some information has been learnt) inefficient rules

This is a strong assumption. Alternative settings:

- **Absence of commitment**: Principal and Agent repeatedly negotiate spot contracts.
- **Commitment with renegotiation**: parties can agree to modify a long-term contract if it is mutually beneficial.

IV.1. Why study dynamics and how?

Classical argument: Commitment is always (weakly) beneficial in a simple Principal - Agent relationship.

Proof: Principal committing to her equilibrium strategy in a non-commitment setting makes her as well off as without commiting, and Agent's best response unchanged.

How necessary is the possibility of commitment ? Possible properties of optimal long term contract:

- **Sequential efficiency, renegotiation-proof:** At any date, no other mechanism and attached equilibrium that is mutually beneficial for Principal (strictly) and Agent
- **Sequential optimality, replication by spot contracts:** Sequential efficient and, at any date, Agent gets his reservation utility

IV.1. Why study dynamics and how?

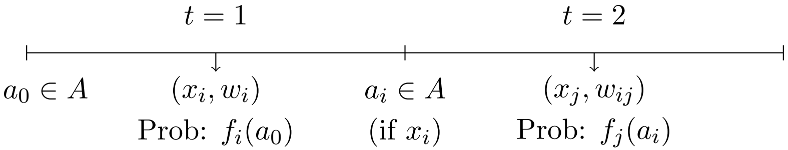
Complete study of dynamics of moral hazard models beyond scope of this introductory course !

Focus on a few properties of the optimal long term contract in one Principal - one Agent framework with **two periods** to account for several transactions, actions, steps of information accrual, and one possible contractual change

Themes:

- Role of memory
- Agent's access to financial markets
- Observable but not verifiable actions

IV.2. The role of memory in repeated moral hazard



Standard model with X discrete and A continuum

- Agent's discount factor δ; risk-neutral Principal's discount factor is market discount factor $\rho = \frac{1}{1+r}$
- **Technological separability**: at t, output only determined by current effort
- a_0 action implemented at 1st period, if observed outcome at 1st period is x_i, let a_i denote the action implemented in 2nd period (in general, incentives may induce a different 2nd period action depending on verifiable variables at 1st period)

IV.2. The role of memory in repeated moral hazard

Contracts:

- Compensation at t contingent on realized states of nature, i.e. on past and current outputs
- w_i or $v_i = u(w_i)$ if x_i at $t = 1$ and w_{ij} or $v_{ij} = u(w_{ij})$ if x_i at $t = 1$ and x_j at $t = 2$

Static benchmark (assume FO approach for simplicity):

- Cost-minimizing condition for a:

$$\frac{1}{u'(w_i)} = \lambda + \mu \frac{f_i'(a)}{f_i(a)}$$

- Binding IR:

$$\sum_i p_i(a) v_i - C(a) = \underline{U}.$$

Role of memory in optimal long term contract

The optimal LT contract exhibits memory provided moral hazard is not degenerate, i.e. if a_0 is not the least costly action; more precisely, for $i \neq i'$, $w_i \neq w_{i'}$ implies $\exists j, w_{ij} \neq w_{i'j}$.

Proof:

- Suppose $i \neq i'$ such that: $v_i \neq v_{i'}$.
- If for all j, $v_{ij} = v_{i'j} = \hat{v}_j$, then necessarily: $a_i = a_{i'} = \hat{a}$; i.e. same action induced (mild caveat if equivalent actions).
- After x_i at $t = 1$, modify: reduce $v_i - \epsilon$ and after x_j at $t = 2$ for all j, increase $\hat{v}_j + \frac{\epsilon}{\delta}$:
    - incentives for 2nd period action after x_i unchanged;
    - Agent's intertemporal utility if x_i unchanged;
    - obviously idem after any other observation

- Optimal intertemporal smoothing for Principal implies:

$$\{\epsilon = 0\} = \arg\min\{u^{-1}(v_i - \epsilon) + \rho \sum_j f_j(\hat{a}) u^{-1}(\hat{v}_j + \frac{\epsilon}{\delta})\}$$

$$\Rightarrow \frac{1}{u'(w_i)} = \frac{\rho}{\delta} \sum_j f_j(\hat{a}) \frac{1}{u'(\hat{w}_j)} = \frac{1}{u'(w_{i'})} \Rightarrow w_i = w_{i'}$$

- Contradiction: memory of first-period incentives
- If no memory at all, it means that v_i independent of i and a_0 is the lowest-cost action.

Therefore, the optimal compensation must depend upon present and past performance for the optimum contract, even though past effort has no impact on current (or future) performance.

IV.2. The role of memory in repeated moral hazard

As a consequence, the optimal LT contract is not sequentially optimal

The period 2 utility provided by the optimal contract depends upon the realization of x_i at $t = 1$, hence cannot be equal to the exogenous reservation utility of the Agent.

However, the optimal LT contract is sequentially efficient (renegotiation proof):

- If not, after history x_i, replace branch of the LT contract by a better sub-contract, subject to the same continuation utility: $\underline{U}_i \equiv \sum_j f_j(a_i)v_{ij} - C(a_i)$.
- Continuation utility constraint is binding: same expected utility after x_i.
- Idem after other $x_{i'}$
- Hence, a better LT contract: contradiction.

IV.3. Role of financial markets

In same framework, suppose the Agent has access to financial markets and can save (or borrow if negative) S_i on his compensation w_i after x_i so as:

$$\max\{u(w_i - S_i) + \delta \sum_j f_j(a_i) u\left(w_{ij} + (1+r)S_i\right)\}$$

Around $S_i = 0$, A would like to save:

- Derivates around $S_i = 0$: $-u'(w_i) + \delta(1+r)\sum_j f_j(a_i)u'(w_{ij})$
- Using same intertemporal transfer as in previous subsection:

$$\frac{1}{u'(w_i)} = \frac{\rho}{\delta} \sum_j f_j(a_i) \frac{1}{u'(w_{ij})}$$

- The function $1/x$ being convex, Jensen inequality implies:

$$\delta(1+r) \sum_j \frac{f_j(a_i)}{(1/u'(w_{ij}))} \geq \delta(1+r) \left(\sum_j \frac{f_j(a_i)}{u'(w_{ij})} \right)^{-1} = u'(w_i)$$

IV.3. Role of financial markets

This is problematic: Agent can easily undo the incentives built in the LT contract !!

One could think of limiting the **borrowing** possibilities of the Agent, but it seems hard to limit his **saving** possibilities !

Nevertheless, if savings are observable and verifiable, i.e. controllable, they should also be included in the optimal moral hazard contract:

$$\min_{(w_i, w_{ij}, S_i)} \sum_{i,j} f_i(a_0) f_{ij}(a_i)(w_i + \rho w_{ij})$$

subject to IR and incentives constraints using

$$\sum_{i,j} f_i(a_0) f_{ij}(a_i) \left[u(w_i - S_i) + \delta u(w_{ij} + (1+r)S_i) - C(a_0) - \delta C(a_i)\right]$$

IV.3. Role of financial markets

With $c_i = w_i - S_i$, $c_{ij} = w_{ij} + (1+r)S_i$, i.e. substitute consumption to earnings, same program as without financial markets:

Optimal LT contract with controlled access to financial markets

Consumption in optimal contract does not depend upon the (controlled) access to financial market; it exhibits memory and optimal LT contract with controlled savings is sequentially efficient.

Stronger result: sequential optimality

Optimal contract with controlled access is sequentially optimal

- Key: at $t = 2$, IR depends upon accumulated savings S_i
- Using S_i, adjust reservation utility (depends on savings) and maintain unchanged continuation utility !

Access to financial markets disconnects the intertemporal smoothing problem from the moral hazard problems (incentives vs insurance) at each t.

When savings cannot be fixed within the contract: savings become one additional moral hazard variable!

Sequential efficiency

In general, the optimal LT contract with non-controlled access to financial market is not sequentially efficient; hence not sequentially optimal.

In the usual proof of renegotiation-proofness (see earlier), when one replaces one branch after x_i with a better sub-contract, this leaves expected utility unchanged but expected marginal utilities may change through wealth effects, which conflicts with intertemporal smoothing

Noticeable case: When $u(w, a) = -\exp\{-r(w - c(a))\}$, utility and marginal utility are aligned: optimal LT is sequentially efficient.

IV.4. Renegotiation after a signal

Not the same dynamics setting here, but issue is intrinsically dynamic: what happens if the Agent's action is observable to the Principal, but not verifiable? Or if some **observable but non-verifiable signal** is observed ?

One cannot write a contract based on this signal, but after observing it, the Principal and the Agent can figure out what kind of contracts would now be better and renegotiate on such a contract !

In the one-period model with risk-neutral Principal, assume that:

- After action a is taken and before outcome x is observed, both Principal and Agent observe signal s
- and Principal and Agent can renegotiate on a new contract if mutually beneficial

IV.4. Renegotiation after a signal

Simple case: the signal s is the action a and Principal has bargaining power at renegotiation stage.

Observing the action:

Any implementable action under standard moral hazard is implementable at full information cost $u^{-1}(C(a))$ under renegotiation.

Proof:

- After any a, gains from trade since risk is not optimally shared and action is already decided: Principal offers new fixed compensation:

$$u^{-1}(\sum_i f_i(a)v_i)$$

- Agent is indifferent
- (IR) implies this is equal to $u^{-1}(C(a))$.

IV.4. Renegotiation after a signal

Renegotiation reduces the cost of implementable action down to its full information value. Therefore, the full information optimal action (if implementable) can be implemented under moral hazard with observability of action and renegotiation !

Renegotiaton is strictly valuable for Principal (if no shifting support)

Compensation is not determined by the initial contract but by the renegotiated contract. Initial contract serves as a threat point if the Agent were to deviate.

Other simple case: the signal s is the action a and Agent has bargaining power at renegotiation stage.

The full information outcome is attainable here, too:

- Consider the contract $w_i = x_i - \Pi^0$, sell out contract at price equal to the Principal's full information profit.
- At renegotiation avec a, Agent proposes full insurance at wage w such that:

$$\sum_i f_i(a)x_i - w = \Pi^0$$

- So, Agent expects: $\sum_i f_i(a)u(w_i) - C(a) = \sum_i f_i(a)x_i - C(a) - \Pi^0$ which is maximized by definition for $a = a^0$ and yields expected utility equal to the reservation value.

Original contract determines the default payoff in the renegotiation, hence here the Principal's profit.

Since renegotiation will bring back full insurance for the Agent, he can behave as if risk-neutral; hence the sell out contract.

This type of result extends to more balanced (but monotonic) renegotiation bargaining processes between Principal and Agent, provided the Principal keeps the bargaining power at the initial stage.

IV.4. Renegotiation after a signal

More elaborate case: s is an imperfect signal and Principal has all bargaining power.

- Signal $s \in \{s_1, ..., s_j, ..., s_m\}$ with marginal probability $g_j(a)$.
- Conditional probability of x_i given a and s_j: $\sigma_{ij}(a)$
- $f(.) = \Sigma(.).g(.)$

Principal knows s while Agent knows a and s: renegotiation potentially under incomplete information.

Except if $\Sigma(a) = \Sigma(a')$ for any two actions: i.e. if s is a sufficient statistics about x for (s, a). Then, renegotiation under symmetric information.

IV.4. Renegotiation after a signal

Under sufficient statistics assumption, renegotiation of a contract v leads to a full insurance contract at wage $u^{-1}(\sum_i \sigma_{ij} u_i)$ after signal s_j, and therefore to an expected utility for Agent:

$$\sum_j g_j(a) \sum_i \sigma_{ij} u_i - C(a) = \sum_i f_i(a) u_i - C(a)$$

So, the same IC and IR constraints hold with or without renegotiation ! The set of implementable actions is the same

And if there is no shifting support for the signal, i.e. $\Sigma >> 0$, then the cost of implementing any action (but the least-cost one) is strictly smaller with renegotiation than without.

Without sufficient statistics condition: see Hermalin-Katz

V. Applications

Remark about general moral hazard models:

- General results about inefficiency and informativeness can be obtained in the general framework
- But it is difficult to obtain explicitly optimal contract and, consequently, almost impossible to use general moral hazard models in more complicated environments.

Applying moral hazard to understand organizations or managerial compensation schemes implies to make additional assumptions so as to work with tractable models, with explicit solutions.

In this section:

- Model with continuum of actions and outcomes and linear schemes
- Model with risk neutrality ... but limited liability.

Holmström - Milgrom setting:

- Agent takes multiple actions: $a = (a_1, ..., a_n) \in \mathbf{R}_+^n$ at cost $C(a)$ convex
- Principal's benefit can be general (concave) $B(a)$
- Agent's actions generate a vector of signals: $x = \mu(a) + \epsilon$, where ϵ is normally distributed with zero mean and var/cov matrix Σ
- Principal is risk neutral
- Agent's preferences: $U(\omega) = -\exp\{-r\omega\}$ with ω is the Agent's wealth: $\omega = w(x) - C(a)$

Central assumption: we restrict attention to linear schemes, i.e. $w(x) = \alpha^t.x + \beta$.

With linear contracts and normally distributed noise term, one has:

$$\mathbf{E}[U(w(\mu(a)+\epsilon) - C(a)] = U\left(\alpha^t.\mu(a) + \beta - C(a) - \frac{r}{2}\alpha^t.\Sigma.\alpha\right)$$

That is, one can reason in terms of certainty equivalent, equal to the expected net wealth minus a risk premium.

The joint surplus to be maximized is: $B(a) - C(a) - \frac{r}{2}\alpha^t.\Sigma.\alpha$ which is independent of β. β simply determines the distribution of the joint surplus between both players.

V.1. Linear schemes with CARA utility

Optimal contract program:

$$\max_{a,\alpha} \quad \{B(a) - C(a) - \frac{r}{2}\alpha^t.\Sigma.\alpha\}$$
$$\text{s.t.} \quad a \in \arg\max_e \{\alpha^t \mu(e) - C(e)\}$$

Assume that $\mu(a) = a$ so that we can follow a FO approach: the incentive constraint writes: $\alpha = \partial C(a)$ (for interior $a >> 0$)

FOC for the optimal contract are thus:

$$\begin{aligned} \partial B(a) - [I_n + r\partial^2 C(a).\Sigma].\partial C(a) &= 0 \\ \alpha &= \partial C(a) \end{aligned}$$

First consider the case of a one-dimensional action: $n = 1$

$$\max_{\alpha} \quad \left(B(a) - C(a) - \frac{r\sigma^2\alpha^2}{2} \right)$$
$$\text{s.t.} \quad \Leftrightarrow C'(a) = \alpha$$

yields the optimal contract: $\alpha = \frac{B'(a)}{1+r\sigma^2 C"(a)}$ and $C'(a) = \alpha$

- more risk-averse (r larger), less performance-based
- higher risk tilts trade-off towards more insurance
- more responsive to incentives (i.e. smaller C'' since $\frac{da}{d\alpha} = \frac{1}{C''(a)}$), more performance-based

The case of two actions: $n = 2$

Assume errors are stochastically independent (Σ is diagonal)

If activities are technologically independent ($\partial^2 C$ and $\partial^2 B$ are diagonal, then:

$$\alpha_i = \frac{\partial_i B(a)}{1 + r\sigma_i^2 \partial_{ii}^2 C(a)}$$

Commissions are set independently of each others. This is the benchmark case.

Assume now that the 2 tasks are not technologically independent.

Moral hazard cost of implementing action $(a_1, a_2) >> (0,0)$:

$$\Gamma(a_1, a_2) = C(a_1, a_2) + \frac{r}{2}(\partial C(a_1, a_2))^t.\Sigma.\partial C(a_1, a_2)$$

so moral hazard marginal cost of a_i is equal to full information marginal cost, i.e. $(1 + r\sigma_i^2 \partial_{ii}^2 C).\partial_i C$, plus $r\sigma_j^2 \partial_{ij}^2 C.\partial_j C$.

- If **complement tasks** wrt costs, the marginal cost is smaller than with independent tasks: tends to lead to higher optimal tasks (working on one reduces the cost of working on the other) and higher commissions
- If **substitutes** wrt costs, tends to lead to lower commissions and lower optimal tasks: if α_i increases, Agent substitutes effort away from task j !

Only one observed activity: $x = a_1 + \epsilon$ (that is, $\sigma_2^2 = \infty$)

$$\alpha_1 = \frac{\partial_1 B - \partial_2 B \frac{\partial_{12} C}{\partial_{22} C}}{1 + r\sigma_1^2(\partial_{11} C - \frac{(\partial_{12} C)^2}{\partial_{22} C})}$$

- If $\partial_{12} C > 0$, more cost-substitutability yields smaller commission
- To provide incentives to a_2, either reward it (but not measured here!) or reduce its opportunity cost, i.e. reduce the reward on the rival activity
- if $\partial_1 B - \partial_2 B \frac{\partial_{12} C}{\partial_{22} C} < 0$, even possible that: $\alpha_1 < 0$
- If output x can be destroyed at no cost, $\alpha_1 = 0$ even if task 1 is perfectly measured (i.e. if $\sigma_1^2 = 0$).

V.1. Linear schemes with CARA utility

Assume:

- **Perfectly substitutable efforts:** $C(a_1 + a_2)$
- $x = a_1 + \epsilon$ (or $\sigma_2^2 = \infty$)
- $C(.)$ has strict minimum at $a = \bar{a} > 0$ and $C(\bar{a}) = 0$: fixed wage contract elicits some effort (enjoyment of work)

If $B(a_1, 0) = 0$ for all a_1 and $B(.)$ is increasing for $(a_1, a_2) >> (0, 0)$, then the optimal contract is characterized by $\alpha_1 = 0$, i.e. fixed wage contract; piece rates rare because of multi-task

Proof:

- $\alpha_1 = 0$ leads to $\max_{a_2} B(\bar{a} - a_2, a_2) - C(\bar{a}) > 0$
- $\alpha_1 > 0$ leads to $a_2 = 0$, hence a negative surplus
- $\alpha_1 < 0$ leads to $a_2 < \bar{a}$ and $a_1 = 0$ with surplus smaller than $B(\bar{a}) - C(\bar{a})$, hence dominated.

Applications: When Agent can allocate effort to production, production output imperfectly observable, and to asset maintainance, asset value difficult to measure.

- "Employment contract": assets belong to the firm. The aggregate surplus maximizing employment contract provides low-power incentives, to avoid reduction in asset value
- "Contract with an independent": assets belong to the Agent. The aggregate surplus maximizing independent contract provides high-power incentives
- Employment contract tends to be optimal for high values of risk-aversion and risks, independent contract for low values: a theory of the firm à la Williamson

Applications for job design:

Allowing or banning external activities that are substitutes to internal activities, that create profit for the Principal: depends upon the availability of signals that can help design high-power incentives for internal activities !

Allocating tasks to different agents or grouping them as a job for one Agent: again, depends upon the observability.

Theory of organizations and hierarchies following Holmström - Milgrom.

V.2. Limited liability models

- Moral hazard models much used in corporate finance: for managerial compensation, shareholders / manager or entrepreneur relationships
- Fact: agents are protected by limited liability, i.e. they are responsible only for the money they put in a venture, not on their personal wealth
- Formally: $w(x) \geq \underline{w}$, often take $\underline{w} = 0$
- Technical consequence: it is not possible to punish Agent as much as wanted to provide incentives
- Partially invalidates our approach
- (Almost) Equivalent to introducing infinite risk aversion of the Agent at level of transfer $\underline{w}$
- So, in fact, address the problem without global risk aversion

V.2. Limited liability models

- Entrepreneur with initial wealth W has a project that requires investment $I > W$
- Project may succeed (profit $R > 0$) or fail (zero profit).
- Effort-dependent probability of success $a \in \{0, 1\}$: $p_0 < p_1$.
- Modelling option:
  - Effort $a = 0$ costless, effort $a = 1$ costs C
  - Effort $a = 0$ means doing something else with non-verifiable private benefits B, $a = 1$ no outside private benefits (choose this interpretation)
- Entrepreneur borrows $I - W$ from investors; all parties risk-neutral
- Assume project is profitable only if $a = 1$:

$$p_1 R - I > 0 > p_0 R + B - I.$$

V.2. Limited liability models

Limited liability assumption

Reimbursements cannot exceed firm's liquidities

- Financial contract: 0 if (verifiable) failure, $R - r$ payback to investors and r as residual compensation for entrepreneur if success : $0 \leq r \leq R$
- Under moral hazard, implementing $a = 1$ imposes:

$$p_1 r \geq p_0 r + B \Longleftrightarrow r \geq \frac{B}{p_1 - p_0}$$

- Limited liability forbids penalizing entrepeneur under failure, hence incentive rent:

$$p_1 r \geq \frac{p_1 B}{p_1 - p_0} > 0 = U_R.$$

- Moral hazard is costly despite risk-neutrality

V.2. Limited liability models

- Look for financial contracts that allow investors to break even (investors' participation constraint):

$$
\begin{aligned}
I - W &\leq p_1 \left(R - \frac{B}{p_1 - p_0} \right) \Longleftrightarrow \\
W &\geq \bar{W} \equiv \frac{p_1 B}{p_1 - p_0} - (p_1 R - I) \\
W &\geq \text{incentive rent - project profitability}
\end{aligned}
$$

- If incentive rent is larger than project profitability, there exist a financial contract that implements $a = 1$ and allows investors to break even only if enough self-financing
- Optimal contract depends on relative bargaining power of both actors
- Moral hazard implies credit market imperfection despite global risk neutrality

Applications of this simple model to many issues in coporate finance: see textbook in corporate finance by Tirole.

Models with limited liability provides a trade-off between rent extraction and incentives, instead of insurance and incentives, but their predictions are very close to more standard models of moral hazard.

And they are much more tractable !

Required readings

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