Principal - Agent model under screening

Microeconomics 2

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I.1. Principal - Agent models

The Principal - Agent model is a simple 2-agent transaction model under asymmetric information:

- The Principal is the Stackelberg leader: she proposes a setting for the transaction (a price, a contract,...) and her offer is a take-it-or-leave-it offer
- The Principal has imperfect / incomplete information on relevant parameters for the transaction (cost, demand, quality)
- The Agent has private information compared to the Principal: the Agent is informed
- The Agent chooses a specific setting to trade or refuses to trade, as a Stackelberg follower, and the transaction is implemented (or the relation ends) according to what was agreed upon

This is the simplest possible two-player framework to analyze transactions under asymmetric information. Building block of any more general model of an economy under asymmetric information

Formalized as a negotiation / bargaining game between two parties: game-theoretical approach justified as private information provides market power (monopoly over the corresponding piece of information), hence a strategic setting

Focus attention: ineffiencies in markets under asymmetric information have their roots at the level of individual transactions

I.2. Information structure: two polar models

Definition: *Moral hazard models*

Also called Principal - Agent models with hidden action, or under imperfect information

- The Agent takes actions, decisions that impact the transaction and (at least) one of the parties' utility
- The Principal cannot perfectly observe all of these actions: she observes only a noisy signal about these actions

Definition: *Screening or adverse selection models*

Also called Principal - Agent models with hidden knowledge, or under incomplete information.

- The Agent has private information on some relevant parameter for the transaction
- The Principal does not know this parameter and has only (non-degenerate) prior beliefs on its value (Bayesian setting)

I.3 Example: Insurance

Insurance and moral hazard

- P, the insurance company, proposes insurance policies and A, the owner of a good that can be damaged chooses one policy
- Suppose the probability of occurrence of an accident depends upon the owner's care, maintenance, caution,...
- Care, caution, maintenance are costly for the owner and very difficult to observe for the insurance company
- If the owner is perfectly insured, he might neglect maintenance and be careless, hence the terminology "moral hazard"; this increases the probability of accident endogenously
- What insurance premium should be charged ? Is full insurance still appropriate ? Which maintainance decisions are chosen ?

Insurance and adverse selection

- P is insurance company, and A, the owner
- Suppose the probability of occurrence of an accident is known by the owner: good's condition, his health condition, his experience / training,...
- The insurance company has only a limited knowledge of these characteristics: perhaps the distribution of risks in the population
- The owner would like to pretend his good is in perfect condition, so as to pay a low premium.
- Can the insurance company select less risky owners ? Is there a way to induce revelation of the owner's private information, perhaps by his choice of a policy ? If full insurance provided?

I.4. Applications under incomplete information

Wide applicability of the basic Principal - Agent model under hidden information:

- Price discrimination: a monopolist tries to extract as much profit from selling a good to a consumer with unknown taste
- Shareholders / manager: pay the manager who has better information about the firm's profitability or opportunities
- Investor / entrepreneur: loan and financial contract for an entrepreneur whose project has unknown profitability
- Optimal regulation: public control over pricing and subsidies to a public utility or a regulated monopolist
- Other ideas ... ?

Basic screening model with binary private information:

- Detailed analysis of the optimal contract and discussion

General framework with richer private information :

- Revelation principle and Taxation Principle
- Implementability analysis
- Characterization and discussion of the optimal contract
- Ex ante vs ex post participation
- Type-dependent reservation utility

Applications: Regulation (Laffont-Tirole), Labor contract, Insurance

- Principal is a monopolist that produces a good of variable quality $q \geq 0$ at cost $C(q)$ per unit of good
- Agent wants to buy one unit of the good; characterized by his taste for quality θ
- If Agent buys quality q at price p,

$$\begin{aligned} \text{Agent/consumer:} \qquad & u(q, p; \theta) = \theta q - p \\ \text{Monopolist:} \qquad & \pi = p - C(q). \end{aligned}$$

- Agent's reservation utility: $U_R = 0$.
- Agent knows his taste θ; private information
- Monopolist has prior beliefs: $\theta \in \{\theta_L, \theta_H\}$ with $\theta_L < \theta_H$ and $\Pr\{\theta = \theta_H\} = f$.

II.2. Complete / full information optimum

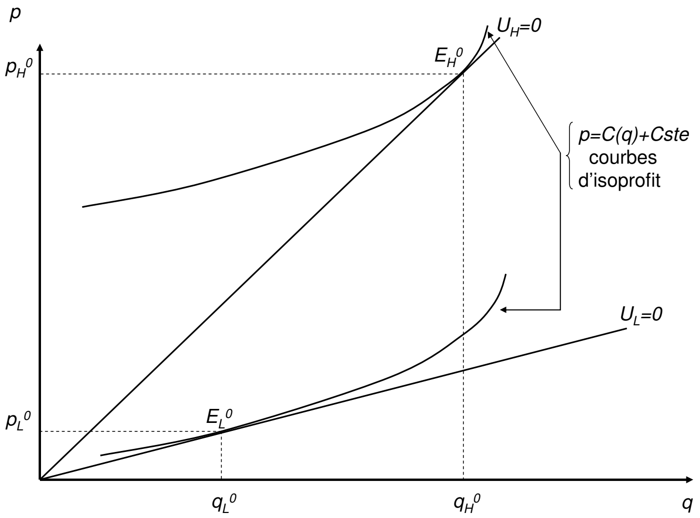
- Under full / complete information (Ex Post Pareto Optimum, i.e. for each θ):

$$\max_{q,p} \{p - C(q)\}$$
$$\text{s.t.:} \qquad U_\theta = \theta q - p \geq 0$$

- Marginal cost of a small increase in quality = marginal benefit: $C'(q^0(\theta)) = \theta$;
- For each θ, consumer has zero net surplus: $p = \theta q$ binding
- Price extracts all surplus from consumer (perfect price discrimination): for each θ, profit equals $\theta q^0(\theta) - C(q^0(\theta))$
- Monopolist proposes 2 products: basic low-quality product $(q_L^0, p_L^0 = \theta_L q_L^0)$, and high-quality product $(q_H^0, p_H^0 = \theta_H q_H^0)$ with:

$$q_L^0 < q_H^0 \text{ and } p_L^0 < p_H^0.$$

II.2. Complete / full information optimum



From now on, assume incomplete information of the monopolist.

- If monopolist proposes same two $(q_L^0, p_L^0 = \theta_L q_L^0)$ and $(q_H^0, p_H^0 = \theta_H q_H^0)$ products, that are optimal under perfect information, despite now incomplete information, consumer θ_H now strictly prefers the low-quality product to the high-quality product:

$$\theta_H q_L^0 - p_L^0 = (\theta_H - \theta_L) q_L^0 > 0 = \theta_H q_H^0 - p_H^0 \quad !!$$

- The monopolist only sells the low-quality product and makes expected profit equal to: $\theta_L q_L^0 - C(q_L^0)$.
- (An alternative for the monopolist is to only offer the high-quality full information optimal product $(q_H^0, p_H^0 = \theta_H q_H^0)$. He would make expected profit equal to: $f(\theta_H q_H^0 - C(q_H^0))$. Depending on parameters, either can dominate)
- Could the monopolist do better than this naive offer?

- The monopolist could rather propose the same low-quality product and the high-quality product charged *at a lower price* (q_H^0, p'_H) with $p'_H < p_H^0$ determined such that:

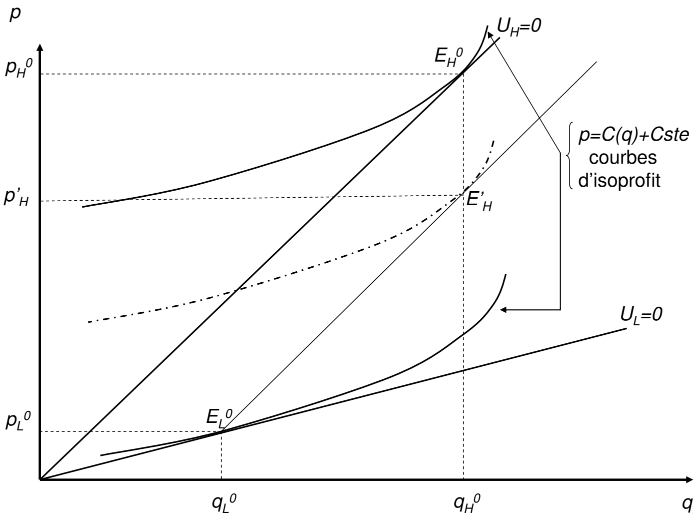
$$\theta_H q_L^0 - p_L^0 = (\theta_H - \theta_L) q_L^0 = \theta_H q_H^0 - p'_H$$

- For this price, market is segmented with both full information optimal qualities sold: θ consumers buy quality $q^0(\theta)$
- On θ_H consumers, monopolist would earn:

$$\begin{aligned} p'_H - C(q_H^0) &= \theta_H q_H^0 - C(q_H^0) - (\theta_H - \theta_L) q_L^0 \\ &= \max_q \left(\theta_H q - C(q) - (\theta_H - \theta_L) q_L^0\right) \\ &> \theta_H q_L^0 - C(q_L^0) - (\theta_H - \theta_L) q_L^0 = p_L^0 - C(q_L^0) \end{aligned}$$

i.e. more than under the full information optimal policy.

- With unchanged profit on θ_L consumers, this policy is better than the naive (full information optimal) policy, when there is incomplete information. Are there even better policies?


p
$p_H{}^0$
p'_H
$p_L{}^0$
$E_H{}^0$
U_H=0
E'_H
$E_L{}^0$
U_L=0
p=C(q)+Cste
courbes
d'isoprofit
$q_L{}^0$
$q_H{}^0$
q

- Price Discrimination: monopolist maximizes profit in proposing 2 products that segment the market, separate consumers with different tastes
- Condition for products (q_L, p_L) and (q_H, p_H) to separate consumers:

$$\begin{aligned} \theta_L q_L - p_L &\geq \theta_L q_H - p_H \quad (\mathrm{IC}_L) \\ \theta_H q_H - p_H &\geq \theta_H q_L - p_L \quad (\mathrm{IC}_H) \end{aligned}$$

- Incentive constraints or revelation constraints: consumer θ picks up the product that he is supposed to pick up rather than another one
- And of course, participation constraints remain:

$$\begin{aligned} \theta_L q_L - p_L &\geq 0 \quad (\mathrm{IR}_L) \\ \theta_H q_H - p_H &\geq 0 \quad (\mathrm{IR}_H) \end{aligned}$$

So, the profit-maximizing market segmentation policy that consists in serving all consumers corresponds to two products, (q_L, p_L) and (q_H, p_H), that solve:

Profit maximizing discrimination

$$\begin{array}{rcl} \max\limits_{q_L, p_L, q_H, p_H} f\left[p_H - C(q_H)\right] & + & (1-f)\left[p_L - C(q_L)\right] \\ \theta_L q_L - p_L & \geq & \theta_L q_H - p_H \quad (\mathrm{IC}_L) \\ \theta_H q_H - p_H & \geq & \theta_H q_L - p_L \quad (\mathrm{IC}_H) \\ \theta_L q_L - p_L & \geq & 0 \quad (\mathrm{IR}_L) \\ \theta_H q_H - p_H & \geq & 0 \quad (\mathrm{IR}_H) \end{array}$$

Note: monopolist could decide not to serve some consumers (exclusion policy) or to offer the same product to all consumers (non-segmentation, non-discriminatory policy); see later.

$$\max_{q_L, p_L, q_H, p_H} f\left[p_H - C(q_H)\right] + (1-f)\left[p_L - C(q_L)\right]$$

$$\begin{aligned}
\theta_L q_L - p_L &\geq \theta_L q_H - p_H \quad (\mathrm{IC}_L)\\
\theta_H q_H - p_H &\geq \theta_H q_L - p_L \quad (\mathrm{IC}_H)\\
\theta_L q_L - p_L &\geq 0 \quad (\mathrm{IR}_L)\\
\theta_H q_H - p_H &\geq 0 \quad (\mathrm{IR}_H)
\end{aligned}$$

II.3. Incomplete information

$$\max_{q_L, p_L, q_H, p_H} f\,[p_H - C(q_H)] + (1-f)\,[p_L - C(q_L)]$$

$$\theta_L q_L - p_L \geq \theta_L q_H - p_H \quad (\mathrm{IC}_L)$$

$$\theta_H q_H - p_H \geq \theta_H q_L - p_L \quad (\mathrm{IC}_H)$$

$$\theta_L q_L - p_L \geq 0 \quad (\mathrm{IR}_L)$$

$$\theta_H q_H - p_H \geq 0 \quad (\mathrm{IR}_H)$$

- IR_H non-binding since:

$$\theta_H q_H - p_H \geq \theta_H q_L - p_L \geq \theta_L q_L - p_L \geq 0.$$

θ_H earns positive surplus: informational rent

$$\max_{q_L,p_L,q_H,p_H} f\left[p_H - C(q_H)\right] + (1-f)\left[p_L - C(q_L)\right]$$
$$\theta_L q_L - p_L \geq \theta_L q_H - p_H \quad (\mathrm{IC}_L)$$
$$\theta_H q_H - p_H \geq \theta_H q_L - p_L \quad (\mathrm{IC}_H)$$
$$\theta_L q_L - p_L \geq 0 \quad (\mathrm{IR}_L)$$

- IR_H non-binding since:

$$\theta_H q_H - p_H \geq \theta_H q_L - p_L \geq \theta_L q_L - p_L \geq 0.$$

θ_H earns positive surplus: informational rent

$$\max_{q_L,p_L,q_H,p_H} f\left[p_H - C(q_H)\right] + (1-f)\left[p_L - C(q_L)\right]$$
$$\theta_L q_L - p_L \geq \theta_L q_H - p_H \quad (\mathrm{IC}_L)$$
$$\theta_H q_H - p_H \geq \theta_H q_L - p_L \quad (\mathrm{IC}_H)$$
$$\theta_L q_L - p_L \geq 0 \quad (\mathrm{IR}_L)$$

- IR_H non-binding since:

$$\theta_H q_H - p_H \geq \theta_H q_L - p_L \geq \theta_L q_L - p_L \geq 0.$$

  θ_H earns positive surplus: informational rent
- IC_H binds necessarily: p_H as large as possible while maintaining the choice of high-quality product

$$\max_{q_L, p_L, q_H, p_H} f\left[p_H - C(q_H)\right] + (1-f)\left[p_L - C(q_L)\right]$$
$$\theta_L q_L - p_L \geq \theta_L q_H - p_H \quad (\mathrm{IC}_L)$$
$$\theta_H q_H - p_H = \theta_H q_L - p_L \quad (\mathrm{IC}_H)$$
$$\theta_L q_L - p_L \geq 0 \quad (\mathrm{IR}_L)$$

- IR_H non-binding since:
  $$\theta_H q_H - p_H \geq \theta_H q_L - p_L \geq \theta_L q_L - p_L \geq 0.$$
  θ_H earns positive surplus: informational rent
- IC_H binds necessarily: p_H as large as possible while maintaining the choice of high-quality product

$$\begin{aligned}
\max_{q_L,p_L,q_H,p_H} f\left[p_H - C(q_H)\right] \quad &+ \quad (1-f)\left[p_L - C(q_L)\right] \\
\theta_L q_L - p_L \quad &\geq \quad \theta_L q_H - p_H \quad (\mathrm{IC}_L) \\
\theta_H q_H - p_H \quad &= \quad \theta_H q_L - p_L \quad (\mathrm{IC}_H) \\
\theta_L q_L - p_L \quad &\geq \quad 0 \quad (\mathrm{IR}_L)
\end{aligned}$$

- IC_L et IC_H imply $q_L \leq q_H$ and IC_L non-binding since:

$$(q_H - q_L)\theta_L \leq p_H - p_L = (q_H - q_L)\theta_H$$

$$\max_{q_L, p_L, q_H, p_H} f\left[p_H - C(q_H)\right] \quad + \quad (1-f)\left[p_L - C(q_L)\right]$$

$$\begin{aligned} \theta_H q_H - p_H &= \theta_H q_L - p_L \quad (\mathrm{IC}_H) \\ \theta_L q_L - p_L &\geq 0 \quad (\mathrm{IR}_L) \end{aligned}$$

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$$(q_H - q_L)\theta_L \leq p_H - p_L = (q_H - q_L)\theta_H$$
- IR_L binding: zero surplus for θ_L.

$$\max_{q_L,p_L,q_H,p_H} f\,[p_H - C(q_H)] \quad + \quad (1-f)\,[p_L - C(q_L)]$$

$$\begin{aligned} \theta_H q_H - p_H \quad &= \quad \theta_H q_L - p_L \quad (\mathrm{IC}_H) \\ \theta_L q_L - p_L \quad &= \quad 0 \quad (\mathrm{IR}_L) \end{aligned}$$

- IC_L et IC_H imply $q_L \leq q_H$ and IC_L non-binding since:
$$(q_H - q_L)\theta_L \leq p_H - p_L = (q_H - q_L)\theta_H$$
- IR_L binding: zero surplus for θ_L.
- Hence the expression of informational rent of θ_H:
$$R(q_L) = \theta_H q_L - p_L = (\theta_H - \theta_L)q_L \nearrow \text{ w.r.t. } q_L.$$

The program boils down to:

$$\begin{array}{ll} \max & f\left[\theta_H q_H - C(q_H) - R(q_L)\right] + (1-f)\left[\theta_L q_L - C(q_L)\right] \\ \text{s.t.} & q_L \leq q_H \text{ (monotonicity constraint).} \end{array}$$

Maximand can be written as the difference between

- the expected aggregate surplus:

$$f\left[\theta_H q_H - C(q_H)\right] + (1-f)\left[\theta_L q_L - C(q_L)\right]$$

- and the expected rent to be left to the Agent (left only for agent of type θ_H): $fR(q_L)$

Efficiency - Rent extraction tradeoff.

Omitting the monotonicity constraint for the moment:

- Maximize aggregate surplus when $\theta = \theta_H$, i.e. $\theta_H q_H - C(q_H)$:
$$C'(q_H) = \theta_H \Leftrightarrow q_H = q_H^0$$
- Maximizes aggregate surplus when $\theta = \theta_L$ corrected by the expected rent to be left, i.e. $\theta_L q_l - C(q_L) - \frac{f}{1-f} R(q_L)$:
$$C'(q_L) = \theta_L - \frac{f}{1-f}(\theta_H - \theta_L) < \theta_L \Rightarrow q_L < q_L^0$$
if interior, otherwise $q_L = 0$.

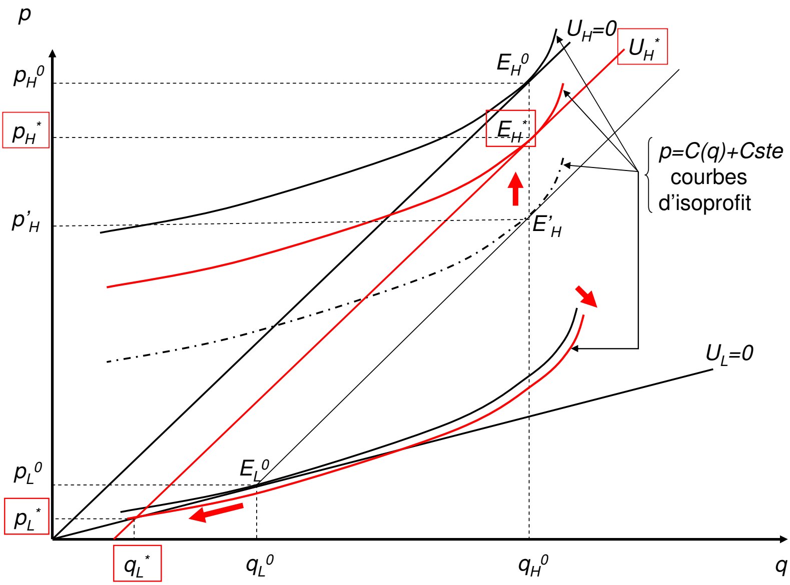
Monotonicity constraint is satisfied since: $q_L < q_L^0 < q_H^0 = q_H$

II.4. Standard results under incomplete information

Classical results from the binary model

- Zero surplus for θ_L-consumers, $U_L = 0$
- **Informational rent** $U_H = R(q_L^*)$ left to θ_H-consumer, necessary to obtain information revelation about θ_H.
- Efficient high quality for high-taste consumer (**no distortion at the top**): $q_H^* = q_H^0$
- **Inefficiency at the bottom**. Low quality is sub-optimal for low-taste consumer: $q_L^* < q_L^0$

Intuition: Conflict between efficiency and rent extraction, to reduce rent $R(q_L^0)$, provide (ex post) suboptimal quality q_L


p
p_H^0
p_H^*
p'_H
p_L^0
p_L^*
U_H=0
U_H^*
E_H^0
E_H^*
E'_H
E_L^0
p=C(q)+Cste
courbes
d'isoprofit
U_L=0
q_L^*
q_L^0
q_H^0
q

Non-discriminatory policies: $q_L = q_H$ is possible in program above and (IC) then imply $p_L = p_H$ and therefore unique product (q, p) for both types.

- Result above proves that the optimal policy is necessarily discriminatory in this model.
- Note: profit-maximizing **non-discriminatory** policy that serves all types of consumers is obviously (q_L^0, p_L^0)

Policies that exclude some type: what if omit (IR_L) and look for a policy (q_H, p_H) that only serves θ_H consumers?

- No difference between exclusion and $(q_L = 0, p_L = 0)$ here!
- If exclusion of θ_L (or $q_L = 0$), θ_H consumers are served their full information optimal high-quality product and get zero surplus (informational rent vanishes)

Exclusion in a more general setting: Consumers's alternative option to buying provides positive reservation utility.

- (IR_θ) becomes: $U_\theta = \theta q_\theta - p_\theta \geq U_R > 0$.
- Now a difference between exclusion and zero-quality!
- Assume full information optimal policy does not exclude consumers: $p^0_\theta = \theta q^0_\theta - U_R \geq C(q^0_\theta)$

Profit-maximizing policy **without exclusion** same as before except that prices lower by U_R and monopolist's profit is:

$$f(\theta_H q^0_H - C(q^0_H)) + (1-f)\max_{q_L}\{\theta_L q_L - C(q_L) - \frac{f}{1-f}R(q_L)\} - U_R$$

Profit maximizing policy **with exclusion** of θ_L consumers is $(q^0_H, p^0_H - U_R)$ and yields monopolist's profit:

$$f(\theta_H q^0_H - C(q^0_H)) - fU_R$$

Optimality of exclusion

Exclusion is profit-maximizing optimal if:

$$\max_{q_L}\{\theta_L q_L - C(q_L) - \frac{f}{1-f}R(q_L)\} < U_R$$

When the outside option cannot be replicated by an admissible policy (e.g. the null policy), incomplete information may lead to an extreme form of inefficiency at the bottom: exclusion, shut-down, ...

Non-exclusion under perfect information and exclusion of θ_L under incomplete information are compatible since:

$$\max_{q_L}\{\theta_L q_L - C(q_L) - \frac{f}{1-f}R(q_L)\} < \max_{q_L}\{\theta_L q_L - C(q_L)\}$$

III. General framework

Extend the basic two-type model to a richer (continuum of types) framework, robustness check of the previous results:

- Discrete model: Informational rent for top type, no rent for bottom type. Richer model: informational for a.a. types ?
- Discrete model: efficiency at top type, inefficiency at bottom type. Richer model: efficiency a.e., inefficiency a.e., both with positive measure?

Extend the basic linear-utility model to a more general framework, robustness check of the previous results:

- Form of the informational rent
- Form of possible inefficiency and interaction with the "monotonicity" constraint

III.1. General model

- Transaction between Principal and Agent is about:
  - A verifiable action / variable, denoted $x \in \mathbf{R}_+$ (in a compact convex set of $\mathbf{R}_+$),
  - a payment $w \in \mathbf{R}$
- Agent has private information about a payoff-relevant parameter $\theta \in \Theta = [\theta_L, \theta_H] \subset \mathbf{R}_+$
- Bayesian approach: Principal does not know θ and has prior beliefs $F(.), f(.)$ over Θ
- Principal's preferences C^2: $V = v(x, \theta) - w$
- Agent's preferences C^3: $U = w + u(x, \theta)$
- Surplus $S(x, \theta) = v(x, \theta) + u(x, \theta)$ assumed concave in x
- Type-independent reservation utility: $U_R(\theta) = U_R = 0$

Benchmark case: θ is public information, known by both parties and by a lawyer: perfect information setting.

- Ex ante Pareto program:

$$(w^0(\theta), x^0(\theta)) \in \quad \arg\max_{w,x} (v(x,\theta) - w) \\ w + u(x,\theta) \geq 0$$

- Participation constraint is obviously binding:

$$w^0(\theta) = -u(x^0(\theta),\theta) \Leftrightarrow U^0(\theta) \equiv w^0(\theta) + u(x^0(\theta),\theta) = 0$$

- Ex post efficiency (maximize surplus from transaction):

$$x^0(\theta) \in \arg\max_x S(x,\theta)$$

III.3. Contracts and revelation principle

- Principal proposes a compensation mode, a contract: how all verifiable variables $y = (w, x)$ are determined
- Agent is informed; good idea to let him "tell" his θ, or leave him some discretion

Definition of a mechanism

A mechanism is game form between Principal and Agent: set of strategies M for the Agent, and outcome function $g(.)$ from M to the set of allocations y: $y(m) = (x(m), w(m))$

- Non-linear price function, i.e. $w = W(x)$, Agent chooses quantity x (M = set of x) under price schedule $W(.)$
- Communication game: Agent sends message in M
- Announcement of information $\tilde{\theta}$ ($M = \Theta$) (direct mech.)
- Designed so that announcement is truthful, i.e. Agent announces the true θ (DRM: direct revelation mech.)

- In mechanism $(M, y(.))$, Agent chooses (pure strategy):

$$m^*(\theta) \in \arg\max_{m \in M} \left(w(m) + u(x(m), \theta)\right)$$

- Consider new mechanism $(\Theta, Y(.) = y(m^*(.)))$; if for some θ, Agent prefers announcing $\tilde{\theta} \neq \theta$ to θ:

$$\begin{aligned} W(\tilde{\theta}) + u(X(\tilde{\theta}), \theta) &> W(\theta) + u(X(\theta), \theta) \Leftrightarrow \\ w(m^*(\tilde{\theta})) + u(x(m^*(\tilde{\theta})), \theta) &> w(m^*(\theta)) + u(x(m^*(\theta)), \theta) \end{aligned}$$

which contradicts m^* as equilibrium !

- Moreover, for any θ, $Y(\theta) = y(m^*(\theta))$ so that the same outcome prevails

III.3. Contracts and revelation principle

Revelation Principle

For any general contract, there exists a DRM that yields the same equilibrium outcome (same ex post utility for A, same expected utility for P, same transaction for each θ)

- Therefore: can restrict attention to contracts such that Agent has incentives to truthfully reveal his type
- Two-step resolution:
  - Implementability: characterize the set of DRM
  - Optimization: characterize the best DRM for Principal

Taxation Principle

For any DRM $(x(.), w(.))$ and associated outcome, there exists an equivalent non-linear schedule $w = \phi(x)$, with $\phi(x) = w(\theta)$ for $x = x(\theta)$ and $\phi(x) = -\infty$ if there exists no θ such that $x = x(\theta)$

Spence-Mirrlees or single-crossing assumption

$\frac{\partial^2 u}{\partial x \partial \theta}$ has a constant sign on the whole domain (say positive, to fix ideas)

- In (x, w)-plane, slope $\frac{dw}{dx} = -\partial_x u(x, \theta)$ of Agent's iso-utility curves decrease when θ increases
- Therefore, iso-utilities for θ and θ' cross only once and always in the same position
- **Intuition**: willingness to increase dx is larger for larger θ, hence the possibility of separating θ' from θ by offering dw large enough for θ' and too small for θ (Draw picture)

A mechanism $(x(.), w(.))$ is truthful iff:

$$\begin{aligned} \forall \theta \in \Theta, \theta \in \quad & \arg\max_{\theta' \in \Theta} \left(w(\theta') + u(x(\theta'), \theta) \right) \\ U(\theta) \equiv w(\theta) + u(x(\theta), \theta) \quad & = \max_{\theta' \in \Theta} \left(w(\theta') + u(x(\theta'), \theta) \right) \end{aligned}$$

These are called the revelation constraints, or the incentive compatibility constraints

Theorem: characterization of implementability

Under the Spence-Mirrlees condition in the general one-dimensional real-valued model, a mechanism $(x(.), w(.))$ is a DRM iff $x(.)$ is non-decreasing and

$$U(\theta) = U(\theta_L) + \int_{\theta_L}^{\theta} \partial_\theta u(x(s), s) ds$$

Proof: if-part

- Write down revelation constraints for 2 types θ and θ':

$$\begin{aligned} U(\theta) &\geq U(\theta') + u(x(\theta'),\theta) - u(x(\theta'),\theta') \\ U(\theta') &\geq U(\theta) + u(x(\theta),\theta') - u(x(\theta),\theta) \end{aligned}$$

- This is equivalent to:

$$u(x(\theta),\theta') - u(x(\theta),\theta) \leq U(\theta') - U(\theta) \leq u(x(\theta'),\theta') - u(x(\theta'),\theta)$$

- Which implies:

$$\int_{x(\theta)}^{x(\theta')} \int_{\theta}^{\theta'} \partial^2_{x\theta} u(x,s)\,ds\,dx \geq 0$$

- hence monotonicity of $x(.)$ and form of $\frac{dU}{d\theta}$

Proof: only-if-part

- Let $U(\theta';\theta)$ denote the utility of pretending to be θ' when the true type is θ. Given the integral form of $U(.)$,

$$\begin{aligned}
U(\theta) - U(\theta';\theta) &= U(\theta) - U(\theta') + u(x(\theta'),\theta') - u(x(\theta'),\theta) \\
&= -\int_{\theta}^{\theta'} \partial_\theta u(x(s),s)ds + \int_{\theta}^{\theta'} \partial_\theta u(x(\theta'),s)ds \\
&= \int_{\theta}^{\theta'} \int_{x(s)}^{x(\theta')} \partial^2_{x\theta} u(x,s)dxds
\end{aligned}$$

- Monotonicity implies that for $\theta < s < \theta'$, $x(s) \leq x(\theta')$. The Spence-Mirrlees condition imples then:

$$U(\theta) - U(\theta';\theta) \geq 0$$

Similar for $\theta > \theta'$, $x(s) \geq x(\theta')$. Overall revelation of θ.

III.4. Implementability analysis

The integral expression for $U(\theta)$ depends upon parameter $U(\theta_L)$, to be characterized in the optimization analysis.

In fact, it is a characterization of the derivative of $U(.)$, which can be directly obtained applying the envelope theorem on the revelation program:

$$U(\theta) = \max_{\theta' \in \Theta} \left(w(\theta') + u(x(\theta'), \theta) \right) \Rightarrow \frac{dU}{d\theta}(\theta) = \partial_\theta u(x(\theta), \theta) \qquad \text{(a.e.)}$$

.

Alternatively, with parameter $U(\theta_H)$:

$$U(\theta) = U(\theta_H) - \int_\theta^{\theta_H} \partial_\theta u(x(s), s) ds$$

Convenient to choose one form or the other depending upon the sign of $\partial_\theta u$ (see below)

Under the Spence - Mirrlees condition, the profit-maximizing mechanism is the solution of:

$$\max_{x(.),U(.)} \quad \int_{\theta_L}^{\theta_H} \left(S(x(\theta),\theta) - U(\theta)\right) f(\theta) d\theta$$

$$U(\theta) = U(\theta_L) + \int_{\theta_L}^{\theta} \partial_\theta u(x(s),s) ds$$

$x(.)$ non-decreasing

$$U(\theta) \geq 0$$

Remark: Efficiency - Rent extraction tradeoff again

Existence: Given the smoothness assumption on $v(.)$ and $u(.)$, if $x(.)$ can be a priori bounded, an optimal mechanism exists

Normalization assumption on θ: $\partial_\theta u(x,\theta) \geq 0$ over the whole domain.

Assumption close to Spence - Mirrlees: utility and marginal utility of an increase of quality co-monotone in θ. It can be viewed as a normalization of θ.

Under this assumption:

- $U(\theta)$ is non-decreasing in θ.
- So, the set of (IR) constraints, $U(\theta) \geq 0$, (one for each θ) can be reduced to the equivalent unique constraint: $U(\theta_L) \geq 0$.

Normalization of θ and the choice of one integral form for the revelation constraint: if assume instead $\partial_\theta u(x,\theta) \leq 0$, the (IR) reduces to $U(\theta_H) \geq 0$.

Plugging the integral form for $U(\theta)$ into the integrand, the objectives becomes:

$$\int_{\theta_L}^{\theta_H} \left(S(x(\theta),\theta) - \int_{\theta_L}^{\theta} \partial_\theta u(x(s),s)ds \right) f(\theta)d\theta - U(\theta_L)$$

After integration-by-parts of the term $\left\{ \int_{\theta_L}^{\theta} \partial_\theta u(x(s),s)ds \right\} \{-f(\theta)\}$, the objectives becomes:

$$\int_{\theta_L}^{\theta_H} \left(S(x(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)} \partial_\theta u(x(\theta),\theta) \right) f(\theta)d\theta - U(\theta_L)$$

III.5. Optimal contract

Define the virtual surplus:

$$\Omega(x,\theta) \equiv S(x,\theta) - \frac{1-F(\theta)}{f(\theta)}\partial_\theta u(x,\theta)$$

The correction captures the (first order) revelation constraint.

The profit-maximizing mechanism solves:

$$\max_{x(.)} \quad \int_{\theta_L}^{\theta_H} \Omega(x(\theta),\theta) f(\theta) d\theta - U(\theta_L)$$
$$x(.) \text{ non-decreasing and } U(\theta_L) \geq 0$$

- Obviously $U(\theta_L) = 0$ at the optimum.
- Let $X(\theta) \equiv \arg\max_x \Omega(x,\theta)$ denote the virtual surplus maximizing allocation, omitting the monotonicity constraint

Optimal contract

Under Spence-Mirrlees and normalization condition, assume that $\Omega(., \theta)$ is quasi-concave, if the unconstrained virtual surplus maximizer $X(.)$ is non-decreasing, the optimal contract is $x^*(\theta) = X(\theta)$ for any θ and:

$$w^*(\theta) = \int_{\theta_L}^{\theta} \partial_\theta u(X(s), s) ds - u(X(\theta), \theta)$$

Natural sufficient conditions for quasi-concavity: $\partial_{xx} u \leq 0$ and $\partial_{xx} v \leq 0$ (standard), and $\partial_{xx\theta} u \geq 0$ (more demanding, usually OK with specifications linear in θ).

Sufficient additional conditions for monotonicity:

- On utilities: $\partial_{x\theta\theta} u \leq 0$ (demanding, OK when utility is linear in θ) and $\partial_{x\theta} v \geq 0$ (Spence - Mirrlees on $v(.)$, demanding, see Guesnerie-Laffont if not satisfied)
- and Monotone Hazard Rate Property (MHRP): $\frac{1-F(\theta)}{f(\theta)}$ decreasing, or $1 - F(.)$ log-concave (OK with usual distributions)
- With strict version of assumptions, optimal contract is strictly separating: $x^*(.)$ is invertible.

In practice, one first solves the relaxed program (omitting the monotonicity constraint) and then one checks a posteriori that the solution satisfies the monotonicity constraint.

More math-oriented route: solve the optimal control problem !

III.5. Optimal contract

Agent earns an informational rent for all $\theta > \theta_L$:

$$U(\theta) = R(\theta) \equiv \int_{\theta_L}^{\theta} \partial_\theta u(x^*(s), s)ds > 0$$

$R(\theta)$ increasing in $x^*(s)$ for (interval of) types $s < \theta$. To reduce informational rent: $x^*(\theta)$ for all $\theta < \theta_H$ is ex post inefficient, downward distortion (but no distortion at the top):

$$\begin{aligned} \partial_x S(x^*(\theta), \theta) &= \frac{1 - F(\theta)}{f(\theta)} \partial^2_{x\theta} u(x^*(\theta), \theta) > 0 \\ \Leftrightarrow x^*(\theta) &< x^0(\theta) \end{aligned}$$

Intuition: decrease dx in x^* around θ

- reduces surplus by: $\partial_x S(x^*(\theta), \theta) f(\theta) dx d\theta$
- reduces rent of all $\theta' > \theta$ by: $(1 - F(\theta))\partial^2_{x\theta} u(x^*(\theta), \theta) dx d\theta$

Exclusion / shutdown of some types

What if $\max_x \Omega(x, \theta) < U_R$ for some θ? The Principal would be better off excluding some type...

Assume θ is excluded, and earns the reservation utility U_R, and θ' is not: incentive compatibility requires:

$$U(\theta') \geq U_R \geq U(\theta') + u(x(\theta'), \theta) - u(x(\theta'), \theta')$$

With $\partial_\theta u > 0$, this double inequality implies that $\theta \leq \theta'$.

The set of excluded type is an interval $[\theta_L, \theta^*]$.

Within $[\theta^*, \theta_H]$, the characterization theorem remains valid so that the program with **optimal** shutdown is:

$$\max_{x(.),\theta^*} \quad \int_{\theta^*}^{\theta_H} \Omega(x(\theta), \theta) f(\theta) d\theta - U(\theta^*)$$

$$x(.) \text{ non-decreasing and } U(\theta^*) \geq U_R$$

Optimal contract with shutdown / exclusion

Under Spence-Mirrlees and normalization condition, assume that $\Omega(., \theta)$ is quasi-concave, that $\partial_\theta \Omega > 0$ and that $\max_x \Omega(x, \theta) < U_R$ for some θ, then if $X(.)$ is non-decreasing, the optimal contract is given by $x^*(\theta) = X(\theta)$ for any $\theta \in [\theta^*, \theta_H]$ and excludes types within $[\theta_L, \theta^*]$, where θ^* solves:

$$\Omega(X(\theta^*), \theta^*) = U_R$$

Non-perfectly separating contract: bunching

What if $X(.)$ is sometimes decreasing in θ ? That is, if $X(.)$ does not satisfy the monotonicity constraint.

- Must take explicitly into account the constraint $\frac{dx}{d\theta}(\theta) \geq 0$
- Optimal control problem:

$$\max_{x(.),c(.))} \quad \int_{\theta_L}^{\theta_H} \Omega(x(\theta), \theta) f(\theta) d\theta$$

$$c(\theta) = \frac{dx}{d\theta}(\theta) \text{ and } c(\theta) \geq 0$$

- Use Hamiltonian technique with co-state variable.

The solution $x^*(.)$ is continuous with our assumptions, and piecewise differentiable.

When $x^*(.)$ strictly increasing on some interval, i.e. when the monotonicity constraint does not bind, then $x^*(.) = X(.)$ the unconstrained solution.

Otherwise, $x^*(.)$ is constant: there is bunching, i.e. locally no perfect discrimination, no perfect separation

The optimal allocation pieces together strictly increasing branches of $X(.)$ and flat (non-discriminatory) parts (See Guesnerie-Laffont). Draw picture.

- The screening model rests on the assumption that Agent is informed when deciding upon participation
- What if Agent is not informed about θ but privately learns θ after having signed the contract
- Example: information bears on external parameters that Agent privately discovers once hired
- Agent decides upon participation ex ante:

$$\int_{\theta_L}^{\theta_H} U(\theta) f(\theta) d\theta \geq U_R$$

- Once contract signed, Agent commits to abide by its ruling, even if ex post it means negative utility

IV.1. Ex ante contracting

- Assume concavity of Ω, MHRP, $X(.)$ non-decreasing ...
- Optimization program:

$$\max_{x(.),U(.)} \quad \int_{\theta_L}^{\theta_H} \left(S(x(\theta),\theta) - U(\theta)\right) f(\theta)d\theta$$

$$U(\theta) = U(\theta_L) + \int_{\theta_L}^{\theta} \partial_\theta u(x(s),s)ds$$

$$x(.) \text{ non-decreasing}$$

$$\int_{\theta_L}^{\theta_H} U(\theta)f(\theta)d\theta \geq U_R$$

- Ex ante IR is binding, this leave:

$$\max_{x(.)} \int_{\theta_L}^{\theta_H} S(x(\theta),\theta)f(\theta)d\theta - U_R$$

IV.1. Ex ante contracting

- Note first that $\partial_{x\theta} S > 0$ implies that $x^0(.)$ is increasing
- So, solution is therefore $x^0(.)$
- And $U(.)$ is determined by

$$U(\theta_L) + \int_{\theta_L}^{\theta_H} \left(\int_{\theta_L}^{\theta} \partial_\theta u(x^0(y), y) dy \right) f(\theta) d\theta = U_R$$

- Ex ante symmetric information, although ex post asymmetric
- $\frac{dU}{d\theta}$ given by IC, but $U(\theta_L)$ free: can be adjusted so that ex ante IR is binding
- Ex ante contracting does not imply ex post inefficiency
- (If however $\partial_{x\theta} S < 0$, $x^0(.)$ has not the right monotonicity (Guesnerie-Laffont): bunching appears !)

Important simplification: outside option for the agent does not depend on his type. We now relax this assumption in the two-type model of section II (See Jullien for the difficult continuous case)

Assume: $U_R(\theta_L) = 0 \leq \mu = U_R(\theta_H)$, i.e. consumer θ_H has better alternative than the $(q - 0, p = 0)$ product.

Profit-maximizing policy solves, using $U_\theta = \theta q_\theta - p_\theta$,

$$\begin{array}{rcl} \max\limits_{q_L, U_L, q_H, U_H} & f & [\theta_H q_H - C(q_H) - U_H] + (1-f)[\theta_L q_L - C(q_L) - U_L] \\ U_L & \geq & U_H - R(q_H) \quad (\mathrm{IC}_L) \\ U_H & \geq & U_L + R(q_L) \quad (\mathrm{IC}_H) \\ U_L & \geq & 0 \quad (\mathrm{IR}_L) \\ U_H & \geq & \mu \quad (\mathrm{IR}_H) \end{array}$$

As long as $\mu \leq R(q_L^*)$, previous solution remains unchanged !

But if $\mu > R(q_L^*)$, our approach does not work: fails at the first step, i.e. (IC_H) and (IRL) do not imply that (IR_H) is slack !

Simple case with efficiency: Assume $R(q_L^0) \leq \mu \leq R(q_H^0)$.

- Full information optimal qualities, q_L^0 and q_H^0, maximize respectively the surplus when θ_L or θ_H
- Binding-participation utilities, $U_L = 0$ and $U_H = \mu$ minimize the rent left to the agent
- Altogether, they satisfy incentive constraints since $U_H - U_L = \mu$ and $R(q_L^0) \leq \mu \leq R(q_H^0)$
- The fully efficient qualities and the reservation utilities constitute the optimal policy. There is no distortion at all !

Intuitive (and sketchy) approach when $R(q_L^*) < \mu < R(q_L^0)$

- Suppose that (IR_L) is binding ($U_L = 0$) and (IC_L) is slack: (IC_H) and (IR_H) write as: $U_H = \sup\{R(q_L), \mu\}$
- If only one of them binds, either standard analysis that leads to $U_H = R(q_L^*)$ or previous efficient case that leads to $U_H = \mu$ and q_L^0.
- If $R(q_L^*) < \mu < R(q_L^0)$, contradiction! Hence, both bind.
- The solution is given by $U_H = \mu$, $U_L = 0$, $R(q_L) = U_H - U_L = \mu$ and obviously $q_H = q_H^0$.
- With these, (IC_L) is indeed slack.
- $q_L = R^{-1}(\mu) > q_L^*$, no better q_L by concavity
- There is still inefficiency as q_L is distorted downwards, but less than when $\mu = 0$

IV.2. Type-dependent reservation utility

What happens when $\mu > R(q_H^0)$?

Suppose μ very large, consumer θ_H has high reservation utility

- U_H cannot be reduced below μ, downward distortion of q_L not needed anymore! I.e. $U_H \geq \mu > U_L + R(q_L)$.
- So, (IC_L) must bind! $U_H - U_L = R(q_H)$.
- Suppose (IR_L) is slack: then $U_H = \mu$, $U_L = \mu - R(q_H)$, plugged into objectives:

$$f\left[\theta_H q_H - C(q_H)\right] + (1-f)\left[\theta_L q_L - C(q_L) + R(q_H)\right] - \mu$$

- Efficient quality q_L^0 for θ_L and upward distortion $q_H^{**} > q_H^0$, i.e. inefficiency, for θ_H with:

$$q_H^{**} = \arg\max_{q_H} \left[\theta_H q_H - C(q_H) + \frac{1-f}{f} R(q_H)\right]$$

(IR_L) has to be checked with this candidate policy:

- If $\mu \geq R(q_H^{**})$, then (IR_L) is indeed satisfied
- If $R(q_H^0) < \mu < R(q_H^{**})$, (IR_L) would be violated
- In this later case, (IR_L), as well as (IC_L) and (IR_H) bind and we obtain: $R(q_H) = \mu$ (Summary on board)

In both cases, the optimal policy involves inefficient quality provision for θ_H (distortion upwards) and efficient quality provision for θ_L: reverse picture than when $\mu = 0$!

"Top" and 'bottom" not determined by index "H" or "L" ! "Top" (i.e. no distortion, rent above reservation utility) corresponds to the type that would deviate from truthfull revelation under the full information, full participation policy $(q_L^0, U_R(\theta_L), q_H^0, U_R(\theta_H))$

Optimal regulation of monopoly (A) by regulatory agency (P), concerned with social welfare: fixing market failure requires knowledge of firm's technology, which is private information.

Fruitfull field for the theory of screening. Baron-Myerson obtain deviation from marginal-cost pricing with:

- marginal cost is private information,
- contract specifies quantity to produce (equivalently price to charge) and subsidy / transfert.

Regulatory agencies observe firms' costs (accounting data); not fit with Baron-Myerson, hence Laffont-Tirole:

- Introduce cost observability and, as additional ingredient, unobserved actions that affect realization of cost
- Relevant results, related to practice (cost-plus, fixed-price, cost-sharing contracts).

- Firm manager (Agent) can implement a project by exerting effort e at personal cost $\phi(e)$;
- The cost of the project is $C = \theta - e$ and it is verifiable by all parties; but the efficiency parameter θ and the effort e are not observable by the Principal (Regulatory agency)
- The Agent's utility: $U = w - \phi(e)$ when he is paid w and exerts effort e.
- Project has value S for Principal: objectives $S - C - w = S - \theta + e - \phi(e) - U$

Perfect information optimum: $\phi'(e^0) = 1$ and $w^0 = \phi(e^0)$

Incomplete information: contract determines all verifiable variables, i.e. $(w(.), C(.))$ mapping the set of all θs into the admissible set of wages and cost realizations.

Implementability characterization:

$$\begin{aligned} U(\theta) &= \max_{\theta'} \left(w(\theta') - \phi(\theta - C(\theta')) \right) \\ &= U(\theta_H) + \int_{\theta}^{\theta_H} \phi'(s - C(s)) ds \end{aligned}$$

and $C(.)$ must be non-decreasing.

Optimum can be written in terms of effort

$$\phi'(e^*(\theta)) = 1 - \frac{F(\theta)}{f(\theta)} \phi''(e^*(\theta)) < \phi'(e^0)$$

with $C^*(\theta) = \theta - e^*(\theta)$, assuming $\phi' > 0$, $\phi'' > 0$ and $\phi''' > 0$ and MHRP ($\frac{F}{f}$ increasing).

- The FOC defines $C = C^*(\theta)$ increasing, hence invertible $\Leftrightarrow \theta = \tau(C)$.
- Differentiating the FOC in θ, with MHRP:

$$(1 - C')(\phi'' + \frac{F}{f}\phi''') = -\frac{d}{d\theta}\left(\frac{F}{f}\right)\phi'' < 0$$

which is equivalent to $1 < C' \Leftrightarrow \tau' < 1$

- Computing $w^*(.)$, one gets:

$$\begin{aligned} w^*(\theta) &= \phi(\theta - C(\theta)) + \int_\theta^{\theta_H} \phi'(s - C(s))ds \\ w^{*'}(\theta) &= -\phi'(\theta - C(\theta))C'(\theta) \end{aligned}$$

- Defining $W(C) \equiv w^*(\tau(C))$, then $W'(C) = -\phi'(\tau(C) - C)$

$$W''(C) = -\phi''(\tau(C) - C)(\tau'(C) - 1) \geq 0$$

V.1. Laffont-Tirole regulation model

- The payment schedule is decreasing convex in realized costs; Agent maximizes preferences (increasing in w and C) on schedule $W(.)$
- $W(.)$ can be replaced by the envelope of its tangents: i.e., schedule can be replaced by $w = a(\hat{\theta}) - Cb(\hat{\theta})$, Agent first chooses $\hat{\theta}$ and then chooses effort e
- Choose $b(\theta) = \phi'(e^*(\theta))$ and:

$$a(\theta) = \phi'(e^*(\theta))(\theta - e^*(\theta)) + \phi(e^*(\theta)) + U^*(\theta)$$

- Then, Agent announces truthfully and chooses $e^*(\theta)$
- Decentralization of the optimum by linear contracts
- Contract is robust to noisy observation of C because of linearity

V.2. Implicit labor contracts

Explaining unemployment as a result of negotiation between firm and trade union under asymmetric information.

Focus here (for pedagogical reasons) on story from the 80s: firms have private information about the demand shocks that impact it, trade union does not observe these shocks.

- Trade union (P) has all bargaining power, firm (A) is informed
- Negotiation bears on level of employement and wages
- Negotiation takes place ex ante, before the realization of the shocks
- Additional twist: trade union (workers) is risk-averse w.r.t. wage risk

- Firm's profit: $\Pi = \theta g(x) - w$ when it employs x workers and pays total wage w while being hit by a productivity shock θ, $g(.)$ increasing concave
- Trade union: $u(w) - \phi(x)$, $u(.)$ increasing concave, $\phi(.)$ increasing convex
- θ_H with probability f, θ_L otherwise with $\theta_L < \theta_H$
- Ex ante bargaining, hence the firm's participation constraint:

$$f(\theta_H g(x_H) - w_H) + (1-f)(\theta_L g(x_L) - w_L) \geq 0 \qquad \text{(eaIR)}$$

  and there is no ex post participation constraint.
- Ex ante participation leads trivially to efficiency in previous model; but not the case here because there is not full transferability (risk aversion)

Perfect information optimum:

- Full insurance of risk averse trade union: $w_H = w_L = w$
- Binding ex ante participation constraint:

$$w = \mathbf{E}[\theta g(x(\theta))] = f\theta_H g(x_H) + (1-f)\theta_L g(x_L)$$

- Plugging into expected trade union's utility: for $\theta \in \{\theta_H, \theta_L\}$,

$$u'\left(\mathbf{E}[\theta g(x(\theta))]\right)\theta g'(x^0(\theta)) = \phi'(x^0(\theta))$$

from which $x_H^0 > x_L^0$.

Incomplete information: The incentive constraints can be written as:

$$\begin{aligned} \Pi_H \equiv \theta_H g(x_H) - w_H &\geq \Pi_L + (\theta_H - \theta_L) g(x_L) \quad (\mathrm{IC}_H) \\ \Pi_L \equiv \theta_L g(x_L) - w_L &\geq \Pi_H - (\theta_H - \theta_L) g(x_H) \quad (\mathrm{IC}_L) \end{aligned}$$

Note that at the full information optimum,

$$\Pi_L^0 = \theta_L g(x_L^0) - w_L^0 < \theta_L g(x_H^0) - w_H^0 = \Pi_H^0 - (\theta_H - \theta_L) g(x_H^0)$$

This suggest to assume that only (IC_L) and (eaIR) bind at the optimum under asymmetric information

The optimum exhibits distortions in both states of nature ! (See Laffont-Martimort for exact expression): in particular, overemployment if good shock: $x_H^* > x_H^0$, a poorly convincing feature...

V.3. Monopolistic insurance

Back to opening example: monopolistic insurance company (P) proposes a insurance policy to a risk-averse owner (A) of a good, who has private information about the risk of an accident θ. Accident is observable and reduces by L the wealth W.

A insurance policy, a contract, provides a net wealth after an accident, A, and a net wealth if there is no accident B, with $W - L \leq A \leq B \leq W$. The company proposes a menu of such policies.

There are two difficulties that are specific to this example:

- The agent' risk aversion, hence non-transferability
- The reservation utility corresponds to the agent not getting any insurance and is therefore type-dependent:

$$U_R(\theta) = \theta U(W - L) + (1 - \theta)U(W)$$

V.3. Monopolistic insurance

Participation constraint for risk θ

$$\theta U(A) + (1-\theta)U(B) \geq U_R(\theta) \quad (\mathrm{IR}\theta)$$

Firm's profit on risk θ: $\quad W - \theta L - [\theta A + (1-\theta)B]$.

Isoprofit lines in plane (B, A) have same slope as agent's iso-utility curves when they cross the 45° line (full insurance line)

Perfect information optimum:

- Perfect insurance: $A^0(\theta) = B^0(\theta)$
- So that binding participation yields: $A^0(\theta) = U^{-1}(U_R(\theta))$, i.e. the certainty equivalent to the lottery $W - L$ with probability θ and W with probability $1-\theta$
- Note that: $A^0_H < A^0_L$

Asymmetric information: The incentive constraints for an insurance contract (A_H, B_H, A_L, B_L) ca be written:

$$\begin{aligned}\theta_H U(A_H) + (1-\theta_H)U(B_H) &\geq \theta_H U(A_L) + (1-\theta_H)U(B_L) \quad \text{(ICH)} \\ \theta_L U(A_L) + (1-\theta_L)U(B_L) &\geq \theta_L U(A_H) + (1-\theta_L)U(B_H) \quad \text{(ICL)}\end{aligned}$$

The full information optimal menu of insurance policies is such that high-risk agents (θ_H) would pretend they have low-risk to get a higher certainty equivalent $A_L^0 > A_H^0$.

Moreover, (ICH) and (IRL) with $\theta_H > \theta_L$ and $A_L < B_L$ imply that (IRH) holds

Look for the optimum with only (IRL) and (ICH) binding

V.3. Monopolistic insurance

The optimal menu of insurance policies under asymmetric information satisfies (Draw picture):

- Low-risk agents are indifferent between their insurance policy and no insurance at all ((IRL) binds)
- High-risk agents are indifferent between their insurance policy and the low-risk insurance policy ((ICH) binds) and their expected utility is larger than their reservation utility
- High-risk agents are fully insured: $A_H^* = B_H^*$
- Low-risk agents are faced with a residual risk: $W - L < A_L^* < B_L^* < W$, hence inefficiency.

Note that here "Top", i.e. the type that gets informational rent and full insurance, corresponds to high-risk θ_H.

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