You should know the definition of the competitive equilibrium market and understand the behavior of a monopoly.

An economy of production and of consumption is a pure perfect competition economy when all firms and all consumers are price taken. In such an economy, the firm 's behavior is stick to the choice of the quantity to produce, *This choice is called offer*, any consummer 's behavior is stick to the choice of the quantity to consume, *This choice is called demand*. Equilibrium Price and quantity coordinate those behavior. In the Samuelson model, the equilibrium allocation maximises the total surplus of the economy.	A set of strategies (that could be mixed strategies) is a Nash equilibrium whenever there is no unilateral deviation of the players.

1 Back to a competitive equilibrium and to the monopoly

Let consider an homogeneous good, seven consumer and six producers. Let suppose that in that market the consumers are interested in consuming 1 or 0, depending on the comparison of the selling price with their reservation price., and the firms are interested in producing 1 or 0, depending on the comparison of the selling price with their (marginal) unit cost. The characteristics of the consumers and of the producers are the following :

$$r^1 = 120 \quad r^2 = 57 \quad r^3 = 18 \quad r^4 = 12 \quad r^5 = 9 \quad r^6 = 8 \quad r^7 = 7$$
$$c^1 = 1 \quad c^2 = 7 \quad c^3 = 8 \quad c^4 = 12 \quad c^5 = 18 \quad c^6 = 20$$

1) Describe the equilibrium if that market is a pure competition one

ROADMAP : first characterizing the equilibrium market allocation, from which follows the equilibrium quantity and the equilibrium price.

We know that the competitive market allocation is efficient, in the sense that this is the allocation that maximises the total surplus of the economy. Then, to built this allocation, we can sequentially associate the best consumer (i.e. the consumer whose reservation price is the highest) with the best firm (i.e. the firm which cost is the smallest)

Then, we match consumer 1 and firm 1 ($120 > 1$), then consumer 2 and firm 2 ($57 > 7$), then consumer 3 and firm 3 ($18 > 8$), then consumer 4 and firm 4 ($12 \geq 12$). After, there is no match possible left. The eequilibrium quantity is $q^* = 4$, the only compatible price is $p^* = 12$.

2) Let suppose that there is only one firm, from which we know her marginal cost

$$c^1 = 1 \quad c^2 = 7 \quad c^3 = 8 \quad c^4 = 12 \quad c^5 = 18 \quad c^6 = 20$$

What is her optimal price, given the demand described before ? [Hint : use a tableur to make computations]

When the monopoly set its price, he choose a maximum price that copes with a demanded quantity. In our case, he will considering choosing $p = 120 \quad p = 57 \quad p = 18 \quad p = 12 \quad p = 9 \quad p = 8 \quad p = 7$ for which he will sell respectively 1, 2, 3, 4, 5, 6 or 7 units of the good. The following excel file finds that in our

	C13		$\times$ $\checkmark$	$f\!x$	=C12*($B12-C$2)							
		A	B	C	D	E	F	G	H	I	J	K
1			Consumers ri	120	57	18	12	9	8	7		
2			Firms ci	1	7	8	12	18	20			
3		Price	120	1	0	0	0	0	0	0		
4			Marginal Prof	119	0	0	0	0	0	0	Agregate profit	119
5												
6		Price	57	1	1	0	0	0	0	0		
7			Marginal Prof	56	50	0	0	0	0	0	Agregate profit	106
8												
9		Price	18	1	1	1	0	0	0	0		
10			Marginal Prof	17	11	10	0	0	0	0	Agregate profit	38
11												
12		Price	12	1	1	1	1	0	0	0		
13			Marginal Prof	11	5	4	0	0	0	0	Agregate profit	20
14												

3) Is that true that the quantity sold under monopoly is less than the quantity sold under market competition?

Yes : $q = 1 \leq q = 4$

4) Redo the Comparison between the pure competition market and the monopoly market when the characteristics of the consumers and the producers are the following

$r^1 = 953$ $\quad r^2 = 788$ $\quad r^3 = 783$ $\quad r^4 = 757$ $\quad r^5 = 695$ $\quad r^6 = 684$ $\quad r^7 = 657$ $\quad r^8 = 605$ $\quad r^9 = 468$ $\quad r^{10} = 349$ $\quad r^{11} = 318$ $\quad r^{12} = 250$ $\quad r^{13} = 181$ $\quad r^{14} = 131$
$c^1 = 44$ $\quad c^2 = 86$ $\quad c^3 = 311$ $\quad c^4 = 345$ $\quad c^5 = 385$ $\quad c^6 = 439$ $\quad c^7 = 495$ $\quad c^8 = 524$ $\quad c^9 = 527$ $\quad c^{10} = 550$ $\quad c^{11} = 574$ $\quad c^{12} = 585$ $\quad c^{13} = 624$ $\quad c^{14} = 709$

We redo the same Tableur file, and we get $q^{**} = 7$ and $p^{**} = 657$

	Consumers ri	953	788	783	757	695	684	657	605	468	349	318	250	181	131		
	Firms ci	44	86	311	345	385	439	495	524	527	550	574	585	624	709		
Price	953	1	0	0	0	0	0	0	0	0	0	0	0	0	0		
	Marginal Prof	909	0	0	0	0	0	0	0	0	0	0	0	0	0	Agregate profit	909
Price	788	1	1	0	0	0	0	0	0	0	0	0	0	0	0		
	Marginal Prof	744	702	0	0	0	0	0	0	0	0	0	0	0	0	Agregate profit	1446
Price	783	1	1	1	0	0	0	0	0	0	0	0	0	0	0		
	Marginal Prof	739	697	472	0	0	0	0	0	0	0	0	0	0	0	Agregate profit	1908
Price	757	1	1	1	1	0	0	0	0	0	0	0	0	0	0		
	Marginal Prof	713	671	446	412	0	0	0	0	0	0	0	0	0	0	Agregate profit	2242
Price	695	1	1	1	1	1									0		
	Marginal Prof	651	609	384	350	310	0	0	0	0	0	0	0	0	Agregate profit	2304
Price	684	1	1	1	1	1	1								0		
	Marginal Prof	640	598	373	339	299	245	0	0	0	0	0	0	0	Agregate profit	2494
Price	657	1	1	1	1	1	1	1							0		
	Marginal Prof	613	571	346	312	272	218	162	0	0	0	0	0	0	Agregate profit	2494
Price	605	1	1	1	1	1	1	1	1						0		
	Marginal Prof	561	519	294	260	220	166	110	81	0	0	0	0	0	Agregate profit	2211
Price	468	1	1	1	1	1	1	1	1	1					0		
	Marginal Prof	424	382	157	123	83	29	-27	-56	-59	0	0	0	0	Agregate profit	1056
Price	349	1	1	1	1	1	1	1	1	1	1				0		
	Marginal Prof	305	263	38	4	-36	-90	-146	-175	-178	-201	0	0	0	Agregate profit	-216

while the pure competition was to set $q^* = 8$ and $p^* \in [524, 605]$.

2 Monopoly

1) Let consider the case such that $c_m = 2q$, $D(p) = 120 - p$, which we rewrite $p(q) = 120 - q$. Compute q^{**} and p^{**} in a monopoly market if the firm is alone. Check that the quantity produced is inferior to the quantity that the firm would have produced under a competitive market.

The representation of this economy is as follows.

p

$p(q) = 120 - q$

$C_m = 2q$

$R_m(q)$

p^{**}

q^{**}

q
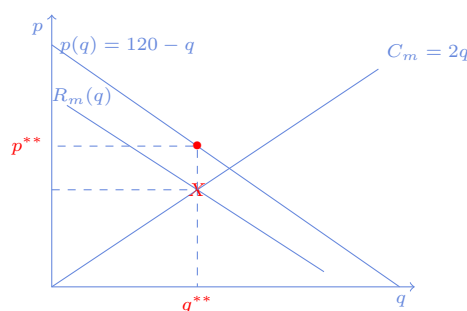
The pure competition reference point is when C_m and $p(q)$ cross, that is when $2q = 120 - q$, i.e, when $q = 40$: $q^* = 40$, $p^* = 80$.

The monopolist tarification is such that $R_m = c_m$; In our example, $R(q) = q(120 - q)$, $R_m = 120 - 2q$, $R_m = c_m$ is equivalent to $120 - 2q = 2q$, and $q = 30$. For that quantity, the maximal price that the firm can charge is $p = 120 - 30 = 90$.

$$p^{**} = 90 \qquad q^{**} = 30$$

2) Let consider a firm that produces two goods in quantity q_1 and q_2, which are substitute, such that the demand of each good is

$$q_1 = 10 - 2p_1 + p2 \qquad q_2 = 10 + p_1 + -2p_2$$

We suppose that the marginal cost is constant equal to c.

The profit of the firm is to set a tarif (p_1, p_2) and to sell at this price $q_1 = 10 - 2p_1 + p2$ and $q_2 = 10 + p_1 - 2p2$. Her profit is then :

$$\pi = q_1(p_1 - c) + q_2(p_2 - c) = \big(10 - 2p_1 + p_2\big)(p_1 - c) + \big(10 + p_1 - 2p_2\big)(p_2 - c)$$

The program of the monopolist is then

$$\max_{p_1, p_2} \pi(p_1, p_2) \tag{1}$$

and the FOC are

$$\frac{\partial \pi(p_1, p_2)}{\partial p_1} = 0 \qquad \frac{\partial \pi(p_1, p_2)}{\partial p_2} = 0$$

OR

$$\frac{\partial \pi(p_1, p_2)}{\partial p_1} = (10 - 2p_1 + p_2) - 2(p_1 - c) + (p_2 - c) = 10 + c - 4p_1 + 2p_2 \tag{2}$$

$$\frac{\partial \pi(p_1, p_2)}{\partial p_2} = (p_1 - c) + (10 + p_1 - 2p_2 - 2(p_2) - c) = 10 + c + 2p_1 - 4p_2 \tag{3}$$

Then, FOC can be written

$$10 + c - 4p_1 + 2p_2 = 0 \tag{4}$$

$$10 + c + 2p_1 - 4p_2 = 0 \tag{5}$$

Symmetry implies that $p_1 = p_2$. (or make the difference between the two equations). Then, it follows

$$p_1^* = p_2^* = \frac{10 + c}{2}$$

3) For the preceding firms comment the variation of the tarif and of the profits with c.

When c increases, then the price increase. That is not so surprising.

As the demand does not change when c increase, this increase of price will decrease the profit. Indeed, with the same c, increasing the price would have lowering the profit. Increasing then c enlarge the the profit 's diminution.

4) Compare the tarif with the situation in which the goods would not be substitute.

Said like that, the comparison is not possible, at least in a precise formulation. Indeed, saying that the goods would not be substitute mean that assumptions about demand are different. We have not any more $q_1 = 10 - 2p_1 + p2 \qquad q_2 = 10 + p_1 + -2p_2$. But at this stage we do not know the new demands.

- - - - - - - - - - - - - - - END OF EXERCISE SET 3 - - - - - - - - - - - - - - -