Pareto improving taxes with externalities*

Van-Quy Nguyen[†a], Jean-Marc Bonnisseau[‡b], and Elena L. del Mercato[§b]

[a] *(Corresponding author) Université Paris-Saclay, Univ Evry, l'EPEE, France and Faculty of Mathematical Economics, College of Technology, National Economics University, Hanoi, Vietnam.*
[b] *Université Paris 1 Panthéon-Sorbonne, CNRS, Centre d'Economie de la Sorbonne and Paris School of Economics, 106-112 Boulevard de l'Hôpital, 75647 Paris Cedex 13, France.*

March 31, 2025

Abstract

We consider a pure exchange economy with consumption externalities in preferences. We study commodity taxes and lump-sum transfers schemes, which lead to equilibrium allocations where all individuals are strictly better off. We extend the result of Geanakoplos and Polemarchakis (2008) on the generic existence of Pareto improving policies with uniform taxes and equal transfers to general non-separable preferences, when the number of individuals is strictly smaller than the number of commodities. We overcome this limitation by considering either uniform taxes with personalized lump-sum transfers, or personalized taxes with uniform lump-sum transfers. As in Geanakoplos and Polemarchakis (2008), we mainly use utility perturbations. We also provide the existence of Pareto improving policies

*The first author is the corresponding one, the other authors follow each other in alphabetical order. This is the revised version of the first version that dates from June 12, 2024. This version is currently under peer review at a major international scientific journal. We are grateful to the anonymous Reviewers and the Editor of that journal for their valuable comments and suggestions. We also acknowledge the financial support of the French National Research Agency ANR-22-CE26-0015, and the LABEX MME-DII (ANR-11-LBX-0023-01)

† quynv@neu.edu.vn

‡ Jean-Marc.Bonnisseau@univ-paris1.fr

§ Elena.delMercato@univ-paris1.fr

for Bergson-Samuelson utilities and two-individual economies, without perturbing utilities.

Keywords: Consumption externalities, commodity taxes, lump-sum transfers, Pareto improvement.

JEL classification: D50, D60, D62.

1 Introduction

The First Welfare Theorem is one of the most powerful results in economics. This theorem postulates that competitive equilibrium allocations are Pareto optimal. However, the First Welfare Theorem is based on a very critical assumption. That is, individuals have self-interested preferences, in such a way that individual utilities do not depend on the choices of other individuals. In the presence of consumption externalities, individual choices affect the utilities of the other individuals. A competitive equilibrium with externalities might not be Pareto optimal. This is because, at a competitive equilibrium, all the individual marginal utilities with respect to own consumptions are positively proportional, since they are positively proportional to the equilibrium price. On the other hand, Pareto optimality conditions (with or without externalities) require that all the "social marginal utilities" are equal.[1]In the absence of externalities, the social marginal utilities are positively proportional to the individual marginal utilities. Hence, for suitable weights of the planner's welfare function, one immediately gets the First Theorem of Welfare Economics. In economies with externalities, this is not necessarily guaranteed at a competitive equilibrium, because the social marginal utilities with respect to the consumption of an individual depend on the marginal utilities of other individuals.[2]This leads to a failure of the First Theorem of Welfare Economics. Actually, as a consequence of our work, this failure is generic.

[1]We call "social marginal utilities" the marginal utilities of the social planner whose welfare function is a weighted sum of the utilities of all the individuals. More precisely, let v_w be a weighted sum of the utility functions of all individuals i for some weights $w = (w_i)_{i\in\mathcal{I}}$. The social marginal utility with respect to the consumption x_i of individual i at the allocation x is defined by $D_{x_i}v_w(x) = \sum_{j\in\mathcal{I}} w_j D_{x_i}u_j(x)$. This gradient has the same dimension as the individual marginal utility $D_{x_i}u_i(x)$ of individual i, but these two gradients do not need to be positively proportional.

[2]This is illustrated in Example (e) in Appendix. For other discussion, see Hochman and Rodgers (1969) and Dufwenberg et al. (2011).

Our paper is built on the solid foundation of general equilibrium models, and it is in line with recent contributions on the analysis of competitive equilibria with various sorts of externalities. Bonnisseau and del Mercato (2010), and del Mercato and Platino (2017b) explore regularity properties of competitive equilibria with consumption and production externalities. Dufwenberg et al. (2011), Balasko (2015), Nguyen (2021) study welfare and regularity features of competitive equilibria with wealths and endowments dependent preferences or other-regarding preferences. More recently, del Mercato and Nguyen (2023) provide new sufficient conditions to decentralize Pareto optima in the presence of consumption externalities. The recent work by Anderson and Duanmu (2025) proposes two generalizations of the Arrow-Debreu model applied to climate change and analyze quota and emission tax equilibria, allowing for very general preferences with consumption, production and price externalities.

As is well known, the introduction of personalized Lindahl prices allows to restore efficiency of equilibrium allocations.[3] Nevertheless, this solution is difficult to implement, because it requires the opening and the organization of many bilateral markets for externalities, where each pair of individuals should have direct interactions.[4] Furthermore, the existence of such equilibria is guaranteed under stronger irreducibility and convexity assumptions. Preferences must be convex, not only in own consumption, but also with respect to the externalities, see for instance Foley (1970), Crès (1996), and Bonnisseau, del Mercato and Siconolfi (2023).

Another stream of the literature focuses on competitive equilibria with externalities and taxation.[5] Aoki (1971) shows that a tax-subsidy system leads to efficiency with external economies of scale, while Osana (1977) demonstrates similar results in economies with Marshallian externalities. Greenwald and Stiglitz (1986) establish that linear taxation can achieve Pareto improvements in economies with imperfect information. More recently, Escobar-Posada and Monteiro (2017) studied optimal taxation in economies with production, consumption, and leisure externalities. Nevertheless, achieving Pareto optimal allocations with specific externalities requires extensive information about preferences and technologies, see Sato (1981) and Tian (2004).

[3] See the seminal works by Samuelson (1954), Arrow (1969), and Foley (1970), and a recent work by Bonnisseau, del Mercato and Siconolfi (2023) on the existence of Arrow-Lindahl equilibria.

[4] The new commodity space has then a huge dimension $L(I-1)I$, and all the individuals must be connected with each other to trade.

[5] Shafer and Sonnenschein (1976) provide the existence of competitive equilibria with externalities and an extremely general scheme of personalized commodity taxes and lump-sum transfers.

In exchange economies with consumption externalities, Geanakoplos and Polemarchakis (2008) adopt a different perspective, because these authors do not focus on reaching full efficiency. They consider tax-transfer schemes to obtain Pareto improvements of competitive equilibrium allocations, that is, equilibrium allocations where all the individuals are strictly better off. Our paper takes place in this line of research.

Our first contribution is to allow for general preferences, that are not supposed to be separable between own consumption and the consumption of others. This means that the externalities may affect not only the utility levels, but also the individual marginal rates of substitutions, and consequently externalities have an impact on equilibrium prices. In the case of separable preferences, equilibrium prices cannot be distinguished from the ones with selfish preferences that depend on own consumption only. Hence, separability is a quite demanding assumption, because it means that externalities are irrelevant in competitive terms.

As usual in the literature on generic Pareto improvements, see Geanakoplos and Polemarchakis (1986), and subsequent contributions by Cass and Citanna (1998), Citanna, Kajii and Villanacci (1998), and Villanacci et al. (2002), we consider regular equilibria to conduct comparative static analysis. Hence, we work with the class of non-separable preferences that ensures the genericity of regular economies, as characterized in Bonnisseau and del Mercato (2010). This covers separable preferences, and also many other non-separable preferences where externalities do not have too strong effects on marginal rate of substitutions.

We first consider the anonymous tax-transfer policy introduced in Geanakoplos and Polemarchakis (2008), in which all individuals face the same tax rates and receive the same lump-sum transfer. To overcome a limitation of the existing results, we also introduce two variants of this policy by considering either uniform tax rates with personalized lump-sum transfers, or personalized taxes with equal transfers.

We show by means of an example, that it could be impossible to get the generic existence of Pareto improving policies in the space of endowments. Hence, as in Geanakoplos and Polemarchakis (2008), we introduce utility perturbations. It is worth noting that our perturbations do not affect individual marginal utilities with respect to own consumption. This ensures that the set of competitive equilibria with or without taxes is not altered by utility perturbations.

In this framework, we recover the result of Geanakoplos and Polemarchakis (2008) on the generic existence of Pareto improving anonymous tax-transfer policies, when the number of individual is strictly smaller than the number of commodities. From a general point of view, to get Pareto improving tax-transfer schemes, we need to have at least as many policy instruments as the number of individuals. Hence, to deal with the case where the number of individual exceeds the number of commodities, we consider either uniform tax rates with personalized transfers, or personalized taxes rates with equal transfers. In both cases, we prove two similar results about the generic existence of Pareto improving policies. It is worth noting that with personalized transfers, one can impose taxes only on specific commodities that generate (negative) externalities, such as fuel and alcohol.[6]

To complete our analysis, we provide a sufficient condition to ensure the existence of Pareto improving tax-transfer policy in economies with Bergson-Samuelson utility functions. We also generalize our assumption to a wider class of utility functions called proportional marginal utilities, which has been introduced by del Mercato and Nguyen (2023). More interestingly, our assumption is trivially satisfied in economies with two individuals, representing a first step to understand the link between the primitives of the economy and Pareto improving taxes.

The paper is organized as follows. Section 2 summarizes notations, definitions, and assumptions regarding the economic framework of the exchange model with tax-transfer policies. In section 3, we prove the genericity of regular economies with full trade equilibrium, and we study the existence and smoothness of the equilibrium with taxes and transfers. In Section 4, we provide our main results on the genericity of Pareto improving tax-transfer policies in the different cases mentioned above, and, finally, we focus on Bergstrom-Samuelson preferences to get Pareto improving tax-transfer policies without utility perturbations.

All the proofs are gathered in Appendix.

2 The model and the assumptions

There is a finite number of commodities labeled by the superscript $\ell \in \mathcal{L} = \{1, ..., L\}$ with $L \geq 2$, and a finite number of individuals labeled by the subscript $i \in \mathcal{I} = \{1, ..., I\}$ with $I \geq 2$. The commodity space is $\mathbb{R}^L$.[7]For every $i \in \mathcal{I}$, the individual consumption set is the positive orthant $\mathbb{R}^L_+$. The consumption by individual i of commodity ℓ is x_i^ℓ, and individual i's

[6]This is usually called *excise* tax.

[7]Without loss of generality, vectors are treated as row matrices. Further, let A be a real matrix with m rows and n columns. It also denotes the linear application $A : v \in \mathbb{R}^n \to A(v) := Av^T \in \mathbb{R}^{[m]}$ where v^T denotes the transpose of v and $\mathbb{R}^{[m]} := \{w^T : w \in \mathbb{R}^m\}$. When $m = 1$, $A(v)$ coincides with the inner product $A \cdot v$, treating A and v as vectors in $\mathbb{R}^n$.

consumption is $x_i = (x_i^\ell)_{\ell \in \mathcal{L}} \in \mathbb{R}_+^L$. The consumption of all individuals other than i is $x_{-i} = (x_j)_{j \neq i} \in \mathbb{R}_+^{L(I-1)}$. The bundle $x = (x_i)_{i \in \mathcal{I}} \in \mathbb{R}_+^{LI}$ is an allocation. With innocuous abuse of notation, x is also denoted by (x_i, x_{-i}). The prices of commodities are denoted by $p = (p^\ell)_{\ell \in \mathcal{L}} \in \mathbb{R}_{++}^L$. We normalize p by choosing commodity L as the *numéraire* commodity, i.e., $p^L = 1$. The set of prices is $\mathbb{S} = \mathbb{R}_{++}^{L-1} \times \{1\}$. For every vector $y = (y^1, \dots, y^L) \in \mathbb{R}^L$, we denote $y^\setminus = (y^1, \dots, y^{L-1}) \in \mathbb{R}^{L-1}$, and then $p \in \mathbb{S}$ is written as $\big(p^\setminus, 1\big)$.

We study consumption externalities in individual i's preferences that are represented by a utility function:

$$u_i : x \in \mathbb{R}_+^{LI} \longrightarrow u_i(x) \in \mathbb{R},$$

where $u_i(x)$ is the utility level of individual i associated with the allocation $x = (x_i, x_{-i}) \in \mathbb{R}_+^{LI}$. The profile of utilities is $u = (u_i)_{i \in \mathcal{I}}$.

The initial endowment of individual i is $e_i = (e_i^\ell)_{\ell \in \mathcal{L}} \in \mathbb{R}_{++}^L$. The bundle of initial endowments is $e = (e_i)_{i \in \mathcal{I}} \in \mathbb{R}_{++}^{LI}$, and $\Omega = \mathbb{R}_{++}^{LI}$ denotes the endowment space. An economy is summarized by $\mathcal{E} = (u, e)$. As usual, $x = (x_i)_{i \in \mathcal{I}} \in \mathbb{R}_+^{LI}$ is a feasible allocation of the economy $\mathcal{E}$ if $\sum_{i \in \mathcal{I}} x_i = \sum_{i \in \mathcal{I}} e_i$. The set F is the set of feasible allocations.

We consider commodity tax (or subsidy) rates. Let t^ℓ be the tax rate on commodity ℓ, $t^\ell > 0$ or $t^\ell < 0$ if it is a subsidy, with $p^\ell + t^\ell > 0$, so that all net prices are strictly positive. As in Geanakoplos and Polemarchakis (2008), taxes affect only the *buyers*. That is, if $x_i^\ell > e_i^\ell$, then individual i faces a unit cost $p^\ell + t^\ell$ on her excess demand $(x_i^\ell - e_i^\ell)$ of commodity ℓ, otherwise she receives the standard payment $p^\ell(x_i^\ell - e_i^\ell)$.[8] Without loss of generality, we impose taxes only on the first $L-1$ commodities, i.e., $t^L = 0$. Indeed, if individuals face a tax $-1 < t^L \neq 0$ on the numéraire commodity L, one easily defines an equivalent system of taxes $(\bar{t}^\ell)_{\ell \in \mathcal{L}}$ and prices $\bar{p}$ with $\bar{t}^L = 0$ and $\bar{p}^L = 1$.[9] Hence, the space of tax rates is $\mathbb{T} = \mathbb{R}^{L-1}$, and $t = (t_1, \dots, t_{L-1}, 0)$ is the vector of tax rates. With innocuous abuse of notation, t is an element of $\mathbb{T}$.

Further, each individual receives a lump-sum transfer. The most simple scheme of lump-sum transfers is the equal transfer policy, as in Geanakoplos and Polemarchakis (2008), where each individual gets the same transfer $T \in \mathbb{R}$. Then, at the prices p, the tax rates t and the lump-sum transfer T,

[8] Notice that imposing taxes on both buyers and sellers has the same effect as taxing only buyers. Indeed, let $\tilde{t}^\ell$ the tax imposed on sellers, that is, individual i faces a cost $p^\ell + \tilde{t}^\ell$ also in the case where $x_i^\ell < e_i^\ell$. Then, it is enough to define new prices $p^\ell + \tilde{t}^\ell$, and new taxes $\bar{t}^\ell = (t^\ell - \tilde{t}^\ell)$ on buyers only.

[9] For sake of clarity, takes $\bar{t}^\ell = \frac{t^\ell}{1+t^L}$ and $\bar{p}^\ell = \frac{p^\ell}{1+t^L}$ for all $\ell \neq L$.

individual i's budget constraint is:[10]

$$(p+t)\cdot(x_i-e_i)_+ - p\cdot(x_i-e_i)_- \leq T. \tag{1}$$

The equal transfer policy is fairly restrictive. This is because the dimension of the policy instruments (t, T) is too low for ensuring a generic Pareto improvement when the number of individuals is equal or greater than the number of policy instruments (i.e., $I \geq L$). We illustrate this issue in Example 2 of Subsection 4.2. To overcome this restriction, we also study two other kinds of tax-transfer systems.

First, we consider uniform tax rates $t \in \mathbb{T}$ as above, but personalized lump-sum transfers. Individual i's transfer is now τ_i, and $\tau = (\tau_i)_{i\in\mathcal{I}} \in \mathbb{R}^I$ is the vector of transfers. Then, individual i's budget constraint becomes:

$$(p+t)\cdot(x_i-e_i)_+ - p\cdot(x_i-e_i)_- \leq \tau_i. \tag{2}$$

At equilibrium, personalized lump-sum transfers can be interpreted as small perturbations of the equal transfer policy.[11]

Second, we consider personalized taxes and equal transfers. Every individual i faces a proportional tax rate $(1+\alpha_i)t$. To ensure that these personalized tax rates are closed to the anonymous taxes rates t, we focus on sufficiently small parameters α_i. That is, we fix an anonymous threshold $\delta > 0$ such that $\alpha = (\alpha_i)_{i\in\mathcal{I}} \in (-\delta, \delta)^I$. Individual i's budget is then:

$$[p+(1+\alpha_i)t]\cdot(x_i-e_i)_+ - p\cdot(x_i-e_i)_- \leq T. \tag{3}$$

Given the prices p, the uniform tax rates t, and the externalities x_{-i}, the maximization problem of individual i with equal transfer T is:

$$\begin{aligned} &\max_{x_i\in\mathbb{R}^L_+} u_i(x_i, x_{-i}) \\ &\text{s.t. } (p+t)\cdot(x_i-e_i)_+ - p\cdot(x_i-e_i)_- \leq T \end{aligned} \tag{$\mathcal{P}_i$}$$

Under the different tax-transfer systems described above, problem $(\mathcal{P}_i)$ changes by replacing its constraint with the budget constraint (2) or (3), respectively.

Following Geanakoplos and Polemarchakis (2008), each individual takes as given her lump-sum transfer as independent of the tax revenues while, at

[10]We recall that for every $z \in \mathbb{R}^L$, $z_+ = (z^\ell_+)_{\ell\in\mathcal{L}} \in \mathbb{R}^L_+$ and $z_- = (z^\ell_-)_{\ell\in\mathcal{L}} \in \mathbb{R}^L_+$ where $z^\ell_+ := \max\{z^\ell, 0\}$ and $z^\ell_- := -\min\{z^\ell, 0\}$.

[11]This is a consequence of a further result stated in Section 3. Indeed, by property (1) of Lemma 2, it is enough to take $T = \dfrac{1}{I}\sum_{i=1}^I \tau_i$ with τ_i small enough, ensuring that the difference between T and τ_i is closed to zero for all i.

equilibrium, the tax-transfer system must satisfy the tax balance condition, i.e., the total tax revenue equals the sum of lump-sum transfers.

From now on, a (t, T)-equilibrium is an equilibrium with uniform taxes and equal transfers, a (t, τ)-equilibrium is an equilibrium with uniform taxes and personalized transfers, and a (t, α, T)-equilibrium is an equilibrium with personalized taxes and equal transfers.

Definition 1 *A vector $(x^*, p^*) \in \mathbb{R}_+^{LI} \times \mathbb{S}$ is a (t, T)-equilibrium of the economy $\mathcal{E}$ if:*

(i) for every $i \in \mathcal{I}$, x_i^* solves problem $(\mathcal{P}_i)$ at (p^*, x_{-i}^*),

(ii) the allocation x^* is feasible, and

(iii) the tax-transfer system satisfies the tax balance condition:

$$T = \frac{1}{I} \sum_{i=1}^{I} t \cdot (x_i^* - e_i)_+.$$

For a (t, τ)-equilibrium, the constraint of problem $(\mathcal{P}_i)$ is replaced by the budget constraint (2). Further, condition (iii) becomes $\sum_{i=1}^{I} \tau_i = \sum_{i=1}^{I} t \cdot (x_i^* - e_i)_+$.

For a (t, α, T)-equilibrium, the constraint of problem $(\mathcal{P}_i)$ is replaced by the budget constraint (3). Further, condition (iii) becomes $T = \frac{1}{I} \sum_{i=1}^{I} (1 + \alpha_i) t \cdot (x_i^* - e_i)_+$.

Remark 1 *Notice that,*

(i) In the case of equal transfers, if $t = 0$, the vector (x^*, p^*) is then a classical competitive Nash equilibrium with externalities, where $p^* \cdot (x_i - e_i) \leq 0$ is the standard budget constraint.[12] The same applies with $t = 0$ and personalized transfers $\tau_i = 0$ for all $i \in \mathcal{I}$

(ii) As in the classical literature on Pareto improving policies in differentiable economies, by "Pareto improvement" we mean that every individual is strictly better off.[13] That is, a feasible allocation $x \in F$ is a Pareto improvement of the feasible allocation $x^* \in F$ if $u_i(x) > u_i(x^*)$ for all $i \in \mathcal{I}$.

[12] This is the classical notion given by Arrow and Hahn (1971), and Laffont (1988).

[13] See Geanakoplos and Polemarchakis (1986), Cass and Citanna (1998), Citanna, Kajii and Villanacci (1998), and Geanakoplos and Polemarchakis (2008).

2.1 Basic assumptions

We introduce basic assumptions on utility functions needed for our analysis.

Assumption 1 *For all $i \in \mathcal{I}$:*

1. *The function u_i is continuous on $\mathbb{R}^{LI}_{+}$.*

2. *For each $x_{-i} \in \mathbb{R}^{L(I-1)}_{+}$, the function $u_i(\cdot, x_{-i})$ is increasing on $\mathbb{R}^{L}_{++}$.*

3. *The function u_i is $\mathcal{C}^3$ on $\mathbb{R}^{LI}_{++}$.*

4. *For each $x_{-i} \in \mathbb{R}^{L(I-1)}_{++}$, the function $u_i(\cdot, x_{-i})$ is differentiably strictly increasing, i.e., for every $x_i \in \mathbb{R}^{L}_{++}$, $D_{x_i} u_i(x_i, x_{-i}) \in \mathbb{R}^{L}_{++}$.*

5. *For each $x_{-i} \in \mathbb{R}^{L(I-1)}_{++}$, the function $u_i(\cdot, x_{-i})$ is differentiably strictly quasi-concave, i.e., for every $x_i \in \mathbb{R}^{L}_{++}$, $D^2_{x_i} u_i(x_i, x_{-i})$ is negative definite on $\operatorname{Ker} D_{x_i} u_i(x_i, x_{-i})$;*

6. *for each $(x_i, x_{-i}) \in \mathbb{R}^{L}_{++} \times \mathbb{R}^{L(I-1)}_{+}$,*
   $cl_{\mathbb{R}^L}\{x'_i \in \mathbb{R}^{L}_{++} : u_i(x'_i, x_{-i}) \geq u_i(x_i, x_{-i})\} \subseteq \mathbb{R}^{L}_{++}$.

Point (2) means that preferences remain increasing even if some consumptions of other individuals vanish. Once externalities are fixed, points (3)-(5) of Assumption 1 are basic assumptions that are needed in any classical smooth economy, see for instance Mas-Colell (1985), Villanacci et al. (2002), del Mercato (2006), and Bonnisseau and del Mercato (2010). As is well known, for the existence and the regularity of competitive equilibria with strictly positive consumptions and prices, one also needs some boundary conditions to control the behavior of the preferences on the boundary of the positive orthant. Point (6) of Assumption 1 is the boundary condition adapted in equilibrium models with externalities when externalities converge to zero.[14]It is satisfied, for instance, by additively separable utilities $u_i(x_i, x_{-i}) = m_i(x_i) + v_i(x_{-i})$, where m_i satisfies the classical boundary condition on $\mathbb{R}^{L}_{+}$, and v_i is defined on $\mathbb{R}^{L(I-1)}_{+}$. Note that points (2) and (6) of Assumption 1 and the strict positivity of prices imply that all consumptions are strictly positive at a competitive Nash equilibrium.

As is well known, the set of regular economies plays a crucial role in comparative statics, as well as for studying Pareto improving policies in differentiable economies. However, generic regularity may fail in presence of

[14]For more discussion on this condition, see for instance del Mercato (2006), Bonnisseau and del Mercato (2010), del Mercato and Platino (2017a), and Nguyen (2021).

externalities, see the example in Section 4 of Bonnisseau and del Mercato (2010). Hence, one needs an additional assumption stated below to ensure the genericity of regular economies.

Assumption 2 (Regularity condition) *Let $x \in \mathbb{R}^{LI}_{++}$ such that all the gradients $\big(D_{x_i}u_i(x)\big)_{i\in\mathcal{I}}$ are positively collinear. Let $v = (v_i)_{i\in\mathcal{I}} \in \mathbb{R}^{LI}$ such that $\sum_{i\in\mathcal{I}} v_i = 0$ and $v_i \in \operatorname{Ker} D_{x_i}u_i(x_i, x_{-i})$ for all $i \in \mathcal{I}$. Then, for all $k \in \mathcal{I}$, $v_k \sum_{i=1}^{I} D^2_{x_i x_k} u_k(x_k, x_{-k})(v_i) < 0$, whenever $v_k \neq 0$.*

The assumption above is in Bonnisseau and del Mercato (2010), and del Mercato and Platino (2017b). Assumption 2 implies that the second order effect of the own consumption dominates the aggregate second order effect of consumption externalities. This assumption is satisfied in economies without externalities, as well as in models where utilities are additively separable in externalities and quasi-concave in own consumption, as for instance in Geanakoplos and Polemarchakis (2008).[15]As explained in del Mercato and Platino (2017b), Assumption 2 does not require utilities to be quasi-concave with respect to externalities. Indeed, it does not involve the whole Hessian matrix of u_k, but only one block of the rows of its Hessian matrix.

$\mathcal{U}$ denotes the set of utility functions $u = (u_i)_{i\in\mathcal{I}}$ satisfying Assumptions 1 and 2. We endow the space $\mathcal{U}$ with the topology of $\mathcal{C}^3$ uniform convergence on compact sets.

2.2 Utility perturbations

Assumptions 1 and 2 are not enough to establish the generic existence of Pareto improving policies in the space of endowments. This is shown below by means of an example.

Example 1. There are three individuals and three commodities. The utility functions are of the Bergson-Samuelson type:

$$u_1(x) = m_1(x_1),\ u_2(x) = m_2(x_2) - m_3(x_3),\ \text{and}\ u_3(x) = m_3(x_3) - m_2(x_2),$$

where m_i is a Cobb-Douglas utility function, $m_i(x_i) = \prod_{\ell=1}^{L} (x_i^\ell)^{1/L}$ for all i. The bundle of initial endowments is $e = (e_1, e_2, e_3) \in \mathbb{R}^{3L}_{++}$. Since the utility functions are separable, the set of competitive Nash equilibria of this economy

[15]Indeed, in both cases, $D^2_{x_i x_k} u_k(x_k, x_{-k}) = 0$ for all $i \neq k$, and then Assumption 2 follows from Assumption 1.(5).

coincides with the set of competitive equilibria of the economy $(m_i, e_i)_{i=1,2,3}$. Every competitive Nash equilibrium is not Pareto optimal. Indeed, for every $e = (e_1, e_2, e_3) \in \mathbb{R}^{3L}_{++}$, there exists a unique competitive Nash equilibrium (x^*, p^*) where $x^* = (x_i^*)_{i=1,2,3} \in \mathbb{R}^{3L}_{++}$. One easily checks that it is possible to transfer a suitable positive consumption from individuals 2 and 3 to individual 1 to obtain a weak Pareto improvement of x^*.[16] However, for every $e = (e_i)_{i=1,2,3} \in \mathbb{R}^{3L}_{++}$, there is no policy that Pareto improves all the individuals. Indeed, it is not possible to strictly increase the utility of both individuals 2 and 3, because $u_2(x) + u_3(x) = 0$ for all x. ■

In order to establish the generic existence of Pareto improving policies, in Section 4, first of all, we follow Geanakoplos and Polemarchakis (2008) by introducing utility perturbations. Note that, at the end of Section 4, we show that perturbations are useless with Bergstrom-Samuelson preferences.

In this subsection we present the utility perturbations. The key property is that these perturbations do not affect the individual marginal utilities with respect to own consumption. Consequently, these perturbations do not change the set of competitive Nash equilibria, nor the set of equilibria with taxes and transfers.

Let $(u, e) \in \mathcal{U} \times \Omega$ be a regular economy with k competitive Nash equilibria $\big((x^*(j), p^*(j))\big)_{j=1}^k$. For every $i \in \mathcal{I}$, define $a_i = (a_{i,j})_{j \neq i} \in \mathbb{R}^{L(I-1)}$. For all $i \in \mathcal{I}$, consider a $\mathcal{C}^\infty$ bump function ρ_i from $\mathbb{R}^L_{++}$ to $[0, 1]$ with a compact support satisfying $\rho_i(x_i) = 1$ on k disjoint open neighborhoods of $\{x_i^*(1), \ldots, x_i^*(k)\}$. The construction of such bump functions is well known in the presence of other sources of market failures, such as incomplete financial markets.[17] Denote $a = (a_1, \ldots, a_I) \in \mathbb{R}^{L(I-1)I}$ the perturbations parameters, and consider an arbitrary open neighborhood $\mathcal{A}^0 \subset \mathbb{R}^{L(I-1)I}$ around 0. For all $i \in \mathcal{I}$, define the following perturbation of u_i associated to the perturbations parameters $a \in \mathcal{A}^0$:

$$u_i^a(x) = u_i(x) + \sum_{j \neq i} \rho_j(x_j) a_{i,j} \cdot x_j \tag{4}$$

Remark that, for all $a \in \mathcal{A}^0$, $u^a = (u_i^a)_{i \in \mathcal{I}}$ belongs to the space $\mathcal{U}$, because for all i, the term $\displaystyle\sum_{j \neq i} \rho_j(x_j) a_{i,j} \cdot x_j$ does not depend on x_i. Further, one easily

[16] A feasible allocation $x \in F$ is a weak Pareto improvement of the feasible allocation $x^* \in F$ if $u_i(x) \geq u_i(x^*)$ for all $i \in \mathcal{I}$ and $u_h(x) > u_h(x^*)$ for some $h \in \mathcal{I}$.

[17] See for instance, Lemma 41 in Chapter 7 and Subsection 5.2.1 in Chapter 15 of Villanacci et al. (2002). However, our perturbations are simpler than the ones in Villanacci et al. (2002), because the latter are quadratic perturbations, while our perturbations are linear.

deduces that a (t, T)-equilibrium ((t, τ)-equilibrium and (t, α, T)-equilibrium, resp.) of the economy (u^a, e) is also a (t, T)-equilibrium ((t, τ)-equilibrium and (t, α, T)-equilibrium, resp.) of the economy (u, e), because the marginal utilities in own consumption coincide, i.e., $D_{x_i} u_i^a(x) = D_{x_i} u_i(x)$ for all i.

3 Equilibria and their properties

In this section, we focus on full trade regular competitive Nash equilibria. This allows to characterize equilibria with taxes and transfers for tax-transfer systems small enough as equilibria with simpler budget constraints, that are linear. Using this characterization, we show that equilibria with taxes and transfers depend smoothly on the tax-transfer system.

Without taxes and transfers, under Assumption 1, the utility maximization problem $(\mathcal{P}_i)$ has a unique solution, and i's individual demand is differentiable with respect to (p, e_i, x_{-i}). On the other hand, i's budget constraint with taxes and transfers is not smooth because of the kink at e_i. Further, in the presence of subsidies, it might also be non-convex. To overcome these difficulties, we follow Citanna, Kajii and Villanacci (1998), Citanna, Polemarchakis and Tirelli (2006), and Geanakoplos and Polemarchakis (2008), who have dealt with this kind of issue. We consider *full trade equilibria*, i.e., equilibria where every individual trades every commodity:

$$x_i^{*\ell} \neq e_i^\ell, \ \forall (i, \ell) \in \mathcal{I} \times \mathcal{L}.$$

We first show that, without taxes and transfers, full trade equilibria happen almost everywhere. That is, there is an open and full Lebesgue measure set of regular economies in the endowment space Ω, where every competitive Nash equilibrium is full trade.

Lemma 1 (Properties of competitive Nash equilibria) *Competitive Nash equilibria exist for all $(u, e) \in \mathcal{U} \times \Omega$. For every $u \in \mathcal{U}$, there exists an open and full Lebesgue measure subset Ω_u^* of Ω such that every $e \in \Omega_u^*$ is a regular economy.*[18] *Further, for every $e \in \Omega_u^*$:*

1. *there is an open neighborhood V of e such that each e' in V is regular and and has a finite, odd, constant number of competitive Nash equilibria.*

2. *At each competitive Nash equilibrium, $x_i^{*\ell} \neq e_i^\ell$ for all $(i, \ell) \in \mathcal{I} \times \mathcal{L}$.*

[18] An economy e is regular if it has a finite (odd) number of equilibria that smoothly depend on the endowments in a neighborhood of e.

From now on, we fix a profile of utilities $u \in \mathcal{U}$, a full trade economy $e \in \Omega^*_u$, and a competitive Nash equilibrium (x^*, p^*) of (u, e). As in Geanakoplos and Polemarchakis (2008), for every vector of tax rates $t \in \mathbb{T}$, we consider the function $t^*_i(t) = (t^{*\ell}_i(t))_{\ell \in \mathcal{L}} \in \mathbb{R}^L$ defined as follows, for all $\ell \in \mathcal{L}$:

$$t^{*\ell}_i(t) = \begin{cases} t^\ell & \text{if } \ x^{*\ell}_i > e^\ell_i \\ 0 & \text{if } \ x^{*\ell}_i < e^\ell_i \end{cases}$$

Remark 2 *The set of solutions of problem ($\mathcal{P}_i$) is upper semi-continuous with respect to (p, t, T) around $(p^*, 0, 0)$, because of the lower and upper semi-continuity of the budget constraint (1) with respect to (p, t, T). Further, we have that $t^*_i(t) \cdot (x_i - e_i) = t \cdot (x_i - e_i)_+$ around x^*_i. Therefore, around $(p^*, 0, 0)$, the set of solutions of problem ($\mathcal{P}_i$) is the same as the one where the budget constraint (1) is replaced by the following linear budget constraint.*[19]

$$(p + t^*_i(t)) \cdot (x_i - e_i) \leq T.$$

Then, there exists a unique solution of problem ($\mathcal{P}_i$), which is differentiable with respect to (p, t, T). The same argument applies to the budget constraint with personalized transfers or with personalized taxes with respect to (t, τ) or (t, α, T), respectively.

Hence, we can characterize (t, T)-equilibria, (t, τ)-equilibria, and (t, α, T)-equilibria, respectively, around (x^*, p^*), through first order conditions with linear budget constraints, market clearing, and tax balance conditions. This allows us to prove the following result.

Lemma 2 (Local properties of equilibria with taxes and transfers) *Let $e \in \Omega^*_u$ be a full trade economy and let (x^*, p^*) be a competitive Nash equilibrium associated with e.*

1. *There exist an open neighborhood $\mathcal{T} \subset \mathbb{T}$ containing $t = 0$, and $\mathcal{C}^1$ mappings $x : \mathcal{T} \to (\mathbb{R}^L_{++})^I$ and $p : \mathcal{T} \to S$ such that $(x(0), p(0)) = (x^*, p^*)$, and for every $t \in \mathcal{T}$, $(x(t), p(t))$ is the unique (t, T)-equilibrium around (x^*, p^*) at the transfer $T = \frac{1}{I} \sum_{i=1}^{I} t \cdot (x_i(t) - e_i)_+$.*

2. *Define $\tau_{-1} = (\tau_i)_{i \neq 1} \in \mathbb{R}^{I-1}$, there exist an open neighborhood $\mathcal{T}' \subset \mathbb{T} \times \mathbb{R}^{I-1}$ containing $(t, \tau_{-1}) = (0, 0)$, and $\mathcal{C}^1$ mappings $x : \mathcal{T}' \to (\mathbb{R}^L_{++})^I$ and $p : \mathcal{T}' \to S$ such that $(x(0, 0), p(0, 0)) = (x^*, p^*)$, and for every $(t, \tau_{-1}) \in$*

[19] For a detailed discussion, see pages 688–689 in Geanakoplos and Polemarchakis (2008).

$\mathcal{T}'$, $(x(t,\tau_{-1}), p(t,\tau_{-1}))$ *is the unique* (t,τ)*-equilibrium around* (x^*, p^*) *at the transfers* $\tau = (\tau_1, \tau_{-1})$ *and* $\tau_1 = \sum_{i=1}^{I} t \cdot (x_i(t,\tau_{-1}) - e_i)_+ - \sum_{i \neq 1} \tau_i$.

3. *There exist an open neighborhood* $\mathcal{T}'' \subset \mathbb{T} \times \mathbb{R}^I$ *containing* $(t,\alpha) = (0,0)$, *and* $\mathcal{C}^1$ *mappings* $x : \mathcal{T}'' \to (\mathbb{R}^L_{++})^I$ *and* $p : \mathcal{T}'' \to S$ *such that* $(x(0,0), p(0,0)) = (x^*, p^*)$, *where for every* $(t,\alpha) \in \mathcal{T}''$, $(x(t,\alpha)), p(t,\alpha))$ *is the unique* (t,α,T)*-equilibrium around* (x^*, p^*) *at the transfer* $T = \frac{1}{I}\sum_{i=1}^{I}(1+\alpha_i)t \cdot (x_i(t,\alpha) - e_i)_+$.

4 Tax-transfer Pareto improving policies

This section presents the key results of the paper with respect to the different tax-transfer systems considered in the previous sections.

In Subsection 4.1, we study the case of uniform taxes and personalized transfers. We get the generic result of these Pareto improving policies without any restriction on the numbers of individuals I and commodities L. In Subsection 4.2, we extend the generic result of Geanakoplos and Polemarchakis (2008) to less restrictive preferences that are not necessarily separable in externalities. Further, we show by means of an example that the restriction $I < L$ is crucial in the case of uniform taxes and equal transfers. In Subsection 4.3, we consider the case of personalized taxes. As in the case with personalized transfers, we get the generic result without any restriction on I and L. Actually, this means that one needs to have enough policy instruments to achieve Pareto improvements.

Finally, in Subsection 4.4, we show that *differentiably Pareto non-optimal allocations* can be improved by uniform taxes and personalized transfers, without perturbing the utility functions, when preferences are of the Bergstrom-Samuelson type. We conclude Subsection 4.4 with an application to a two-individual economy, where we explicitly compute the tax-transfer policy with personalized transfers.

4.1 Uniform taxes and personalized transfers

The following results state that, generically in the space of utility functions, every competitive Nash equilibrium can be Pareto improved by an equilibrium with uniform taxes and personalized transfers.

Theorem 1 *Let $(u, e) \in \mathcal{U} \times \Omega_u^*$ be a full trade economy. There exists an open and full Lebesgue measure $\mathcal{A}^0_{(u,e)}$ of $\mathcal{A}^0$ such that, for all $a \in \mathcal{A}^0_{(u,e)}$, every competitive Nash equilibrium of the economy (u, e) can be Pareto improved by a (t, τ)-equilibrium of the economy (u^a, e).*

As a consequence of Theorem 1, one obtains the following corollary.

Corollary 1 *There exists an open and dense subset of $\mathcal{U} \times \Omega$ such that every competitive Nash equilibrium can be Pareto improved by a (t, τ)-equilibrium.*

4.2 Uniform taxes and equal transfers

In this subsection, we study Pareto improvement policies with uniform taxes and equal transfers for all individuals. The following theorem is obtained by adapting the proofs of Theorems 1 and Corollary 1 to this framework, when there are more commodities than individuals.

Theorem 2 *Assume that $I < L$.*

i) Let $(u, e) \in \mathcal{U} \times \Omega_u^*$ be a full trade economy. There exists an open and full Lebesgue measure $\mathcal{A}^1_{(u,e)}$ of $\mathcal{A}^0$ such that, for all $a \in \mathcal{A}^1_{(u,e)}$, every competitive Nash equilibrium of the economy (u, e) can be Pareto improved by a (t, T)-equilibrium of the economy (u^a, e).

ii) There exists an open and dense subset of $\mathcal{U} \times \Omega$ such that every competitive Nash equilibrium can be Pareto improved by a (t, T)-equilibrium.

Note that the restriction $I < L$ is also made in Geanakoplos and Polemarchakis (2008). This condition ensures that the dimension of the tax policy is large enough to improve the welfare of all individuals.[20]

This restriction is crucial for the genericity of Pareto improving policies with equal transfers. If $I \geq L$, even with utility perturbations, it is impossible to obtain a generic result with anonymous taxes and equal transfers. We show this by means of an example.

Example 2. There are two individuals and two commodities. The individual utility functions are $u_1(x) = m_1(x_1) + m_2(x_2)$ and $u_2(x) = m_2(x_2) - m_1(x_1)$, where $m_i(x_i) = x_i^1 - \frac{1}{2}(x_i^1)^2 + x_i^2$ for all i. For all $i = 1, 2$, Assumption 1.(4) is satisfied for every $0 < x_i^1 < 1$. Therefore, in what follows, we focus

[20] This kind of restriction is usually made also in the literature on the constrained suboptimality with incomplete markets. See, for example, Cass and Citanna (1998), Citanna, Kajii and Villanacci (1998), and Villanacci et al. (2002).

on equilibria where the individual consumptions of commodity 1 are strictly lower than 1.

The initial endowments are $e_1 = (\varepsilon_1, 1-\varepsilon_2)$ and $e_2 = (1-\varepsilon_1, \varepsilon_2)$ for $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ small enough. Without taxes and transfers, there is a unique full trade competitive Nash equilibrium $x_1^* = \left(\frac{1}{2}, \frac{3}{4} + \frac{\varepsilon_1}{2} - \varepsilon_2\right)$, $x_2^* = \left(\frac{1}{2}, \frac{1}{4} + \varepsilon_2 - \frac{\varepsilon_1}{2}\right)$, and $p^* = \left(\frac{1}{2}, 1\right)$. This equilibrium is not Pareto optimal because, by transferring commodity 2 from individual 1 to individual 2, one can improve the utility of individual 2 without decreasing the utility of individual 1.

Let t be the tax rate for commodity 1. The price for individual 1 is $(p+t, 1)$, because individuals 1 is a buyer for commodity 1. The price for individual 2 is $(p, 1)$ since individual 2 is a seller for commodity 1. After simple computation, the tax equilibrium is $\tilde{x}_1^1 = \dfrac{1-t}{2}$, $\tilde{x}_1^2 = \dfrac{3}{4} + \dfrac{\varepsilon_1}{2} - \varepsilon_2 + \dfrac{t}{4}$, $\tilde{x}_2^1 = \dfrac{1+t}{2}$ and $\tilde{x}_2^2 = \dfrac{1}{4} + \varepsilon_2 - \dfrac{\varepsilon_1}{2} - \dfrac{t}{4}$. Hence, we get $u_1(\tilde{x}) - u_1(x^*) = -\dfrac{t^2}{4}$ and $u_2(\tilde{x}) - u_2(x^*) = 0$. Therefore, there is no t such that $u_i(\tilde{x}) - u_i(x^*) \geq 0$ for all i with at least one strict inequality.

Importantly, even with linear perturbations $u_i^a(x) = u_i(x) + a_{i,j}^1 x_j^1 + a_{i,j}^2 x_j^2$, it is impossible to obtain Theorem 1. Indeed, one gets:

$$u_1^a(\tilde{x}) - u_1^a(x^*) = \frac{t}{4}(-t + 2a_{1,2}^1 - a_{1,2}^2) \text{ and } u_2^a(\tilde{x}) - u_2^a(x^*) = \frac{t}{4}(-2a_{2,1}^1 + a_{2,1}^2).$$

Now consider the set $A = \{a \in \mathbb{R}^4 : a_{1,2}^1 > 0, a_{1,2}^2 < 0, a_{2,1}^1 > 0, a_{2,1}^2 < 0\}$. Then for any $a \in A$, one gets that $u_1^a(\tilde{x}) - u_1^a(x^*) < 0$ for any $t < 0$ and $u_2^a(\tilde{x}) - u_2^a(x^*) < 0$ for any $t > 0$. Therefore, it is impossible to obtain a Pareto improvement where $u_i^a(\tilde{x}) - u_i^a(x^*) > 0$ for all i. Importantly, the set A has positive Lebesgue measure as it is an open set.

4.3 Personalized taxes and equal transfers

In this case of personalized taxes and equal transfers, the number of policy instruments is large enough, because $(L-1) + I > I$. This allows to obtain the result for all $I, L \geq 2$.

Theorem 3

i) Let $(u, e) \in \mathcal{U} \times \Omega_u^*$ be a full trade economy. There exists an open and full Lebesgue measure $\mathcal{A}_{(u,e)}^2$ of $\mathcal{A}^0$ such that, for all $a \in \mathcal{A}_{(u,e)}^2$, every competitive Nash equilibrium of the economy (u, e) can be Pareto improved by a (t, α, T)-equilibrium of the economy (u^a, e).

ii) There exists an open and dense subset of $\mathcal{U} \times \Omega$ such that every competitive Nash equilibrium can be Pareto improved by a (t, α, T)-equilibrium.

Note that as a consequence of Theorems 1, 2, and 3, we get that for almost all the economies, all competitive Nash equilibria allocations are not Pareto optimal.

4.4 Pareto improving policies with Bergson-Samuelson utilities

In this subsection, we consider Bergson-Samuelson utilities. That is, for every $i \in \mathcal{I}$, $u_i(x) = V_i(m_1(x_1), \ldots, m_I(x_I))$, with V_i strictly increasing in component i, and m_k continuous, strictly increasing, and strictly quasi-concave in own consumption x_k. This class of preferences has been introduced by Bergson and Samuelson, and it has been largely studied in welfare economics. Further, Bergson-Samuelson utilities have recently attracted the attention of Dufwenberg et al. (2011), and Bourlès, Bramoullé and Perez-Richet (2017).

As for this class of utilities, we prove that, at a *differentiably Pareto non-optimal* equilibrium, the result on Pareto improving policies does not require utilities perturbations. A feasible allocation x^* is differentiably Pareto non-optimal if there exists $z = (z_i)_{i \in \mathcal{I}} \in \mathbb{R}^{LI}$, $z \neq 0$, with $\displaystyle\sum_{i=1}^{I} z_i = 0$ such that $Du_i(x^*) \cdot z > 0$ for all $i \in \mathcal{I}$. This implies that, for all $\varepsilon > 0$ small enough, $u_i(x^* + \varepsilon z) > u_i(x^*)$ for all i. Hence, $x^* + \varepsilon z$ is a Pareto improvement of x^*.

The following proposition is the main result in this subsection.

Proposition 1 *Let $(u, e) \in \mathcal{U} \times \Omega$ such that $e \in \Omega^*_u$ is a full trade economy, where u_i belongs to the class of Bergson-Samuelson utilities for all i. Let x^* be a differentiably Pareto non-optimal competitive Nash equilibrium allocation of this economy. Then, x^* can be improved by a (t, τ)-equilibrium of the economy (u, e).*

Remark 3 *Actually, the result above holds true for the class of utility functions satisfying the following assumption, which is slightly larger than the Bergson-Samuelson class.*

Assumption 3 *For all $v \in \mathbb{R}^{LI}$ such that $v_i \in \operatorname{Ker} D_{x_i} u_i(x^*)$ for all $i \in \mathcal{I}$, $\sum_{i=1}^{I} v_i = 0$ and for all $\ell \neq L$, $\sum_{i=1}^{I} v_i^\ell \mathbf{1}_{\{x_i^\ell - e_i^\ell > 0\}} = 0$, then $D_x u_k(x^*) \cdot v = 0$ for all $k \in \mathcal{I}$,*

(i) Assumption 3 holds true in the class of proportional marginal utilities introduced by del Mercato and Nguyen (2023), which contains the class

of Bergson-Samuelson utilities. Indeed, since, by definition, there exists $c_{i,j} \in \mathbb{R}$ such that $D_{x_j} u_i(x^*) = c_{i,j} D_{x_j} u_j(x^*)$, for all $(i,j) \in \mathcal{I} \times \mathcal{I}$. Then, $v_i \in \operatorname{Ker} D_{x_i} u_i(x^*)$ for all $i \in \mathcal{I}$ entails $D_x u_i(x^*) \cdot v = 0$ for all $i \in \mathcal{I}$.

(ii) *Assumption 3 is also satisfied in any two-individual economy. This is because, at a full-trade equilibrium, for every commodity ℓ, only one individual is a seller and the other one is a buyer. Then, from $\sum_{i=1}^{2} v_i = 0$ and $\sum_{i=1}^{2} v_i^\ell \mathbf{1}_{\{x_i^\ell - e_i^\ell > 0\}} = 0$ all ℓ, one deduces that $v = 0$.*

We end this subsection by showing how we can compute a tax-transfer Pareto improving equilibrium for a two-individual economy.

Let (x_1^*, x_2^*, p^*) be a full-trade equilibrium such that (x_1^*, x_2^*) is differentiably Pareto non-optimal. Then $D_{x_1} u_1(x^*) - D_{x_2} u_1(x^*)$ is not positively collinear to $D_{x_2} u_2(x^*) - D_{x_1} u_2(x^*)$.[21] Thus $z = (z_1, z_2) \in \mathbb{R}^{2L}$ with

$$z_1 = \frac{D_{x_1} u_1(x^*) - D_{x_2} u_1(x^*)}{\|D_{x_1} u_1(x^*) - D_{x_2} u_1(x^*)\|} - \frac{D_{x_2} u_2(x^*) - D_{x_1} u_2(x^*)}{\|D_{x_2} u_2(x^*) - D_{x_1} u_2(x^*)\|}$$

and $z_2 = -z_1$ satisfies $D_x u_i(x^*) \cdot z > 0$ for all $i = 1, 2$. For $\varepsilon > 0$ small enough, consider:

$$\tilde{x} = x^* + \varepsilon z,$$

that is a full trade Pareto improvement of x^*. We now define the price p and the tax rate t as follows, for all $\ell \neq L$:

$$p^\ell = \frac{D_{x_i^\ell} u_i(\tilde{x})}{D_{x_i^L} u_i(\tilde{x})} \text{ for the unique individual } i \text{ such that } \tilde{x}_i^\ell < e_i^\ell,$$

$$t^\ell = \frac{D_{x_j^\ell} u_j(\tilde{x})}{D_{x_j^L} u_j(\tilde{x})} - p^\ell \text{ for the unique individual } j \text{ such that } \tilde{x}_j^\ell > e_j^\ell.$$

Since $\tilde{x}$ is a full-trade allocation, p and t are well-defined. At equilibrium, the individual transfers are defined by $\tau_i = (p + t) \cdot (\tilde{x}_i - e_i)$ for all i. By construction, for $i = 1, 2$, $\tilde{x}_i$ is a unique solution of i's individual problem:

$$\begin{aligned} &\max_{x_i \in \mathbb{R}^L_{++}} u_i(x_i, \tilde{x}_j) \\ &\text{s.t } (p + t) \cdot (x_i - e_i) \leq \tau_i \end{aligned}$$

Since u_i is strictly quasi-concave on x_i, p is near to p^* and τ_i is near to 0 for all $i = 1, 2$, $\tilde{x}_i$ is also the solution for the problem below.

$$\begin{aligned} &\max_{x_i \in \mathbb{R}^L_{++}} u_i(x_i, \tilde{x}_j) \\ &\text{s.t } (p + t) \cdot (x_i - e_i)_+ - p \cdot (x_i - e_i)_- \leq \tau_i \end{aligned}$$

[21] Since in a two individuals economy, first order conditions for Pareto optimality are equivalent to have $D_{x_1} u_1(x^*) - D_{x_2} u_1(x^*) = \theta(D_{x_2} u_2(x^*) - D_{x_1} u_2(x^*))$ for some $\theta > 0$.

Therefore, $(p, t, \tilde{x})$ is a (t, τ)-equilibrium, which Pareto improves the competitive Nash equilibrium (x^*, p^*). ■

Appendix

Example (e). We consider the two-individual, two-commodity economy in Subsection 6.1 of del Mercato and Nguyen (2023), where the global consumption of commodity 2 exhibits positive externalities. Individual i's consumption is $x_i = (x_i^1, x_i^2)$, and for each $i = 1, 2$ and $j \neq i$, the individual log-linear utility function is:[22]

$$u_i(x_i, x_j^2) = \ln x_i^1 + \ln \left[x_i^2 + \varepsilon_i(x_i^2 + x_j^2)\right],$$

where $0 < \varepsilon_i < 1$ measures how much individual i positively cares about the global consumption of commodity 2.

Each individual i maximizes u_i on his budget constraint $p \cdot x_i \leq p \cdot e_i$, by taking as given the consumption choice x_j^2 of the other individual j. At a competitive equilibrium, the individual marginal utilities of individuals 1 and 2 must be positively proportional to the competitive price, and consequently $D_{x_1} u_1(x_1, x_2^2) = \gamma D_{x_2} u_2(x_2, x_1^2)$ for some $\gamma > 0$. This is equivalent to have:

$$\text{MRS}_{2,1}^1 = \frac{x_1^1(1 + \varepsilon_1)}{x_1^2 + \varepsilon_1(x_1^2 + x_2^2)} = \text{MRS}_{2,1}^2 = \frac{x_2^1(1 + \varepsilon_2)}{x_2^2 + \varepsilon_2(x_1^2 + x_2^2)}. \tag{5}$$

On the other hand, consider the social planner who maximizes a weighted sum of the utilities of the two individuals:

$$v_w(x_1, x_2) = w_1 u_1(x_1, x_2^2) + w_2 u_2(x_2, x_1^2),$$

under the classical feasibility constraint $x_1 + x_2 = r$. Using first order necessary and sufficient conditions associated with the latter problem, one easily gets that a feasible allocation $x = (x_1, x_2) \in \mathbb{R}^4_{++}$ is Pareto optimal if and only if there exist strictly positive weights $w = (w_1, w_2)$ such that:

$$D_{x_1} v_w(x_1, x_2) = D_{x_2} v_w(x_1, x_2).$$

This means that the social marginal utility with respect to the consumption x_1 equals the social marginal utility with respect to the consumption x_2. It turns out that the equality above holds true if and only if

$$\frac{x_1^1}{x_1^2 + \varepsilon_1(x_1^2 + x_2^2)} = \frac{x_2^1}{x_2^2 + \varepsilon_2(x_1^2 + x_2^2)}. \tag{6}$$

[22] Up to a strictly increasing transformation, u_i represents the same preferences as the restriction on $\mathbb{R}^3_{++}$ of the function $\widetilde{u}_i(x_i, x_j^2) = x_i^1 \left[x_i^2 + \varepsilon_i(x_i^2 + x_j^2)\right]$. Notice that $\widetilde{u}_i$ satisfies all the basic conditions given in Assumption 1.

Therefore, by comparing (5) and (6), one deduces that a competitive equilibrium allocation is never Pareto optimal if $\varepsilon_1 \neq \varepsilon_2$. ■

For the proofs of our results, we adopt the standard characterization of equilibria by necessary and sufficient first order conditions. Note that we need to be careful with equilibria with tax policy since the budget constraint is not smooth and the budget set may be non convex.

We first present the result for (t,τ)-equilibrium. Then we deal with anonymous tax and equal transfer, which leads to the same kind of equations with less variables, and we finally show how to adapt the proof for the case of personalized taxes and equal transfer. We write the conditions for a pseudo (t,τ)-equilibrium where the true budget constraint is replaced by the linear one as described in Remark 2. Let us consider the Lagrange multiplier λ_i associated with the budget constraint $(p + t_i^*(t)) \cdot (x_i - e_i) \leq \tau_i$, and define the set of endogenous variables as $\Xi := \mathbb{R}^{LI}_{++} \times \mathbb{R}^{I}_{++} \times \mathbb{R}^{L-1}_{++}$ with generic element $\xi := (x, \lambda, p^\backslash) := \left((x_i, \lambda_i)_{i\in\mathcal{I}}, p^\backslash\right)$. The equilibrium function is defined by

$$\Gamma_e : \Xi \times \mathbb{R}^{L-1+I} \to \mathbb{R}^{\dim \Xi + 1},\ \Gamma_e(\xi, t, \tau) = \left(T_e(\xi, t, \tau), R_e(\xi, t, \tau)\right), \tag{7}$$

where $T_e(\xi, t, \tau)$ is determined by the first order conditions associated with i's utility maximization under the budget constraint $(p + t_i^*(t)) \cdot (x_i - e_i) \leq \tau_i$, and market clearing conditions, i.e.,

$$T_e(\xi, t, \tau) = \left(\left(T_e^{(i,1)}(\xi, t, \tau), T_e^{(i,2)}(\xi, t, \tau)\right)_{i\in\mathcal{I}}, T_e^M(\xi, t, \tau)\right),$$

with:

$$\begin{aligned} T_e^{(i,1)}(\xi, t, \tau) &= D_{x_i} u_i(x) - \lambda_i \left(p + t_i^*(t)\right), \\ T_e^{(i,2)}(\xi, t, \tau) &= (p + t_i^*(t)) \cdot (x_i - e_i) - \tau_i, \\ T_e^M(\xi, t, \tau) &= \sum_{i\in\mathcal{I}} x_i^\backslash - \sum_{i\in\mathcal{I}} e_i^\backslash, \end{aligned} \tag{8}$$

while the tax balance condition is $R_e(\xi, t, \tau) = \sum_{i\in\mathcal{I}} \tau_i - \sum_{i\in\mathcal{I}} t_i^*(t) \cdot (x_i - e_i)$

Remark 4 *Remark that, by Assumption 1,*

i) the equilibrium function for the competitive Nash equilibrium is $T_e(\xi, 0, 0)$ since $t_i^*(0) = 0$ for all $i \in \mathcal{I}$, and for every $(\xi, t, \tau) \in \Gamma_e^{-1}(0)$, τ_1 is completely determined by the equation $R_e(\xi, t, \tau) = 0$.

ii) If we consider a perturbation u^a of the utility functions, as given in the analytical form (4), then this perturbation does not affect the individual's marginal utilities since $D_{x_i} u_i(x) = D_{x_i} u_i^a(x)$, and consequently it has no effects on the competitive equilibria.

Note that, in the presence of perturbations, the domain of equilibrium function Γ_e changes, because it now depends on the perturbation parameters $a \in \mathcal{A}^0$. However, its components are the same. Then, we use the notation $\widehat{\Gamma}_e$ for the function from $\Xi \times \mathbb{R}^{L-1+I} \times \mathcal{A}^0$ to $\mathbb{R}^{\dim \Xi+1}$, that has the same components as in (8) and where u_i replaced by u_i^a.

Proof of Lemma 1. (1) The existence of equilibria is a consequence of del Mercato (2006). The generic regularity follows from Bonnisseau and del Mercato (2010). (2) As for the genericity of full-trade equilibrium, it suffices to remark that if one adds to the equilibrium equations a new condition $x_i^\ell - e_i^\ell = 0$ for a given (i, ℓ), then 0 is still a regular value of the new equilibrium function. The dimension of the range space of the new equilibrium function is also strictly greater than the number of equilibrium variables ξ. Hence, one concludes that for an open and full Lebesgue measure set of endowments $\Omega_{i,\ell}^*$, no equilibrium satisfies the equation $x_i^\ell - e_i^\ell = 0$. Then, by taking the intersection over all pairs (i, ℓ), we get the result. See, Chapter 4 of Nguyen (2022) for a detailed proof. ■

Proof of Lemma 2. We prove the result for (t, τ)-equilibrium, i.e., the result in (2). Let $\xi^* = (x^*, \lambda^*, p^{*\backslash})$ be the (extended) competitive Nash equilibrium associated with e. We first prove that Γ_e is $\mathcal{C}^2$ around $(\xi^*, 0, 0)$, and the mapping $D_{\xi,t,\tau}\Gamma_e(\xi^*, 0, 0)$ is onto. Note that t_i^* is a linear function of t. Consequently, Γ_e is $\mathcal{C}^2$ around $(\xi^*, 0, 0)$. The Jacobian matrix of Γ_e at the point $(\xi^*, 0, 0)$ is

$$\begin{array}{c} \begin{array}{cc} \xi \qquad\qquad\qquad & \tau \end{array} \\ \begin{pmatrix} D_\xi T_e(\xi^*, 0, 0) & D_\tau T_e(\xi^*, 0, 0) \\ D_\xi R_e(\xi^*, 0, 0) & D_\tau R_e(\xi^*, 0, 0) \end{pmatrix} \end{array}$$

So, the Jacobian matrix $D_{\xi,\tau_1}\Gamma_e(\xi^*, 0, 0)$ is equal to

$$\begin{pmatrix} D_\xi T_e(\xi^*, 0, 0) & D_{\tau_1} T_e(\xi^*, 0, 0) \\ 0 & 1 \end{pmatrix}$$

and it has full row rank since (x^*, λ^*, p^*) is a regular competitive Nash equilibrium, hence the Jacobian matrix $D_\xi T_e(\xi^*, 0, 0)$ has full row rank. From Remarks 2, 4 and the Implicit Function Theorem, one obtains the result in (2). Results in (1) and (3) are proved using the similar argument. ■

4.5 The methodology

Before presenting the proof of Theorems 1, 2 and 3, we present the common structure of these proofs. Then, we also give the proof of a preliminary lemma, which is presented hereafter as Lemma 3.

By Lemma 2, (t, τ)-equilibria smoothly depend on the tax rates and transfers (t, τ_{-1}) around $(0,0)$. For every $(t, \tau_{-1}) \in \mathcal{T}$, let us consider the indirect utility levels of all individuals at a (t, τ)-equilibrium, that is:

$$\tilde{G}(t, \tau_{-1}) = \left(u_i\left(x(t, \tau_{-1})\right)\right)_{i \in \mathcal{I}}$$

In order to achieve a Pareto improvement, it is enough to show that there exists $(t^*, \tau^*_{-1}) \in \mathbb{T} \times \mathbb{R}^{I-1}$ such that

$$\left[D_{t,\tau_{-1}} \tilde{G}(0,0)\right](t^*, \tau^*_{-1}) \gg 0.$$

Indeed, using directional derivatives, one gets that every individual is strictly better-off at $(\varepsilon t^*, \varepsilon \tau^*_{-1})$ for every $\varepsilon > 0$ small enough. By Gordan's Theorem (Mangasarian, 1969, page 31), the above condition is equivalent to prove that there is no $\pi = (\pi_i)_{i \in \mathcal{I}} \in \mathbb{R}^I_+ \setminus \{0\}$ such that $\pi D_{t,\tau_{-1}} \tilde{G}(0,0) = 0$.

Working with the Jacobian matrix $D_{t,\tau_{-1}} \tilde{G}(0,0)$ is not an easy task, because it combines the direct effect $D_{t,\tau_{-1}} x(t, \tau_{-1})$ of tax policy changes on equilibrium consumptions with the indirect effects $D_{x_{-i}} u_i(x(t, \tau_{-1})))$ of tax policy changes on external marginal utilities.

This difficulty does not arise in Geanakoplos and Polemarchakis (2008), because individual utilities are linear in consumption externalities. Thus, the indirect effects are constant for all tax policies. In order to overcome this difficulty, we use the methodology developed in Cass and Citanna (1998) and Citanna, Kajii and Villanacci (1998).[23]We consider the Jacobian matrix of the mapping (Γ_e, G), where the mapping $G : \Xi \times \mathbb{T} \times \mathbb{R}^I \to \mathbb{R}^I$ is

$$G(\xi, t, \tau) = \left(u_1(x), \ldots, u_I(x)\right). \tag{9}$$

This allows to handle the direct and indirect effects of tax policy changes separately. The following lemma shows that the two approaches are equivalent.

Lemma 3 *Let $\xi^* \in \Xi$ be a competitive Nash equilibrium associated with the full trade economy $(u, e) \in \mathcal{U} \times \Omega^*_u$. The two following properties are equivalent.*

1. *There is no $\pi \in \mathbb{R}^I_+ \setminus \{0\}$ such that $\pi D_{t,\tau_{-1}} \tilde{G}(0,0) = 0$.*
2. *There is no $(c_\xi, c_{\tau_1}, \pi) \in \mathbb{R}^{\dim \Xi} \times \mathbb{R} \times \mathbb{R}^I_+ \setminus \{0\}$ such that:*

$$(c_\xi, c_{\tau_1}, \pi) D_{\xi,t,\tau}(\Gamma_e, G)(\xi^*, 0, 0) = 0.$$

[23]The reader can also find a survey on this approach in Villanacci et al. (2002).

By Lemma 3, in order to prove the existence of a tax-transfer Pareto improving policy of ξ^*, it is enough to prove that $D_{\xi,t,\tau}(\Gamma_e, G)(\xi^*, 0, 0)$ has full row rank.

Proof of Lemma 3. Applying the Inverse Function Theorem to $(\xi^*, 0, 0) \in \Gamma_e^{-1}(0)$ as in the proof of Lemma 2, there exist $\mathcal{C}^1$ functions $\lambda : \mathcal{T} \to \mathbb{R}^I$ and $\tau_1 : \mathcal{T} \to \mathbb{R}$ such that $\chi(t, \tau_{-1}) = (x(t, \tau_{-1}), \lambda(t, \tau_{-1}), p(t, \tau_{-1}), \tau_1(t, \tau_{-1})) \in \Gamma_e^{-1}(0)$ is the unique extended (t, τ)-equilibrium around (ξ^*, τ_1^*) for all $(t, \tau_{-1}) \in \mathcal{T}$.

$$D_{t,\tau_{-1}}(\chi, \tau_1)(0, 0) = -(D_{\xi,\tau_1}\Gamma_e(\xi^*, 0, 0))^{-1} D_{t,\tau_{-1}}\Gamma_e(\xi^*, 0, 0)$$

and $\tilde{G}(t, \tau_{-1}) = G(\chi(t, \tau_{-1}), t, \tau_1(t, \tau_{-1}), \tau_{-1})$.

Let $\pi \in \mathbb{R}_+^I$ such that $\pi D_{t,\tau_{-1}}\tilde{G}(0, 0) = 0$.

Since $D_{t,\tau_{-1}}\tilde{G}(0, 0) = D_{\xi,\tau_1}G(\xi^*, 0, 0) D_{t,\tau_{-1}}(\chi, \tau_1)(0, 0) + D_{t,\tau_{-1}}G(\xi^*, 0, 0)$ and $D_{t,\tau_{-1}}G(\xi^*, 0, 0) = 0$,

$$\pi D_{\xi,\tau_1}G(\xi^*, 0, 0) D_{t,\tau_{-1}}(\chi, \tau_1)(0, 0) = 0$$

Now let $c = (c_x, c_\lambda, c_{p\setminus}, c_{\tau_1}) \in \mathbb{R}^{\dim \Xi + 1}$ defined by:

$$c = -\pi D_{\xi,\tau_1}G(\xi^*, 0, 0)(D_{\xi,\tau_1}\Gamma_e(\xi^*, 0, 0))^{-1}$$

Then,

$$(c, \pi) D_{\xi,t,\tau}(\Gamma_e, G)(\xi^*, 0, 0) = c D_{\xi,t,\tau}\Gamma_e(\xi^*, 0, 0) + \pi D_{\xi,t,\tau}G(\xi^*, 0, 0)$$

Note that

$$c D_{\xi,t,\tau}\Gamma_e(\xi^*, 0, 0) = \begin{pmatrix} c D_{\xi,\tau_1}\Gamma_e(\xi^*, 0, 0) & c D_{t,\tau_{-1}}\Gamma_e(\xi^*, 0, 0) \end{pmatrix}$$

and

$$\pi D_{\xi,t,\tau}G(\xi^*, 0, 0) = \begin{pmatrix} \pi D_{\xi,\tau_1}G(\xi^*, 0, 0) & \pi D_{t,\tau_{-1}}G(\xi^*, 0, 0) \end{pmatrix}$$

From the definition of c, $c D_{\xi,\tau_1}\Gamma_e(\xi^*, 0, 0) = -\pi D_{\xi,\tau_1}G(\xi^*, 0, 0)$, and

$$\begin{aligned} c D_{t,\tau_{-1}}\Gamma_e(\xi^*, 0, 0) &= -\pi D_{\xi,\tau_1}G(\xi^*, 0, 0)(D_{\xi,\tau_1}\Gamma_e(\xi^*, 0, 0))^{-1} D_{t,\tau_{-1}}\Gamma_e(\xi^*, 0, 0) \\ &= \pi D_{\xi,\tau_1}G(\xi^*, 0, 0) D_{t,\tau_{-1}}(\chi, \tau_1)(0, 0) = 0 \end{aligned}$$

Since $D_{t,\tau_{-1}}G(\xi^*, 0, 0) = 0$, one obtains $(c, \pi) D_{\xi,t,\tau}(\Gamma_e, G)(\xi^*, 0, 0)$.

Conversely, let (c, π) such that $(c, \pi) D_{\xi,t,\tau}(\Gamma_e, G)(\xi^*, 0, 0) = 0$. Using the same equations as above, one deduces that

$$c = -\pi D_{\xi,\tau_1}G(\xi^*, 0, 0)(D_{\xi,\tau_1}\Gamma_e(\xi^*, 0, 0))^{-1}$$

and

$$c D_{t,\tau_{-1}}\Gamma_e(\xi^*, 0, 0) = \pi D_{t,\tau_{-1}}\chi(0, 0) D_{t,\tau_{-1}}G(\xi^*, 0, 0) = 0$$

so, $\pi D_{t,\tau_{-1}}\tilde{G}(0,0) = 0$. ∎

Proof of Theorem 1. In order to take into account the perturbation parameter a, as we have done by extending the mapping Γ_e to $\widehat{\Gamma}_e$, we define the mapping $\widehat{G}$ as the mapping G, just extending its domain and replacing the utility functions u_i by the perturbed utility functions u_i^a.

We aim to prove that, at every regular full-trade extended competitive Nash equilibrium ξ^*, for almost all $a \in \mathcal{A}^0$, there is no non-zero solution $c = (c_x, c_\lambda, c_{p\backslash}, c_{\tau_1}) \in \mathbb{R}^{\dim \Xi + 1}$ and $\pi = (\pi_1, \ldots, \pi_I) \in \mathbb{R}^I_+$ of the following system:

$$(c,\pi) D_{\xi,t,\tau}(\widehat{\Gamma}_e, \widehat{G})(\xi^*, 0, 0, a) = 0.$$

To achieve this result, we consider the mapping Ψ defined on $\mathbb{R}^{\dim \Xi + 1} \times \mathbb{R}^I \times \mathcal{A}^0$ by

$$\Psi(c,\pi,a) = \begin{pmatrix} (c,\pi) D_{\xi,t,\tau}(\widehat{\Gamma}_e, \widehat{G})(\xi^*, 0, 0, a) \\ \sum_{i\in\mathcal{I}} \pi_i - 1 \end{pmatrix} \tag{10}$$

and we prove the following result.

Lemma 4 *0 is a regular value of Ψ.*

Proof of Lemma 4. We aim to show that for each $(c,\pi,a) \in \Psi^{-1}(0)$, the Jacobian matrix $D_{c,\pi,a}\Psi(c,\pi,a)$ has full row rank. The computation of the Jacobian matrix of Ψ is described below.

$$\begin{array}{cc} & \begin{array}{ccc} \qquad c \qquad & \qquad \pi \qquad & a \end{array} \\ \begin{array}{c} (c,\pi) D_{\xi,t,\tau}(\widehat{\Gamma}_e, \widehat{G}) \\ \sum_{i\in\mathcal{I}} \pi_i - 1 \end{array} & \begin{pmatrix} \left[D_{\xi,t,\tau}\widehat{\Gamma}_e\right]^T & \left[D_{\xi,t,\tau}\widehat{G}\right]^T & N \\ 0 & \mathbf{1}_I^T & 0 \end{pmatrix} \end{array}$$

The Jacobian matrix $D_{\xi,t,\tau}(\widehat{\Gamma}_e, \widehat{G})(\xi^*, 0, 0, a)$ and the one of $D_{c,\pi,a}\Psi(c,\pi,a)$ are given on pages 31 and 32. In these matrices, for all i, we denote by M_i the $L \times (L-1)$-matrix of the linear mapping $t \to t_i^*(t)$. Note also that $t_i^*(0) = 0$, then some terms disappear in the formula.

Now let $\delta = \big((\delta_{x_i}, \delta_{\lambda_i})_{i\in\mathcal{I}}, \delta_{p\backslash}, (\delta_{\tau_i})_{i\in\mathcal{I}}, \delta_t, \mu\big) \in \mathbb{R}^{\dim \Xi + I + L}$. We show that

$$\delta D_{c,\pi,a}\Psi(c,\pi,a) = 0 \Rightarrow \delta = 0$$

We first remark that $\sum_{i\in\mathcal{I}} \pi_i = 1$ implies that there exists j such that $\pi_j \neq 0$. Now, considering the product of the columns associated to $c_{p\backslash}$, c_τ, and $a_{i,j}$, we get

$$\begin{cases} \sum\limits_{i\in\mathcal{I}} (\delta_{x_i})^\backslash = 0 \ \text{(S.1)} \\ \sum\limits_{i\in\mathcal{I}} \delta_{x_i} \cdot p^* = 0 \ \text{(S.2)} \\ \pi_j \delta_{x_i} = 0 \ \forall i \neq j \ \text{(S.3)} \end{cases} \tag{S}$$

Since $\pi_j \neq 0$ for some j, it follows that $\delta_{x_i} = 0$ for any $i \neq j$. Then $(\delta_{x_i})^{\setminus} = 0$ and $\delta_{x_i} \cdot p^* = 0$ for all i. Combining with $\sum_i (\delta_{x_i})^{\setminus} = 0$, we get $\delta_{x_i} = 0$ for all i. One then remarks that the product of the column associated to π_i becomes $\mu = 0$.

Then, for all i, the product of the columns associated to c_{x_i} becomes:

$$-\delta_{\lambda_i} p^* + \lambda_i^* (\delta_{p^{\setminus}},\ 0) - \lambda_i^* \delta_t (M_i)^T = 0,$$

Since the last column of matrix $(M_i)^T$ contains only 0 and $p_L^* = 1$, we get $\delta_{\lambda_i} = 0$ for all i. Therefore, we get $(\delta_{p^{\setminus}}, 0) = t_i^*(\delta_t)$, for all i. For all $\ell = 1, \ldots, L-1$, since (u, e) is a full trade equilibrium, there is a j such that $x_j^{*\ell} - e_j^\ell > 0$ and a k such that $x_k^{*\ell} - e_k^\ell < 0$. Then, $t_{k\ell}^*(\delta_t) = 0$, hence $\delta_{p^{\setminus}\ell} = 0$. Furthermore, $t_{j\ell}^*(\delta_t) = \delta_{t\ell} = 0$. Consequently, $\delta_t = 0$ and $\delta_{p^{\setminus}} = 0$. Hence, we have $\delta = 0$. ■

Now, let $\big(\xi^*(k)\big)_{k\in\kappa}$ be the finite set of equilibria of the economy (u, e). For a given k, as a consequence of Lemma 4 and the Transversality Theorem, there exists a full Lebesgue measure subset $\mathcal{A}^k_{(u,e)}$ of $\mathcal{A}^0$ such that for all $a \in \mathcal{A}^k_{(u,e)}$, 0 is a regular value for the mapping $\Psi(\cdot,\cdot,a)$ for ξ_k^*. The dimension of the domain of $\Psi(\cdot,\cdot,a)$ is $\dim \Xi + 1 + I$, which is strictly less than the dimension of the range of $\Psi(\cdot,\cdot,a)$, $\dim \Xi + L + I$. Therefore, for any $a \in \mathcal{A}^k_{(u,e)}$ the system $\Psi(c, \pi, a) = 0$ has no solution.

We now show that the set $\mathcal{A}^k_{(u,e)}$ is open since $a \in \mathcal{A}^k_{(u,e)}$ if and only if $D_{\xi,t,\tau}(\widehat{\Gamma}_e, \widehat{G})(\xi_k^*, 0, 0, a)$ has full row rank, which means that a determinant is different from 0. Since the determinant is continuous with respect to a, we get the result.

The proof of Theorem 1 is then complete by taken $\mathcal{A}^0_{(u,e)} = \cap_{k\in\kappa} \mathcal{A}^k_{(u,e)}$. ■

Proof of Corollary 1. With innocuous abuse of notation, we extend the domains of the mappings Γ_e and G to encompass the spaces of utilities and of endowments.

We consider the subset $\mathcal{E}^{PI}$ of $\mathcal{U} \times \Omega$ defined as the economy (u, e) such that $e \in \Omega_u^*$ and $D_{\xi,t,\tau}(\Gamma_e, G)(\xi, 0, 0)$ has full row rank for all competitive Nash equilibrium ξ. We first prove that the set $\mathcal{E}^{PI}$ is dense and open. Then, the corollary is an immediate consequence of Theorem 1 since, for all $(u, e) \in \mathcal{E}^{PI}$, every competitive Nash equilibrium of (u, e) can be Pareto improved by a tax policy with lump-sum transfers.

Claim 1. $\mathcal{E}^{PI}$ is dense in $\mathcal{U} \times \Omega$

Let $(\bar{e}, \bar{u}) \in \Omega \times \mathcal{U}$ and an arbitrary open neighborhood $\mathcal{O}$ of (e, u). We show that there exists a perturbation parameter a such that $(e, \bar{u}^a) \in \mathcal{O} \cap \mathcal{E}^{PI}$.

By definition of product topology, there exists two open neighborhood of $\mathcal{O}_e$ of $\bar{e}$ and $\mathcal{O}_u$ of $\bar{u}$ such that $\mathcal{O}_e \times \mathcal{O}_u \subset \mathcal{O}$. From Lemma 1, there exists an endowment $e \in \Omega^*_{\bar{u}} \cap \mathcal{O}_e$. Then since the mapping $a \to \bar{u}^a$ is continuous and $\mathcal{A}^0_{(\bar{u},e)}$ is dense in $\mathcal{A}^0$, there exists $a \in \mathcal{A}^0_{(\bar{u},e)}$ close enough to 0 such that $\bar{u}^a \in \mathcal{O}_u$. Consequently, $(e, \bar{u}^a) \in \mathcal{O} \cap \mathcal{E}^{PI}$.

Claim 2. $\mathcal{E}^{PI}$ is open.

To get the result, we first prove the properness of the canonical projection pr from the equilibrium set Eq $\subset \Xi \times \Omega \times \mathcal{U}$ to $\Omega \times \mathcal{U}$ defined by $\mathrm{pr}(\xi, e, u) = (e, u)$. Let $(\xi^n, e^n, u^n)_{n\in\mathbb{N}}$ be a sequence of Eq such that the sequence $(e^n, u^n)_{n\in\mathbb{N}}$ converges to $(e^*, u^*) \in \Omega \times \mathcal{U}$. We prove that a subsequence converges to an element in Eq.

One easily shows that $(x^n)_{n\in\mathbb{N}}$ admits a subsequence converging to $(x^*) \in \mathbb{R}^{LI}_+$ thanks to the market clearing conditions and the compactness of S. Then, the key step is to show that $x^* \in \mathbb{R}^{LI}_{++}$, mainly as a consequence of the boundary condition in Assumption 1.

Define $\mathbf{1}_L := (1, 1, \ldots, 1) \in \mathbb{R}^L_{++}$ and $\epsilon > 0$. For all $i \in \mathcal{I}$, we consider the following compact sets $C^e_i = \{e^n_i : n \in \mathbb{N}\} \cup \{e^*_i\}$, $C^x_i = \{x^n_i : n \in \mathbb{N}\} \cup \{x^*_i\}$, $C^{x,\epsilon}_i = \{x^n_i + \epsilon\mathbf{1}_L : n \in \mathbb{N}\} \cup \{x^*_i + \epsilon\mathbf{1}_L\}$. From point (2) of Assumption 1, $u^*_i(\cdot, x^*_{-i})$ is increasing, so there exists $\delta \in \left]0, \frac{1}{2}\left(u^*_i(e^*_i, x^*_{-i}) - u^*_i(\frac{1}{2}e^*_i, x^*_{-i})\right)\right[$.

Since u^n converges uniformly on compact sets, we have that there exists $\bar{n} \in \mathbb{N}$ such that for any $n > \bar{n}$ and for any $x \in \prod_{j\neq i}^I C^x_j \times C^{x,\epsilon}_i$, $u^*_i(x) > u^n_i(x) - \delta$. In particular, we have

$$\forall\ n > \bar{n},\ u^*_i(x^n_i + \epsilon\mathbf{1}_L, x^n_{-i}) > u^n_i(x^n_i + \epsilon\mathbf{1}_L, x^n_{-i}) - \delta$$

From the utility maximisation at x^n_i, we have $u^n_i(x^n_i + \epsilon\mathbf{1}_L, x^n_{-i}) \geq u^n_i(e^n_i, x^n_{-i})$ for any n and any i. Once again, since u^n converges uniformly on compact sets, there exists $\bar{m} \in \mathbb{N}$ such that for any $m > \bar{m}$ and for any $x \in \prod_{j\neq i}^I C^x_j \times C^e_i$, $u^m_i(x) > u^*_i(x) - \delta$. In particular, we have

$$\forall\ m > \bar{m},\ u^m_i(e^m_i, x^m_{-i}) > u^*_i(e^m_i, x^m_{-i}) - \delta$$

Then, for any n greater than $\bar{n}$ and $\bar{m}$, we have

$$u^*_i(x^n_i + \epsilon\mathbf{1}_L, x^n_{-i}) > u^*_i(e^n_i, x^n_{-i}) - 2\delta$$

Taking the limit on n, since (x^n, e^n) converges to $(x^*, e^*) \in \mathbb{R}^{LI}_+ \times \Omega$, and u^*_i is continuous, we get $u^*_i(x^*_i + \epsilon\mathbf{1}_L, x^*_{-i}) \geq u^*_i(e^*_i, x^*_{-i}) - 2\delta$.

With ϵ converging to 0, we note that x^*_i belongs to the set $cl_{\mathbb{R}^L}\{x_i \in \mathbb{R}^L_{++} : u_i(x_i, x^*_{-i}) \geq u^*_i(\frac{1}{2}e^*_i, x^*_{-i})\}$. Then, by point (6) of Assumption 1, $x^*_i \in \mathbb{R}^L_{++}$.

Finally, ξ^* is an equilibrium of the economy (u^*, e^*) thanks to the continuity of the equilibrium function Γ_e and the uniform convergence of Du^n_i to Du^*_i on compact set for all i.

Now $\mathcal{E}^{PI}$ is open since this is the complement of the image by the proper projection pr of the union of the following closed sets: $\{(\xi, u, e) \in \Gamma_e^{-1}(0) \mid$

$\text{rank} D_{\xi,t,\tau}\left(\Gamma, \tilde{G}\right) < \dim \Xi + 1 + I\}$ and $\{(\xi, u, e) \in \Gamma_e^{-1}(0) \mid \exists (i, \ell) \in \mathcal{I} \times \mathcal{L} : x_{i\ell} = e_{i\ell}\}$.[24] ■

Proof of Theorem 2. The proof follows the same structure as the one of Theorem 1. We consider a regular full trade equilibrium ξ^* of the economy (u, e) such that $e \in \Omega_u^*$. With equals transfer T, the tax policy depends only on the tax rates $t \in \mathbb{T}$. The changes are the budget constraints and tax balance equation:

$$T_e^{(i,2)}(\xi, t, T) = (p + t_i^*(t)) \cdot (x_i - e_i) - T,$$
$$R_e(\xi, t, T) = T - \frac{1}{I} \sum_{i \in \mathcal{I}} t_i^*(t) \cdot (x_i - e_i)$$

The function Ψ, from $\mathbb{R}^{\dim \Xi + I} \times \mathcal{A}^0$ to $\mathbb{R}^{\dim \Xi + 1 + I}$ is defined in a similar way, where the only change comes from the fact that we have no more derivatives with respect to τ_i for all i, but the derivatives with respect to T.

$$\Psi(c, \pi, a) = \begin{pmatrix} (c, \pi) D_{\xi,t,\tau}(\widehat{\Gamma}_e, \widehat{G})(\xi^*, 0, 0, a) \\ \sum_{i \in \mathcal{I}} \pi_i - 1 \end{pmatrix} \tag{11}$$

Therefore we have that the dimension of the domain of $\Psi(\cdot, \cdot, a)$, $\dim \Xi + I + 1$ is strictly less than the dimension of the range of $\Psi(\cdot, \cdot, a)$, $\dim \Xi + L + 1$, if $I \leq L - 1$.

The equation $(c, \pi) D_{\xi,\tau,t}(\widehat{F}, \widehat{G})(\xi_k^*, 0, 0, a) = 0$ becomes

$$\begin{cases} \sum_{k \in \mathcal{I}} c_{x_k} D^2_{x_i x_k} u_k(x^*) + c_{\lambda_i} p^* + (c_{p^\backslash}, 0) + \sum_{k \in \mathcal{I}} \pi_k \left(D_{x_i} u_k(x^*) + a_{i,k}\right) = 0, \ \forall i \\ -c_{x_i} \cdot p^* = 0, \ \forall i \\ -\sum_{k \in \mathcal{I}} \lambda_k (c_{x_k}, 0) + \sum_{k \in \mathcal{I}} c_{\lambda_k} (x_k^\backslash - e_k^\backslash) = 0 \\ -\sum_{k \in \mathcal{I}} c_{\lambda_k} + c_\tau = 0 \\ -\sum_{k \in \mathcal{I}} \lambda_k c_{x_k} M_k + \sum_{k \in \mathcal{I}} c_{\lambda_k} (x_k - e_k) M_k - c_\tau \frac{1}{I} \sum_{k \in \mathcal{I}} (x_k - e_k) M_k = 0 \end{cases}$$

Now let $\delta = \left((\delta_{x_i}, \delta_{\lambda_i})_{i \in \mathcal{I}}, \delta_{p^\backslash}, \delta_\tau, \delta_t, \mu\right) \in \mathbb{R}^{\dim \Xi + 1 + L - 1 + 1}$ such that $\delta D_{c,\pi,a} \Psi(c, \pi, a) = 0$. Then considering the products associated to the columns c_{λ_i}, $c_{p^\backslash}$, c_τ, and

[24] Note that $\text{rank} D_{\xi,t,\tau}\left(\Gamma_e, \tilde{G}\right) = \dim \Xi + 1 + I$ implies that $\text{rank} D_{\xi,t,\tau} \Gamma_e = \dim \Xi + 1$.

$a_{i,j}$, we have

$$\begin{cases} \delta_{x_i} \cdot p^* - \delta_\tau + \delta_t M_i(x_i - e_i) = 0 \ \forall i \ \text{(S2.1)} \\ \sum_{i \in \mathcal{I}} (\delta_{x_i})^{\setminus} = 0 \ \text{(S2.2)} \\ \delta_\tau - \frac{1}{I} \sum_{i \in \mathcal{I}} \delta_t M_i(x_i - e_i) = 0 \ \text{(S2.3)} \\ \pi_j \delta_{x_i} = 0 \ \forall i \neq j \ \text{(S2.4)} \end{cases} \tag{S}$$

Sum up $(S2.1)$ over i and combining with $(S2.3)$, one get $\left(\sum_{i \in \mathcal{I}} \delta_{x_i} \right) \cdot p^* = 0$. Since $\sum_i \pi_i = 1$, $\pi_j \neq 0$ for some j. It follows $\delta_{x_i} = 0$ for any $i \neq j$. Then $\delta_{x_i} \cdot p^* = 0$ for all i, combining with $\sum_i (\delta_{x_i})^{\setminus} = 0$, we get $\delta_{x_i} = 0$ for all i. The rest of the proof is exactly the same as the proofs of Theorems 1 and Corollary 1. ■

Proof of Theorem 3. Now the tax policy depends on the tax rates $t \in \mathbb{T}$ and $\alpha = (\alpha_i)_{i \in \mathcal{I}} \in (-\delta, \delta)^I$. The changes are as follows:

$$\begin{aligned} T_e^{(i,1)}(\xi, t, T, \alpha) &= D_{x_i} u_i(x) - \lambda_i \left(p + (1 + \alpha_i) t_i^*(t)\right), \\ T_e^{(i,2)}(\xi, t, T, \alpha) &= (p + (1 + \alpha_i) t_i^*(t)) \cdot (x_i - e_i) - T, \\ R_e(\xi, t, T, \alpha) &= T - \frac{1}{I} \sum_{i \in \mathcal{I}} (1 + \alpha_i) t_i^*(t) \cdot (x_i - e_i) \end{aligned}$$

The function Ψ is defined as in the previous proof with the change arising from the derivatives with respect to α. Therefore the dimension of the domain of $\Psi(\cdot, \cdot, a)$, $\dim \Xi + I$ is strictly less than the dimension of the range of Ψ, $\dim \Xi + I + L - 1$. The proof is identical to the one of Theorem 2 since at $\alpha_i = 0$ for all i, the two systems $(c, \pi) D_{\xi,t}(\widehat{\Gamma}_e, \widehat{G})(\xi_k^*, 0, 0, 0, a) = 0$ and $\delta D_{\xi,c,\pi,a} \Psi = 0$ do not change. ■

Proof of Proposition 1. Let (x^*, p^*) be a competitive Nash equilibrium such that x^* is differentiably Pareto non-optimal. Let us assume that the utility functions satisfy Assumption 3. It suffices to prove that the following system has no solution $(c, \pi) \in \mathbb{R}^{\dim \Xi + 1} \times \mathbb{R}_+^I$.

$$\begin{cases} (c, \pi) D_{\xi,t,\tau}(\Gamma_e, G)(\xi^*, 0, 0) = 0 \\ \sum_{i \in \mathcal{I}} \pi_i = 1 \end{cases}$$

The corresponding equations with the multipliers (λ_i^*) are:

$$\begin{cases} \sum_{k\in\mathcal{I}} c_{x_k} D^2_{x_i x_k} u_k(x^*) + c_{\tau_1} p^* + (c_{p\backslash}, 0) + \sum_{k\in\mathcal{I}} \pi_k D_{x_i} u_k(x^*) = 0, \ \forall i \text{ (i.1)} \\ -c_{x_i} \cdot p^* = 0, \ \forall i \text{ (i.2)} \\ \sum_{k\in\mathcal{I}} \lambda_k^* c_{x_k} = 0 \ (i.3) \\ -c_{\lambda_i} + c_{\tau_1} = 0, \ \forall i \text{ (i.4)} \\ -\sum_{k\in\mathcal{I}} \lambda_k^* c_{x_k} M_k = 0 \ (i.6) \\ \sum_{i\in\mathcal{I}} \pi_i = 1 \end{cases} \tag{12}$$

where M_i is defined in the proof of Theorem 1.

Let $(c, \pi) \in \mathbb{R}^{\dim \Xi + 1} \times \mathbb{R}^I_+$ be a solution of System (12). Since x^* is an equilibrium, for all i, there exists $\lambda_i^* > 0$ such that $D_{x_i} u_i(x^*) = \lambda_i^* p^*$. For all i, let $v_i = \lambda_i^* c_{x_i}$. From (i.3) and (i.6), $\sum_{i=1}^{I} v_i = 0$ and for all $\ell \in \mathcal{L}$, $\sum_{i=1}^{I} v_i^\ell \mathbf{1}_{\{x_i^\ell - e_i^\ell > 0\}} = 0$. Multiplying $(i.1)$ with $\lambda_i^* c_{x_i}$ and summing up over i, we get

$$\sum_{k\in\mathcal{I}} \left(\frac{1}{\lambda_k^*} \sum_{i\in\mathcal{I}} v_k D^2_{x_i x_k} u_k(x^*)(v_i) + \pi_k D_x u_k(x^*) \cdot v \right) = 0$$

Therefore, Assumption 3 implies that $D_x u_i(x^*) \cdot v = 0$ for all $i \in \mathcal{I}$, which follows

$$\sum_{k\in\mathcal{I}} \frac{1}{\lambda_k} \sum_{i\in\mathcal{I}} v_k D^2_{x_i x_k} u_k(x^*)(v_i) = 0$$

Then Assumption 2 implies $v_k = 0$ for all k. Since $\lambda_k^* > 0$ for all k, one gets $c_{x_k} = 0$ for all k. Then equation $(i.1)$ implies that $\sum_{k\in\mathcal{I}} \pi_k D_{x_i} u_k(x^*) = -c_{\tau_1} p^* - (c_{p\backslash}, 0)$ for all i. Now since x^* is differentiably Pareto non-optimal, there is $z \in \mathbb{R}^{LI}$ with $\sum_{i\in\mathcal{I}} z_i = 0$ such that $D_x u_k(x^*) \cdot z > 0$ for all k. Multiplying with π_k and summing up over k, and note that $\pi_k > 0$ for at least one k, we have

$$\sum_{k\in\mathcal{I}} \pi_k \sum_{i\in\mathcal{I}} D_{x_i} u_k(x^*) \cdot z_i = \sum_{i\in\mathcal{I}} \left(\sum_{k\in\mathcal{I}} \pi_k D_{x_i} u_k(x^*) \right) \cdot z_i > 0$$

Then, one get $0 < \big(-c_\tau p^* - (c_{p\setminus}, 0)\big) \cdot \sum_{i \in \mathcal{I}} z_i = 0$, a contradiction. Therefore, system (12) has no solution $(c, \pi) \in \mathbb{R}^{\dim \Xi + L} \times \mathbb{R}^I_+$. ■

Topology of space $\mathcal{U}$. In the space $\mathcal{C}^3(\mathbb{R}^{LI}_{++} \times \mathbb{S}, \mathbb{R})$ of $\mathcal{C}^3$ functions with domain $\mathbb{R}^{LI}_{++} \times \mathbb{S}$ and the codomain $\mathbb{R}$, we consider the following metric. Let $(K_n)_{n \in \mathbb{N}}$ be a family of compact sets of $\mathbb{R}^{LI}_{++} \times \mathbb{S}$ such that $\bigcup_n K_n = \mathbb{R}^{LI}_{++} \times \mathbb{S}$. For each n, define the following norm on $\mathcal{C}^3(K_n, \mathbb{R})$.

$$\|f\|_n = \max_{x \in K_n} |f(x)| + \max_{x \in K_n} \|Df(x)\| + \max_{x \in K_n} \|D^2 f(x)\| + \max_{x \in K_n} \|D^3 f(x)\|$$

Then the metric on $\mathcal{C}^3(\mathbb{R}^{LI}_{++} \times \mathbb{S}, \mathbb{R})$ is defined as

$$d(f, g) = \sum_{n=1}^{+\infty} \frac{1}{2^n} \min\{\|f - g\|_n, 1\}$$

Note that the space $\big(\mathcal{C}^3(\mathbb{R}^{LI}_{++} \times \mathbb{S}, \mathbb{R}), d\big)$ is a metric space. Therefore, the space $\mathcal{U}$ is a metric subspace of the space of $\prod_{i=1}^I \mathcal{C}^3(\mathbb{R}^{LI}_{++} \times \mathbb{S}, \mathbb{R})$. In this space, compactness and sequential compactness are equivalent.

$$
\begin{array}{cc}
 & \begin{array}{cccccccc} x_i & \lambda_i & x_j & \lambda_j & p^{\setminus} & \tau_i & \tau_j & t \end{array} \\
\begin{array}{c} c_{x_i} \\ c_{\lambda_i} \\ c_{x_j} \\ c_{\lambda_j} \\ \vdots \\ c_{p^{\setminus}} \\ c_{\tau_1} \\ \pi_i \\ \pi_j \end{array} &
\left(\begin{array}{cccccccc}
D^2_{x_i} u_i & -p^T & D^2_{x_j x_i} u_i & 0 & -\lambda_i [I_{L-1}|0]^T & 0 & 0 & -\lambda_i M_i \\
p & 0 & 0 & 0 & x_i^{\setminus} - e_i^{\setminus} & -1 & 0 & (x_i - e_i) M_i \\
D^2_{x_i x_j} u_j & 0 & D^2_{x_j} u_j & -p^T & -\lambda_j [I_{L-1}|0]^T & 0 & 0 & -\lambda_j M_j \\
0 & 0 & p & 0 & x_j^{\setminus} - e_j^{\setminus} & 0 & -1 & (x_j - e_j) M_j \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
[I_{L-1}|0] & 0 & [I_{L-1}|0] & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & -\sum_i (x_i - e_i) M_i \\
D_{x_i} u_i & 0 & D_{x_j} u_i + a_{i,j} & 0 & 0 & 0 & 0 & 0 \\
D_{x_i} u_j + a_{j,i} & 0 & D_{x_j} u_j & 0 & 0 & 0 & 0 & 0
\end{array}\right)
\end{array}
$$

Matrix of $D_{\xi,t,\tau}(\widehat{\Gamma}_e, \widehat{G})(\xi^*, 0, 0, a)$

$$
\begin{array}{c c}
 & \begin{array}{cccccccccc} c_{x_i} & c_{\lambda_i} & c_{x_j} & c_{\lambda_j} & c_{p\setminus} & c_\tau & \pi_i & \pi_j & a_{i,j} & a_{j,i} \end{array} \\
\begin{array}{c} \Delta c_{x_i} \\ \Delta c_{\lambda_i} \\ \Delta c_{x_j} \\ \Delta c_{\lambda_j} \\ \vdots \\ \Delta c_{p\setminus} \\ \Delta c_{\tau_i} \\ \Delta c_{\tau_j} \\ \Delta c_t \\ \mu \end{array} &
\begin{pmatrix}
D^2_{x_i} u_i & 0 & D^2_{x_j x_i} u_j & 0 & [I_{L-1}|0]^T & p^{*T} & (D_{x_i} u_i)^T & (D_{x_i} u_j + a_{j,i})^T & 0 & \pi_j I_L \\
-p^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
D^2_{x_i x_j} u_i & 0 & D^2_{x_j} u_j & 0 & [I_{L-1}|0]^T & p^{*T} & (D_{x_j} u_i + a_{i,j})^T & (D_{x_i} u_i)^T & \pi_i I_L & 0 \\
0 & 0 & -p^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\lambda_i [I_{L-1}|0] & 0 & \lambda_j [I_{L-1}|0] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
-\lambda_i (M_i)^T & 0 & -\lambda_j (M_j)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{pmatrix}
\end{array}
$$

Matrix of $D_{c,\pi,a}\Psi(c, \pi, a)$

References

Anderson, R.M., Duanmu, H., 2025. Cap-and-trade and carbon tax meet Arrow-Debreu, Econometrica 93(2), 357–393.

Aoki, M., 1971. Marshallian external economies and optimal tax-subsidy structure, Econometrica 39(1), 35–53.

Arrow, K.J. (1969). The organization of economic activity: Issues pertinent to the choice of market versus non-market allocation. In *The Analysis and Evaluation of Public Expenditures: The PPB System.* Joint Economic Committee, Congress of the United States, Washington, D.C., 47–64.

Arrow, K.J., Hahn, F., 1971. General Competitive Analysis. San Francisco: Holden-Day.

Balasko, Y., 2015. Wealth concerns and equilibrium, Journal of Mathematical Economics 59, 92–101.

Bonnisseau, J.-M, del Mercato, E.L., 2010. Externalities, consumption constraints and regular economies, Economic Theory 44, 123-147.

Bonnisseau, J.-M., del Mercato, E.L., Siconolfi, P., 2023. Existence of an equilibrium in arrowian markets for consumption externalities, Journal of Economic Theory 209.

Bourlès, R., Bramoullé, Y., Perez-Richet, E., 2017. Altruism in networks, Econometrica 85(2), 675–689.

Cass, D., Citanna, A., 1998. Pareto improving financial innovation in incomplete markets, Economic Theory 11, 467–494.

Citanna, A., Kajii, A., Villanacci, A., 1998. Constrained suboptimality in incomplete markets: A general approach and two applications, Economic Theory 11, 495–521.

Citanna, A., Polemarchakis, H.M., and Tirelli, M., 2006. The taxation of trades in assets, Journal of Economic Theory 126(1), 299–313.

Crès, H., 1996. Symmetric smooth consumption externalities, Journal of Economic Theory 69, 334–366.

del Mercato, E.L., 2006. Existence of competitive equilibria with externalities: A differential viewpoint, Journal of Mathematical Economics 42, 525–543.

del Mercato, E.L., and Platino, V., 2017a. Private ownership economies with externalities and existence of competitive equilibria: A differentiable approach, Journal of Economics 121, 75–98.

del Mercato, E.L., and Platino, V., 2017b. On the regularity of smooth production economies with externalities: Competitive equilibrium à la Nash, Economic Theory 63, 287–307.

del Mercato, E.L., and Nguyen, Van-Quy, 2023. Sufficient conditions for a "simple" decentralization with consumption externalities, Journal of Economic Theory 209.

Dufwenberg, M., Heidhues, P., Kirchsteiger, G., Riedel, F., Sobel, J., 2011. Other-regarding preferences in general equilibrium, Review of Economic Studies 78, 613–639.

Escobar-Posada, R.A., and Monteiro, G., 2017. Optimal tax policy in the presence of productive, consumption, and leisure externalities, Economic Letters 152, 62–65.

Foley, D.K., 1970. Lindahl's solution and the core of an economy with public goods, Econometrica 38, 66–72.

Geanakoplos, J.D., Polemarchakis, H.M., 1986. Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete. In: Heller, W.P., Starr, R.M., Starrett, D.A. (Eds.), Uncertainty, Information and Communication: Essays in Honor of K.J. Arrow, vol. III., Cambridge University Press, Cambridge, pp. 65–96.

Geanakoplos, J.D., Polemarchakis, H.M., 2008. Pareto improving taxes, Journal of Mathematical Economics 44, 682–696.

Greenwald, B.C., and Stiglitz, J.E., 1986. Externalities in economies with imperfect information and incomplete markets, Quarterly Journal of Economics 101, 229–264.

Hochman, H.M., and Rodgers, J.D., 1969. Pareto optimal redistribution, American Economic Review 59, 542–557.

Laffont, J.-J., 1988. Fundamentals of Public Economics. The MIT Press Cambridge.

Mangasarian, O., 1969. Nonlinear Programming. McGraw-Hill, New York.

Mas-Colell, A., 1985. The Theory of General Economic Equilibrium. A Differentiable Approach. Cambridge University Press.

Nguyen, Van-Quy, 2021. Endowment-regarding preferences, Journal of Mathematical Economics 94, 102454.

Nguyen, Van-Quy, 2022. Welfare theory and other regarding preferences, PhD Thesis, Université Paris 1 Panthéon-Sorbonne.

Osana, H., 1977. Optimal tax-subsidy system for an economy with Marshallian externalities, Econometrica 45(2), 329–340.

Samuelson, P.A., 1954. The pure theory of public expenditure, Review of Economic Studies 36, 387–389.

Sato, F., 1981. On the informational size of message spaces for resource allocation processes in economies with public goods, Journal of Economic Theory 24, 48–69.

Shafer, W., Sonnenschein, H., 1976. Equilibrium with externalities, commodity taxation, and lump sum transfers, International Economic Review 17(3), 601–611.

Tian, G., 2004. A unique informationally efficient allocation mechanism in economies with consumption externalities, International Economic Review 45, 79–111.

Villanacci, A., Carosi, L., Benevieri, P., Battinelli, A., 2002. Differential Topology and General Equilibrium with Complete and Incomplete Markets. Kluwer Academic Publishers.