Online Appendix

The cost of contract renegotiation: Evidence from the local public sector

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Proof of Proposition 1. Intertemporal welfare under full commitment is:

$$W^F(b_1,b_2) = S-(1+\lambda)\left((\beta b_1 + (1-\beta)b_2)\,F(\beta b_1 + (1-\beta)b_2 + k) + \int_{\beta b_1+(1-\beta)b_2+k}^{\bar{\theta}} \theta f(\theta)d\theta\right)$$

$$+\alpha \int_{\underline{\theta}}^{\beta b_1+(1-\beta)b_2+k} (\beta b_1 + (1-\beta)b_2 + k - \theta) f(\theta)d\theta.$$

The term $(\beta b_1 + (1-\beta)b_2)F(\beta b_1 + (1-\beta)b_2 + k)$ represents the expected subsidy under a long-term fixed-price contract knowing that only a mass of those types worth $F(\beta b_1+(1-\beta)b_2+k)$ takes such contract. The term $\int_{\beta b_1+(1-\beta)b_2+k}^{\bar{\theta}} \theta f(\theta)d\theta$ is the expected payment under a cost-plus contract. Finally, the last term represents the expected information rent which is left only to the most efficient types under the fixed-price contract.

The principal's problem under full commitment can be rewritten as:

$$(\mathcal{P}^F): \quad \max_{(b_1,b_2)} W^F(b_1,b_2)$$

The monotone hazard rate property ensures quasi-concavity of this objective.[1] The corresponding first-order conditions characterize the optimal subsidy under full commitment which is constant over time. ■

Renegotiation-Proof Scenario. We first describe the timing of the game with the possibility of renegotiation. Proposition 3 then shows the validity of the Renegotiation-Proofness Principle in our context. We finally characterize renegotiation-proof profiles in Proposition 4.

[1] See for instance Bagnoli and Bergstrom (2005).

Timing.

• Date 0: The firm learns its efficiency parameter θ.

• Date 0.25: The principal commits to a menu $(C_1^0, C_2^0, C_3^0) \equiv C^0 = (b_1, b_2^0, b_3^0)$.

• Date 0.50: The firm makes its choice among those three possible options. The principal updates his beliefs on the firm's innate cost taking into account whether a fixed-price or a cost-plus contract is chosen in the first period.

• Date 1.00: First-period costs and payments are realized.

• Date 1.25: The principal may offer a renegotiation with new (fixed-price) subsidies. Let these new subsidies be $\tilde{b}_2$ and $\tilde{b}_3$ depending on the first-period history.

• Date 1.50: The firm chooses whether to accept the new offer or not and the second-period effort accordingly. If the offer is refused, the initial contract is enforced. Otherwise, the renegotiated offer supersedes the initial contract.

• Date 2: Second-period costs and payments are realized.

Renegotiation. Let denote $\tilde{R} = (\tilde{C}_2, \tilde{C}_3) \equiv (\tilde{b}_2, \tilde{b}_3)$ a subsidy profile offered at the renegotiation stage following an initial offer C^0.[2] Renegotiation takes place if those new subsidies increase the operator's payoff, i.e., if the following inequalities hold:

$$\tilde{b}_2 \geq b_2^0 \text{ and } \tilde{b}_3 \geq b_3^0. \tag{1}$$

The first inequality in (1) says that types in Θ_G accept the renegotiation that takes place after the choice of a fixed-price contract in the first-period if it increases the subsidy above b_2^0. The second inequality is similar for types in Θ_I who chose to operate with a cost-plus contract earlier on.

Renegotiation-proofness. We are now ready to prove:

Proposition 3 There is no loss of generality in restricting the analysis to contracts of the form $C = (b_1, R)$ that come unchanged through the renegotiation process, i.e., such that $R = (b_2, b_3)$ maximizes the principal's second period welfare subject to the following acceptance conditions:

$$\tilde{b}_2 \geq b_2 \quad \text{and} \quad \tilde{b}_3 \geq b_3. \tag{2}$$

Proof: Fix any initial contract C^0 and consider renegotiated offers $\tilde{R} = (\tilde{b}_2, \tilde{b}_3)$ that

[2] We omit the dependence of $\tilde{R}$ on C^0 for notational simplicity.

satisfy (1). Given the agent's conjectures about the renegotiated subsidies $R = (b_2, b_3)$ (which are correct at equilibrium), the principal's expected welfare for date 2 becomes:

$$W_2(C^0, \tilde{R}, R) = \int_{\underline{\theta}}^{b_1+k+\frac{1-\beta}{\beta}(b_2-b_3)} \left(S - (1+\lambda)\tilde{b}_2 + \alpha(\tilde{b}_2 + k - \theta)\right) f(\theta)d\theta \tag{3}$$

$$+ \int_{b_1+k+\frac{1-\beta}{\beta}(b_2-b_3)}^{\tilde{b}_3+k} \left(S - (1+\lambda)\tilde{b}_3 + \alpha(\tilde{b}_3 + k - \theta)\right) f(\theta)d\theta \tag{4}$$

$$+ \int_{\tilde{b}_3+k}^{\bar{\theta}} (S - (1+\lambda)\theta)\, f(\theta)d\theta. \tag{5}$$

Note that this expression is 'unconditional', i.e., it is a weighted sum of the welfares following each possible first-period scenario with the weights being the corresponding probabilities $F\left(b_1 + k + \frac{1-\beta}{\beta}(b_2 - b_3)\right)$ of choosing the two-period fixed-price contract (i.e., C_1^0) earlier on and $1 - F\left(b_1 + k + \frac{1-\beta}{\beta}(b_2 - b_3)\right)$ of operating under a cost-plus contract in the first period (i.e., either C_2^0 or C_3^0).

The 'conditional' second-period welfares following the first-period choice to operate under either a fixed-price (following C_1^0) or a cost-plus (following either C_2^0 or C_3^0) contract are respectively

$$\mathcal{W}_2(C^0, \tilde{R}, R|FP) = \int_{\underline{\theta}}^{b_1+k+\frac{1-\beta}{\beta}(b_2-b_3)} \left(S - (1+\lambda)\tilde{b}_2 + \alpha(\tilde{b}_2 + k - \theta)\right) \frac{f(\theta)}{F\left(b_1 + k + \frac{1-\beta}{\beta}(b_2 - b_3)\right)} d\theta$$

and

$$\mathcal{W}_2(C^0, \tilde{R}, R|CP) = \int_{b_1+k+\frac{1-\beta}{\beta}(b_2-b_3)}^{\tilde{b}_3+k} \left(S - (1+\lambda)\tilde{b}_3 + \alpha(\tilde{b}_3 + k - \theta)\right) \frac{f(\theta)}{1 - F\left(b_1 + k + \frac{1-\beta}{\beta}(b_2 - b_3)\right)} d\theta$$

$$+ \int_{\tilde{b}_3+k}^{\bar{\theta}} (S - (1+\lambda)\theta) \frac{f(\theta)}{1 - F\left(b_1 + k + \frac{1-\beta}{\beta}(b_2 - b_3)\right)} d\theta.$$

Maximizing $\mathcal{W}_2(C^0, \tilde{R}, R|FP)$ with respect to $\tilde{b}_2$ and $\mathcal{W}_2(C^0, \tilde{R}, R|CP)$ with respect to $\tilde{b}_3$ is clearly equivalent to maximizing $W_2(C^0, \tilde{R}, R)$ with respect to $\tilde{R} = (\tilde{b}_2, \tilde{b}_3)$. Because it is more compact, this latter ('unconditional') approach is privileged here.

The expression of $W_2(C^0, \tilde{R}, R)$ takes into account that operators with types in $\Theta_G = [\underline{\theta}, b_1 + k + \frac{1-\beta}{\beta}(b_2 - b_3)]$ are already committed to a two-period fixed-price contract anticipating second-period equilibrium subsidies. These types nevertheless welcome any increase in the second-period subsidy $\tilde{b}_2$ above b_2 at the renegotiation stage. The principal's payoff from such deviation must be computed with this new subsidy. This yields a contribution to expected second period welfare equal to the first-term in (3).

Types in $\Theta_I = [b_1 + k + \frac{1-\beta}{\beta}(b_2 - b_3), b_3 + k]$ are committed to operate under a fixed-price contract only in the second period. But increasing this second-period subsidy from b_3 to $\tilde{b}_3$ attracts some even less efficient types who are now willing to operate under a fixed-price contract for the second period. The least efficient types in $[\tilde{b}_3 + k, \bar{\theta}]$ remain on a cost-plus contract. This yields the expressions of the last two terms (4) and (5).

The principal maximizes the second-period welfare $W_2(C^0, \tilde{R}, R)$ subject to the acceptance condition (1). The renegotiated offers $R = (b_2, b_3)$ must solve:

$$(\mathcal{R}^0): \quad R = \arg\max_{\tilde{R}} W_2(C^0, \tilde{R}, R) \text{ subject to (1).}$$

Take any initial contract offer $C^0 = (b_1, R^0)$ and define R as the solution to $(\mathcal{R}^0)$. Consider now the new contract $C = (b_1, R)$. We want to prove that the history of the firm's types self-selection and the principal's second-period payoff are both unchanged with this new offer. Several observations lead to that result.

1. Since the operator perfectly anticipates the issue of renegotiation and makes his first-period choices accordingly, self-selection among the three different options takes place exactly in the same way with C as when C^0 is initially offered.

2. By definition, any offer $\tilde{R} = (\tilde{b}_2, \tilde{b}_3)$ that is feasible at the renegotiation-stage given R is feasible given R^0. Indeed, that b_2 satisfies the first condition in (1) and $\tilde{b}_2$ satisfies the first condition in (2) implies
$$\tilde{b}_2 \geq b_2^0. \tag{6}$$
Similarly, that b_3 satisfies the second condition in (1) and $\tilde{b}_3$ satisfies the second condition in (2) implies
$$\tilde{b}_3 \geq b_3^0. \tag{7}$$

3. By definition, R solves $(\mathcal{R}^0)$ and thus for any $\tilde{R} = (\tilde{b}_2, \tilde{b}_3)$ that is feasible given R^0, we have:
$$W_2((b_1^0, R), R, R) \geq W_2((b_1^0, R), \tilde{R}, R). \tag{8}$$

This condition is true, in particular, for any $\tilde{R} = (\tilde{b}_2, \tilde{b}_3)$ that is feasible if R is offered at the renegotiation-stage. This shows that R comes unchanged through the renegotiation process, i.e., solves the following problem:

$$(\mathcal{R}): \quad R = \arg\max_{\tilde{R}} W_2((b_1, R), \tilde{R}, R) \text{ subject to (2).}$$

This ends the proof of Proposition 3. ■

Let us now characterize renegotiation-proof allocations.

Proposition 4 A first-period menu of contracts $C = (b_1, b_2, b_3)$ is renegotiation-proof if and only if the following two conditions hold:

$$b_3 \geq \beta b_1 + (1-\beta) b_2, \tag{9}$$

$$kf(b_3+k) - \left(1 - \frac{\alpha}{1+\lambda}\right)\left(F(b_3+k) - F\left(b_1 + k + \frac{1-\beta}{\beta}(b_2 - b_3)\right)\right) \leq 0. \tag{10}$$

Condition (9) ensures that the interval Θ_I is non-empty. It is just a feasibility condition on the possible subsidies profiles that are relevant to generate the pattern of histories found in our data set. The intuition behind Condition (10) is given in the text.

Proof. Turning now to problem $(\mathcal{R})$, first note that $\alpha < 1+\lambda$ implies that the maximum of the integral in (3) is obtained when (2) is binding.

Second, consider (unexpected) renegotiation offers with $\tilde{b}_3 \geq b_3$. Types in $[b_3 + k, \tilde{b}_3 + k]$ which were expecting to operate on a second-period cost-plus contract are now adopting the fixed-price contract with the new greater subsidy $\tilde{b}_3$ at the renegotiation stage. Optimizing $(\mathcal{R})$ which is quasi-concave in $\tilde{b}_3$ and taking into account that b_3 must be the solution yields condition (10). This ends the proof of Proposition 4. ■

Proof of Proposition 2. Define now the principal's intertemporal welfare when offering $C = (b_1, b_2, b_3)$ as:

$$\mathcal{W}(C) = \int_{\underline{\theta}}^{b_1+k+\frac{1-\beta}{\beta}(b_2-b_3)} \left(S - (1+\lambda)(\beta b_1 + (1-\beta)b_2) + \alpha(\beta b_1 + (1-\beta)b_2 + k - \theta)\right) f(\theta) d\theta$$

$$+ \int_{b_1+k+\frac{1-\beta}{\beta}(b_2-b_3)}^{b_3+k} \left(S - (1+\lambda)(\beta\theta + (1-\beta)b_3) + \alpha(1-\beta)(b_3 + k - \theta)\right) f(\theta) d\theta$$

$$+ \int_{b_3+k}^{\bar{\theta}} \left(S - (1+\lambda)\theta\right) f(\theta) d\theta.$$

The optimal renegotiation-proof menu solves the following optimization problem:

$$(\mathcal{P}^R): \quad \max_C \mathcal{W}(C) \text{ subject to (9) and (10).}$$

We shall assume quasi-concavity in (b_1, b_2, b_3) of the corresponding Lagrangean. The solution $C^R = (b_1^R, b_2^R, b_3^R)$ to problem $(\mathcal{P}^R)$ is then straightforward. The first-order optimality conditions for b_1^R and b_2^R are the same so that, it is optimal to set $b_1^R = b_2^R = \underline{b}^R$. Taking into account this fact and optimizing with respect to $(\underline{b}^R, \bar{b}^R)$ yields the following first-order conditions:

$$k = \left(1 - \frac{\alpha}{1+\lambda}\right)\left(R\left(\underline{b}^R + k + \frac{1-\beta}{\beta}(\underline{b}^R - \bar{b}^R)\right) + \frac{\mu}{\beta(1+\lambda)}\right), \tag{11}$$

$$k = \left(1 - \frac{\alpha}{1+\lambda}\right)\left(\frac{F(\bar{b}^R + k) - F\left(\underline{b}^R + k + \frac{1-\beta}{\beta}(\underline{b}^R - \bar{b}^R)\right)}{f(\bar{b}^R + k) - f\left(\underline{b}^R + k + \frac{1-\beta}{\beta}(\underline{b}^R - \bar{b}^R)\right)}\right)$$
$$-\mu\frac{\left(\left(1 - \frac{\alpha}{1+\lambda}\right)\left(\frac{f(\bar{b}^R+k)}{1-\beta} + \frac{f\left(\underline{b}^R+k+\frac{1-\beta}{\beta}(\underline{b}^R-\bar{b}^R)\right)}{\beta}\right) - \frac{kf'(\bar{b}^R+k)}{1-\beta}\right)}{(1+\lambda)\left(f(\bar{b}^R + k) - f\left(\underline{b}^R + k + \frac{1-\beta}{\beta}(\underline{b}^R - \bar{b}^R)\right)\right)} \tag{12}$$

where $\mu > 0$ is the Lagrange multiplier of the renegotiation-proof constraint defined in Section II.B.

Moreover, this constraint implies that

$$F(\bar{b}^R + k) - F\left(\underline{b}^R + k + \frac{(1-\beta)}{\beta}(\underline{b}^R - \bar{b}^R)\right) \geq 0$$

which itself implies $\underline{b}^R \leq \bar{b}^R$. ■

Welfare Estimates. Using our estimates from the case where renegotiation-proof contracts are considered, we get the following expression of welfare in network i:

$$\mathcal{W}_i^R = S - (1+\lambda)\,T_i^R + \widehat{\alpha}_i^R U_i^R, \tag{13}$$

where

$$T_i^R = \int_{\underline{\theta}}^{\underline{b}_i^R + \widehat{k}_i^R + \frac{1-\widehat{\beta}}{\widehat{\beta}}(\underline{b}_i^R - \bar{b}_i^R)} \underline{b}_i^R f(\theta)d\theta + \int_{\underline{b}_i^R + \widehat{k}_i^R + \frac{1-\widehat{\beta}}{\widehat{\beta}}(\underline{b}_i^R - \bar{b}_i^R)}^{\bar{b}_i^R + \widehat{k}_i^R} (\widehat{\beta}\theta + (1-\widehat{\beta})\bar{b}_i^R) f(\theta)d\theta$$
$$+ \int_{\bar{b}_i^R + \widehat{k}_i^R}^{\bar{\theta}} \theta f(\theta)d\theta,$$

and

$$U_i^R = \int_{\underline{\theta}}^{\underline{b}_i^R+\widehat{k}_i^R+\frac{1-\widehat{\beta}}{\widehat{\beta}}(\underline{b}_i^R-\bar{b}_i^R)} \left(\underline{b}_i^R + \widehat{k}_i^R - \theta\right) f(\theta)d\theta + \int_{\underline{b}_i^R+\widehat{k}_i^R+\frac{1-\widehat{\beta}}{\widehat{\beta}}(\underline{b}_i^R-\bar{b}_i^R)}^{\bar{b}_i^R+\widehat{k}_i^R} (1-\beta)\,(\bar{b}_i^R+\widehat{k}_i^R-\theta)f(\theta)d\theta.$$

Likewise, from our full commitment program, we define welfare as the weighted sum of surplus S, expected taxes T_i^F and operator's expected rent U_i^F weighted by the corresponding weight $\widehat{\alpha}_i^R$:

$$\mathcal{W}_i^F = S - (1+\lambda)\,T_i^F + \widehat{\alpha}_i^R U_i^F, \tag{14}$$

where

$$T_i^F = \widehat{b}_i^F F\left(\widehat{b}_i^F + \widehat{k}_i^R\right) + \int_{\widehat{b}_i^F+\widehat{k}_i^R}^{\bar{\theta}} \theta f(\theta)d\theta,$$

and

$$U_i^F = \int_{\underline{\theta}}^{\widehat{b}_i^F+\widehat{k}_i^R} (\widehat{b}_i^F + \widehat{k}_i^R - \theta)f(\theta)d\theta.$$

Note that the gross surplus S vanishes when one computes the difference between both welfare measures $\mathcal{W}_i^R$ and $\mathcal{W}_i^F$. Hence, we evaluate the welfare differential between both renegotiation-proof and perfect commitment situations as

$$\Delta\mathcal{W}_i = \mathcal{W}_i^F - \mathcal{W}_i^R. \tag{15}$$

Similar definitions follow for ΔT_i and ΔU_i. ■

Short-Term Contracts. This Appendix characterizes an equilibrium sequence of short-term contracts. We are again looking for a cut-off equilibrium where, in the first period, types $\theta > \theta_1^*$ choose a cost-plus contract whereas types $\theta \leq \theta_1^*$ choose a fixed-price contract with subsidy b_1. Continuation contracts depend on what happened in the first period. Again following any first-period history and after having updated beliefs accordingly, the principal offers the choice between a fixed-price and a cost-plus contract. We are first solving for such continuations before finding the equilibrium cut-off θ_1^* in the first period. To make the comparison between the cases of short-term contracting and renegotiation relevant, we isolate below conditions under which the same three patterns that arise in our dataset under a renegotiation scenario also occur under short-term contracting: fixed-price contracts in both periods, a cost-plus followed by a fixed-price contract and finally cost-plus contracts in both periods.

Second-period contracts. Suppose that a fixed-price contract has been chosen in the first period, the principal now offers a subsidy b_2 that again might split $[\underline{\theta}, \theta_1^*]$ into two sub-intervals. In the second period, types with a cost parameter $\theta \in [\underline{\theta}, \theta_2^*]$ (where $\theta_2^* \leq \theta_1^*$) choose this fixed-price contract whereas types $\theta \in [\theta_2^*, \theta_1^*]$ operate under a cost-plus contract. Of course, θ_2^* is again defined as $\theta_2^* = b_2 + k$. Following such history, the second-period welfare becomes:

$$\mathcal{W}_2(b_2|FP) = \int_{\underline{\theta}}^{b_2+k} (S - (1+\lambda)b_2 + \alpha(b_2 + k - \theta)) \frac{f(\theta)}{F(\theta_1^*)} d\theta + \int_{b_2+k}^{\theta_1^*} (S - (1+\lambda)\theta) \frac{f(\theta)}{F(\theta_1^*)} .d\theta \tag{16}$$

Optimizing this expression with respect to b_2, we find:

$$\theta_2^* = \min\{\theta^F, \theta_1^*\}. \tag{17}$$

When $\theta_1^* \leq \theta^F$, all types in $[\underline{\theta}, \theta_1^*]$ operate under a fixed-price contract also in the second period. The corresponding subsidy is thus:

$$\theta_1^* = b_2 + k.$$

This scenario replicates a segmentation of the types set which is similar to that arising in our renegotiation scenario. The difference is that of course the level of the second-period subsidy might change. It is lower with short-term contracts because the principal cannot make any commitment to a second-period subsidy in order to compensate the firm for an earlier choice of a fixed-price contract.

Following the choice of a cost-plus contract in the first period, the principal offers a subsidy b_3 that now splits the set $[\theta_1^*, \bar{\theta}]$ into two sub-intervals. Operators with a type $\theta \in [\theta_1^*, \theta_3^*]$ (where $\theta_3^* \geq \theta_1^*$) choose a fixed-price contract for the second period whereas those with a type $\theta \in [\theta_3^*, \bar{\theta}]$ still operate under a cost-plus contract. Again, we have $\theta_3^* = b_3 + k$. The second-period 'conditional' welfare becomes:

$$\mathcal{W}_2(b_3|CP) = \int_{\theta_1^*}^{b_3+k} (S - (1+\lambda)b_3 + \alpha(b_3 + k - \theta)) \frac{f(\theta)}{1 - F(\theta_1^*)} d\theta + \int_{b_3+k}^{\bar{\theta}} (S - (1+\lambda)\theta) \frac{f(\theta)}{1 - F(\theta_1^*)} d\theta. \tag{18}$$

Optimizing this expression yields the cut-off θ_3^* as

$$k = \left(1 - \frac{\alpha}{1+\lambda}\right) \frac{F(\theta_3^*) - F(\theta_1^*)}{f(\theta_3^*)} \tag{19}$$

which can be rewritten as the conditional optimality equation defined in Section V.B.

Assuming that not only $R(\theta) = \frac{F(\theta)}{f(\theta)}$ but also $S(\theta) = \frac{F(\theta)-1}{f(\theta)}$ are increasing with θ, the right-hand side of (19) is proportional to $(1 - F(\theta_1^*))R(\theta_3^*) + F(\theta_1^*)S(\theta_3^*)$ which is also an increasing function of θ_3^*. Hence, (19) admits a unique solution $\theta_3^* \in (\theta_1^*, \bar{\theta})$ when:

$$k < \left(1 - \frac{\alpha}{1+\lambda}\right) \frac{1 - F(\theta_1^*)}{f(\bar{\theta})}.$$

Note also that (19) implies that $k < \left(1 - \frac{\alpha}{1+\lambda}\right) R(\theta_3^*)$ and thus $\theta_3^* > \theta^F$.

The optimality condition (19) implies that, viewed as a function of θ_1^*, θ_3^* satisfies:

$$\frac{d\theta_3^*}{d\theta_1^*} = \frac{f(\theta_1^*)}{f(\theta_3^*) - \frac{k}{1-\frac{\alpha}{1+\lambda}} f'(\theta_3^*)}. \tag{20}$$

But $R'(\theta_3^*) > 0$ implies $\frac{f'(\theta_3^*)}{f(\theta_3^*)} < \frac{1}{R(\theta_3^*)}$ and thus $f(\theta_3^*) - \frac{k}{1-\frac{\alpha}{1+\lambda}} f'(\theta_3^*) > f(\theta_3^*)\left(1 - \frac{k}{\left(1-\frac{\alpha}{1+\lambda}\right)R(\theta_3^*)}\right) > 0$ which holds since $\theta_3^* > \theta^F$. Therefore, we have:

$$\frac{d\theta_3^*}{d\theta_1^*} > 0. \tag{21}$$

First-period subsidy. The cut-off type θ_1^* must be indifferent between choosing a first-period fixed-price contract b_1 followed by a second-period subsidy $b_2 = \theta_1^* - k$ that leaves no extra rent to that type or choosing a cost-plus contract followed by a fixed-price with subsidy b_3. This leads to the indifference condition:

$$\beta(b_1 + k - \theta_1^*) = (1-\beta)(b_3 + k - \theta_1^*)$$

or using the definition of θ_3^*

$$\beta(b_1 + k - \theta_1^*) = (1-\beta)(\theta_3^* - \theta_1^*). \tag{22}$$

By incentive compatibility, more efficient types $\theta \leq \theta_1^*$, then also prefer the first-period fixed-price contract when $\beta \geq \frac{1}{2}$.

Conditions (19) and (22) define the pair (θ_1^*, θ_3^*) as a function of b_1 only. We shall make this dependence explicit in what follows. The same remark applies to the second-period subsidies that are also functions of b_1 only. We will thus have:

$$\theta_1^*(b_1) = b_2(b_1) + k \text{ and } \theta_3^*(b_1) = b_3(b_1) + k. \tag{23}$$

We deduce from the first of those conditions, taken together with (19) and (22) that:

$$\beta(b_1 - b_2(b_1)) = (1-\beta)(\theta_3^* - \theta_1^*) > 0$$

and thus

$$b_2(b_1) < b_1.$$

The intertemporal welfare can be written as a function of b_1 also as:

$$\mathcal{W}(b_1) = \beta \left(\int_{\theta_1^*(b_1)}^{\bar{\theta}} (S - (1+\lambda)\theta) f(\theta) d\theta + \int_{\underline{\theta}}^{\theta_1^*(b_1)} (S - (1+\lambda)b_1 + \alpha(b_1 + k - \theta)) f(\theta) d\theta \right)$$
$$+ (1-\beta)(F(\theta_1^*(b_1))W_2(b_2(b_1)|FP) + (1 - F(\theta_1^*(b_1)))W_2(b_3(b_1)|CP)). \quad (24)$$

Using the Envelope Theorem to simplify the impact of b_1 on the second period subsidy, we get:

$$\frac{d\mathcal{W}}{db_1}(b_1) = \beta \left(\frac{d\theta_1^*}{db_1} (\alpha(b_1 + k - \theta_1^*(b_1)) - (1+\lambda)(b_1 - \theta_1^*(b_1))) f(\theta_1^*(b_1)) + (\alpha - 1 - \lambda) F(\theta_1^*(b_1)) \right)$$
$$+ (1-\beta) \frac{d\theta_1^*}{db_1} (((1+\lambda)(b_3(b_1) - \theta_1^*(b_1)) - \alpha(b_3(b_1) + k - \theta_1^*(b_1)))\, f(\theta_1^*(b_1)).$$

Simplifying further using (22) yields the following expression:

$$\frac{d\mathcal{W}}{db_1}(b_1) = \beta \left((\alpha - 1 - \lambda))\, F(\theta_1^*(b_1)) + (1+\lambda) \frac{2\beta - 1}{\beta} \frac{d\theta_1^*}{db_1} f(\theta_1^*(b_1)) k \right). \quad (25)$$

Assuming quasi-concavity of the objective, the optimal first-period subsidy should solve:

$$k = \frac{\beta}{2\beta - 1} \left(1 - \frac{\alpha}{1+\lambda} \right) \frac{R(\theta_1^*(b_1))}{\frac{d\theta_1^*}{db_1}(b_1)}. \quad (26)$$

Differentiating (22) with respect to b_1 yields:

$$\frac{d\theta_1^*}{db_1} = \frac{1}{1 + \frac{1-\beta}{\beta} \left(\frac{d\theta_3^*}{d\theta_1^*} - 1 \right)}.$$

Therefore, we get:

$$\frac{\frac{\beta}{2\beta - 1}}{\frac{d\theta_1^*}{db_1}(b_1)} = 1 + \frac{1-\beta}{2\beta - 1} \frac{d\theta_3^*}{d\theta_1^*}.$$

Since we posit $\beta \geq 1/2$ (to replicate a dynamic pattern of choices similar to the renegotiation-proof scenario) and (21) holds, the right-hand side above is greater than one. Inserting this condition into (26), the solution satisfies $\theta_1^* \leq \theta^F$ as conjectured.

Altogether, (19), (22) and (26) define the cut-offs θ_1^* and θ_3^*. From (22) and (23), we finally get the expression of the subsidies (b_1, b_2, b_3). We can evaluate the probabilities of each different regimes. Inserting into (24) yields then the expression of the intertemporal welfare with short-term contracts. ■