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Optimal indirect taxation with a restricted number of tax rates ☆

Pascal Belan [a,b], Stephane Gauthier [c,d,*]

[a] *LEN (Université de Nantes), France*
[b] *EUREQua (Université de Paris 1), France*
[c] *CREM (Université de Caen), France*
[d] *CREST-LMA, 15 bd Gabriel Péri, 92245 Malakoff Cedex, France*

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Abstract

This paper analyzes the optimal structure of indirect taxation when the number of available tax rates is smaller than the number of taxable commodities. Such a constraint requires to choose the levels of tax rates and the groups of commodities that will be taxed at equal rates (or exempted). In a partial equilibrium framework, with a single agent and a low amount of tax collection, it is shown that the process of allocation of commodities to groups depends on both price elasticities and consumption spendings. Still, the optimal tax structure displays a weak form of the inverse elasticity rule; consumption spendings influence the size of the fiscal base, and may lead to many tax exemptions.

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1. Introduction

The theory of optimal indirect taxation emphasizes that tax systems should meet two fundamental properties for the collection of a given amount of fiscal liabilities to induce the least

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* Corresponding author. CREST-LMA, 15 bd Gabriel Péri, 92245 Malakoff Cedex, France. Tel.: +33 141173738; fax: +33 141177666.
E-mail address: gauthier@ensae.fr (S. Gauthier).

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welfare loss. First, in accordance with Lipsey and Lancaster's (1956) second-best principle, every consumption good should be taxed, since a larger fiscal base allows for reducing tax rates, which alleviates distortions caused by public intervention into the economy. Second, as underlined by Ramsey (1927) or Deaton (1979), tax rates should generally not be uniform. As an example, in a partial equilibrium framework, the inverse elasticity rule does recommend to set tax rates in inverse proportion of price elasticities.

Still, actual tax schemes display generous exemptions or a quite restricted number of different tax rates. In the U.S., for instance, the Retail Sales Tax only concerns 50% of sales at the State level, in clear violation with the second-best principle (Cornia et al., 2000). Instead, in European countries, the Value-Added Tax imposes one main rate, the standard rate, to most of consumption goods. Practical issues in the administration of the tax may often justify such departures (Slemrod, 1990). Actually, some exemptions are socially beneficial as soon as taxation entails significant administrative costs (Yitzhaki, 1979). Moreover, if it is difficult to clearly define commodities, a uniform tax structure can be used to prevent firms to evade taxes through suitable labelling of goods.

Such departures can also be viewed as a consequence of a constraint on the number of available tax rates. Indeed, when the number K of possible rates equals the number n of commodities, it is straightforward to accommodate simultaneously the two fundamental principles of taxation. If, on the contrary, K is less than n, there may exist a tension between the two desired properties: in an economy where Ramsey taxation would call for the inverse elasticity rule, one extreme possibility consists in taxing the K commodities with least price elasticities, but one could as well gather the n goods into K groups and tax all the groups at K different rates, which would give us a broad tax base.

This paper is an exploration into the problem of the optimal allocation of commodities to groups. The issue is how n goods should be taxed when there is an ad hoc constraint on the number of available tax rates.

The tax rates associated to a given arbitrary allocation of commodities to groups have been first derived by Diamond (1973). In fact, the usual rules then hold, provided that they refer to groups rather than single commodities. That is, in a partial equilibrium framework, the tax rate on a group of commodities should be set in inverse proportion to the price elasticity of demand for this group. Early insights about commodity grouping itself have been given in Gordon (1989) by appealing to a tax reform methodology. Nevertheless, under the assumption that all the consumption goods must be taxed, it turns out that many different allocations of commodities to groups may be welfare improving.

We highlight that, in a partial equilibrium framework, and for low levels of tax collection, the grouping process relies on both price elasticity of demand and consumer spendings. As is usual, in order to dampen the effects of price distortions on the decentralized allocation of resources, there is a tendency toward taxing more heavily commodities whose price elasticity is low. Still, in the same time, it would also be socially preferable to tax commodities that are highly demanded, as this enables lower tax rates.

It is possible, however, to disentangle the roles played by each of these two characteristics. Our main result shows that goods should be grouped successively as ranked by the elasticities: a commodity should be more heavily taxed than another only if it has a lower price elasticity. In this sense, at social optimum, a weak version of the inverse elasticity rule has to be satisfied. Consumer spendings influence the extent to which two commodities with close elasticities should be exempted, taxed at the same rate or allocated to different groups. Namely, if, on the one hand, there is a low demand for commodities with low price elasticity, the social planner will be forced to enlarge the fiscal base in order to levy the required level of taxes. If, on the other hand, these commodities are highly consumed, the second-best principle will be typically violated, i.e., it is then optimal to restrict the fiscal base to goods with low price elasticity and large demand, while all the other goods will be tax free.

Such a property may partly explain some differences between actual indirect tax schemes. Actually, the prediction would be that countries in which there is a negative correlation between price elasticity of demand and consumer spendings tend to adopt narrower fiscal bases.

The paper is organized as follows. Section 2 describes the framework, and Sections 3–6 offer a characterization of the optimal tax structure. A few insights about possible extensions and some concluding comments are given in Sections 7 and 8. All the proofs are in Appendix.

2. The framework

Our attention is focused on the efficiency viewpoint. We study a standard partial equilibrium economy with a single consumer, n commodities ($n \geq 1$) and labor as numeraire. The preferences of the representative agent are described by

$$U(X_1, \ldots, X_n, L) = \sum_{i=1}^{n} U_i(X_i) - L, \tag{1}$$

where $X_i (i=1,\ldots,n)$ is the amount of consumption good i purchased, and L is the amount of labor supplied. In Eq. (1), $U_i(\cdot)$ is increasing and strictly concave.

The utility function (1) embodies strong separability assumptions which ensure that the demand for each good only depends on its own price; weaker assumptions will be adopted in Section 7. The fact that there is a discrete number of goods can be viewed as the result of a primary grouping of commodities whose physical or usage characteristics are sufficiently close, e.g., food, public transport or leisure goods. One can therefore consider that the task of defining goods as separate commodities has been already done by tax authorities. This enables us to avoid the possibility that similar goods may be redesigned by producers and that new commodities could be introduced in response to the tax rules.[1]

We also assume that the production side of the economy is characterized by a constant return to scale technology. Producer prices are fixed; without loss of generality, they are set at unity, so that the consumer price of commodity i writes $(1+t_i)$, where t_i is an ad valorem tax. The problem of the consumer is to maximize Eq. (1), given these prices, and subject to the budget constraint

$$\sum_{i=1}^{n} (1 + t_i) X_i \leq L. \tag{2}$$

Let $X_i(t_i)$ and $L(\boldsymbol{t})$ solve this problem, where $\boldsymbol{t}$ stands for the n-dimensional vector whose ith component is t_i $(i=1,\ldots,n)$.

[1] The same kind of argument could be applied to other considerations which may shape how bundles of goods are formed, such as tax evasion or compliance costs.

In the standard Ramsey framework,[2] a benevolent social planner chooses tax rates $(t_1,\ldots,t_n)$ maximizing social welfare, measured by the indirect utility of the representative consumer

$$V(\mathbf{t}) = \sum_{i=1}^{n} U_i(X_i(t_i)) - L(\boldsymbol{t}), \tag{3}$$

subject to the constraint that a certain fiscal revenue $R(R>0)$ is collected,

$$\sum_{i=1}^{n} t_i X_i(t_i) \geq R. \tag{4}$$

The Ramsey tax rates solution to this problem are characterized by two important properties. First, all the commodities must be taxed. Second, as required by the inverse elasticity rule, tax rates must be set in inverse proportion to the price elasticity of demand. See, e.g., Atkinson and Stiglitz (1980).

In practice, however, tax authorities employ a low number of tax rates, due to, e.g., the presence of administration costs that grow with the number of different rates.

In the sequel, we depart from Ramsey's framework in assuming that there are $K \leq n$ available tax rates τ_k $(k=1,\ldots,K)$. Let G_k represent the group of commodities taxed at rate τ_k $(k=1,\ldots, K+1)$,with τ_{K+1} equal to 0. Hence, the consumer price of commodity i is $(1+\tau_k)$ if G_k comprises i. When K is less than n, the task of the government does no longer reduce to the mere determination of the optimal level of each tax rate. It involves in addition to specify the groups of commodities that will be taxed at common rates or exempted; namely, it involves a partition Γ of the set of the n consumption goods into $(K+1)$ groups G_k $(k=1,\ldots,K+1)$. Such a partition is called a *grouping architecture*.

A grouping architecture Γ and the corresponding vector $(\tau_1,\ldots,\tau_K)$ define a *tax structure*. An *optimal tax structure* yields by definition the highest social welfare under the requirement that tax collection is at least R. In order to characterize such a structure, we first derive the vector $(\tau_1^\Gamma(R),\ldots,\tau_K^\Gamma(R))$ of *Diamond tax rates* which maximizes welfare for a given arbitrary grouping architecture Γ, under the government budget constraint (4). The resulting level of welfare is $V(\mathbf{t}(R;\Gamma))$, where the ith component of $\mathbf{t}(R;\Gamma)$ is $\tau_k^\Gamma(R)$ if i belongs to G_k $(k=1,\ldots,K+1)$. Then, in Sections 4, 5, and 6, we shall turn our attention to the optimal grouping architecture Γ^*, which satisfies $V(\mathbf{t}(R;\Gamma^*)) \geq V(\mathbf{t}(R;\Gamma))$ for any possible architecture Γ.

3. Diamond tax rates

Let the grouping architecture Γ be arbitrarily given, in the sense that consumption goods are arbitrarily allocated to groups. By definition, Diamond tax rates maximize social welfare $V(\mathbf{t})$ under the government budget constraint

$$\sum_{k=1}^{K} \sum_{i \in G_k} \tau_k X_i(\tau_k) \geq R. \tag{5}$$

[2] In this framework, there is no joint taxation of income and commodities (Atkinson and Stiglitz, 1980, Lecture 12). It is well known that income taxation may render indirect taxes useless (Atkinson and Stiglitz, 1976; Mirrlees, 1976). Still, if both types of taxes are managed by different authorities, as in the U.S., where some taxes are defined at local levels and others at the state level, differences in the valuation of the marginal cost of public funds justify the use of indirect taxes. This is an instance of imperfect coordination among tax authorities, such as discussed by Belan et al. (2005).

The solution $(\tau_1^{\Gamma}(R),\ldots,\tau_K^{\Gamma}(R))$ to this problem is such that Eq. (5) holds atequality, and

$$\sum_{i\in G_k} X_i\left(\tau_k^{\Gamma}\right) = \lambda \sum_{i\in G_k}\left(\tau_k^{\Gamma}\frac{dX_i}{dt_i}\left(\tau_k^{\Gamma}\right) + X_i\left(\tau_k^{\Gamma}\right)\right) \tag{6}$$

for $k=1,\ldots,K$. In Eq. (6), the Lagrange multiplier λ represents the social value of one marginal unit of tax liability collected in a lump-sum fashion. It is greater than 1 in a second-best optimum. Let $\varepsilon_i(t_i)$, $\varepsilon_i(t_i)>0$, stand for the (absolute value of the) price elasticity of commodity i with respect to $(1+t_i)$. Let also

$$m_k(\tau_k) = \sum_{i\in G_k}\frac{X_i(\tau_k)}{\sum_{i\in G_k} X_i r}\varepsilon_i(\tau_k) \tag{7}$$

stand for the price elasticity of the kth group, measured by the average of price elasticities of commodities assigned to G_k weighted by their budget shares in this group. With these notations, Eq. (6) reads

$$\frac{\tau_k^{\Gamma}}{1+\tau_k^{\Gamma}} = \left(1-\frac{1}{\lambda}\right)\frac{1}{m_k\left(\tau_k^{\Gamma}\right)} \tag{8}$$

for $k=1,\ldots,K$. These K relations form the *Diamond tax rule*, which generalizes the usual Ramsey rule, provided that price elasticities are suitably applied to bundles of commodities, such as defined by Eq. (7). Therefore, both the second best principle and the inverse elasticity rule apply to groups: all the groups should be taxed in inverse proportion of their price elasticities.

4. The optimal tax structure

This section shows that the decision to tax or to exempt some commodity, as well as the choice of commodities that should be taxed at common rates, involve an interplay between two different characteristics. On the one hand, there is a social incentive for gathering commodities with similar elasticities into the same group. On the other hand, consumption spendings also matter. In fact, the social planner would prefer to tax more heavily a small sample of commodities that are highly consumed, thereby collecting most of the required level of taxes from these commodities.

In the remainder of this paper, we shall concentrate on the case of a small government, where the amount R of tax collection is positive and arbitrarily close to 0. In this setting, the welfare loss caused by taxation $V(\mathbf{0})-V(\mathbf{t}(R;\Gamma))$ can be approximated at first-order by the marginal cost of public funds $\lambda(R;\Gamma)$ at $R=0$. Since all the Diamond tax rates are equal to 0 when $R=0$, the Diamond tax rule (8) implies that $\lambda(0;\Gamma)=1$: any tax structure thus leads to the same welfare loss at first-order (Samuelson, 1964). Therefore, the goal of tax authorities will be to determine the tax structure which minimizes the second-order term of the welfare loss function $V(\mathbf{0})-V(\mathbf{t}(R;\Gamma))$,

$$\frac{R^2}{2}\frac{d\lambda}{dR}(0;\Gamma). \tag{9}$$

Let X_i be the quantity $X_i(0)$ under *laissez-faire* (X_i also coincides with the amount of numeraire devoted to commodity i) and $X(G_k)$ the consumer expenditures for group G_k. Let also $m(G_k)$ stand for the price elasticity $m_k(0)$ of group G_k such defined by Eq. (7). Then, differentiating the government budget constraint (5) and the Diamond tax rule (8) at $R=0$ leads to

$$\frac{d\lambda}{dR}(0;\Gamma) = \left(\sum_{k=1}^{K}\frac{X(G_k)}{m(G_k)}\right)^{-1} \tag{10}$$

which provides the desired criterion for the optimal tax structure.

Proposition 1. For a small amount R of tax collection, a tax structure $(\Gamma^1,\tau(R;\,\Gamma^1))$ welfare dominates a tax structure $(\Gamma^2,\,\tau(R;\,\Gamma^2))$ if and only if

$$\sum_{k=1}^{K}\frac{X\left(G_k^1\right)}{m\left(G_k^1\right)} > \sum_{k=1}^{K}\frac{X\left(G_k^2\right)}{m\left(G_k^2\right)}.$$

Proposition 1 indicates that the social planner should favor tax structures where the price elasticity of groups of taxed commodities are low enough. This is usual in the tax literature, since this allows the government to collect fiscal revenue without any significant distortions. A difference is, however, that the amount of consumer spendings plays a particular role in the analysis. For the optimum to be reached, the budget share of taxed commodities have actually to be high enough, which may be surprising at first sight. In fact, this is the result of two conflicting effects. On the one hand, social welfare will clearly deteriorate when highly consumed commodities are taxed. Nevertheless, on the other hand, a larger quantity means that a lower tax rate is sufficient to generate the required tax revenue, which implies a smaller excess burden.

Still, at this stage, there is no simple hint for designing the optimal tax structure. The social planner may choose to impose low common rates to groups formed by commodities with high budget shares. It could as well form groups on the basis of price elasticities, with high rates applied to groups of commodities with low price elasticity.

5. The scope of taxation

Let us study which commodities should be taxed or exempted. Since, as usual, the deadweight loss is proportional to the square of the tax rate, social welfare improves whenever one uses many small taxes. Proposition 1, on the contrary, suggests that a negative correlation between price elasticity and budget share provides a social incentive for levying taxes from a small subset of commodities, leaving most of the consumption goods untaxed.

Our first result shows that price elasticities play a special role in the decision to tax.

Lemma 1. Consider two grouping architectures Γ^1 and Γ^2 which only differ in that some consumption good j is taxed in Γ^1, whereas it is exempted in Γ^2. If the tax structure $(\Gamma^1,\,\tau(R;\,\Gamma^1))$ is better for social welfare than the tax structure $(\Gamma^2,\,\tau(R;\,\Gamma^2))$, there would be a further welfare improvement by taxing all the exempted commodities whose price elasticity is less than ε_j.

This lemma has a striking consequence for the design of the optimal tax structure. Indeed, if some commodity j is taxed at a positive rate while another commodity i with $\varepsilon_i<\varepsilon_j$ remains untaxed, there is necessarily a welfare improving reform. This reform either consists to release j from any tax, or to tax it, in which case, by Lemma 1, i should be also taxed. As a result, we have:

Proposition 2. In the optimal tax structure, the set of taxed commodities comprises all the commodities with lowest price elasticities, while every remaining commodities, with highest price elasticities, are possibly untaxed.

The fact that a weak version of the inverse elasticity rule applies in the optimal tax structure suggests that the second-best principle may fail. Nevertheless, even in the simple framework under scrutiny, it is difficult to clearly assess the a priori plausible intuition that one should always form groups with many goods taxed at low common rates. The relevant forces which influence the size of the optimal fiscal base seem clear, however.

First, the distribution of price elasticities matters. Indeed, a large fiscal base would obtain when consumption goods have similar price elasticities: in the polar configuration where they are all identical, Proposition 1 indicates that the optimal tax structure simply maximizes the aggregate consumption of taxed commodities, so that every good should be taxed (at the same rate).

It follows that a narrow fiscal base arises only if price elasticities differ sufficiently across commodities. Of course, the optimal base always comprises at least K commodities; otherwise, one should tax some commodities that are currently exempted. Since any enlargement of the fiscal base to more than K goods necessarily involves an additional welfare loss (by the Diamond tax rule, in a given group, goods with highest (resp. lowest) elasticities will be too heavily (resp. lightly) taxed), the only reason why these commodities should be still taxed is to be found in their budget shares.

By Proposition 1, the fact that an increase in the demand of taxed commodities is welfare improving actually implies that two commodities should be grouped only if the budget share of the good whose demand is the most sensitive to tax is (relatively) high enough. Hence, in addition to the distribution of price elasticities, the optimal fiscal base has to be shaped by the correlation between price elasticity and budget share. A negative (resp. positive) correlation between these two variables tends to favor a narrow (resp. large) fiscal base. The next result stresses that only K consumption goods should be taxed when this correlation is strongly negative.

Lemma 2. If $\varepsilon_j/\varepsilon_i$ is greater than $2+X_j/X_i$ for any $j>i$, then any group of taxed commodities should comprise only one commodity, i.e., only K commodities are taxed in the optimal tax structure.

One can now easily be convinced that the two driving forces identified above play distinct roles in the extent to which the second-best principle holds. For instance, when all the budget shares are equal (so that there is no correlation between elasticity and budget share), a broad fiscal base should be implemented if all the price elasticities are identical, whereas a narrow base obtains if, by Lemma 2, the ratio $\varepsilon_j/\varepsilon_i$ is high enough for any $j>i$.

6. The inverse elasticity rule

It is surprising that the social planner should never violate a weak form of the inverse elasticity rule in order to benefit from a high demand. It seems natural, therefore, to analyze whether such a property could be a particular instance of a principle of broader application. For the inverse elasticity rule to extend to the whole optimal tax structure, a commodity i must be more heavily taxed than a commodity j only if $\varepsilon_i<\varepsilon_j$ $(i,j=1,\ldots,n)$. Equivalently, by the Diamond tax rule, any group must be connected with respect to price elasticity, in the sense of Definition 1.

Definition 1. A group G_k $(k=1,\ldots,K+1)$ which comprises at least two different commodities i and j, with $\varepsilon_i<\varepsilon_j$, is connected with respect to price elasticity if and only if it also comprises any commodity whose price elasticity lies between ε_i and ε_j. A group which comprises a single commodity is always connected.

The starting point of this section is to assume that the optimal tax structure fails to be connected (with respect to price elasticity), i.e., there are two groups of taxed commodities $G_1 \cup \{j\}$and $G_2 \cup \{i\}$ such that both

$$m(G_1 \cup \{j\}) < m(G_2 \cup \{i\}) \tag{11}$$

and

$$\varepsilon_j \equiv \sup\{\varepsilon_s; s \in G_1 \cup \{j\}\} > \varepsilon_i \equiv inf\{\varepsilon_s; s \in G_2 \cup \{i\}\}. \tag{12}$$

The issue is whether there exists in this configuration some tax reform allowing the government to collect the required amount of tax liabilities and giving rise to a social welfare improvement. The class of reforms under scrutiny consists to form two new groups from $G_1 \cup \{j\}$ and $G_2 \cup \{i\}$ while the composition of any other group is left unchanged and tax rates are adjusted according to the Diamond rule.

As Proposition 3 shows, if Eqs. (11) and (12) are satisfied, some of these reforms are welfare improving.

Proposition 3. Consider a partial equilibrium framework with a small government. Then, the optimal tax structure is connected with respect to price elasticity, i.e., each group is connected with respect to price elasticity in this structure.

Since, by Proposition 2, the group of exempted commodities is connected, Proposition 3 implies that every group of taxed commodities is connected in the optimal tax structure; equivalently, by the Diamond rule, $t_i \geq t_j$ if and only if $\varepsilon_i < \varepsilon_j$ whatever i and j are $(i, j = 1, \ldots, n)$.

One can illustrate this result by considering the reform which consists to introduce i into $G_1 \cup \{j\}$. By Proposition 1, this reform is welfare improving if and only if

$$\frac{X(G_1 \cup \{j\}) + X_i}{m(G_1 \cup \{i,j\})} + \frac{X(G_2)}{m(G_2)} > \frac{X(G_1 \cup \{j\})}{m(G_1 \cup \{j\})} + \frac{X(G_2) + X_i}{m(G_2 \cup \{i\})}. \tag{13}$$

The pure effect of budget share is clear in Eq. (13). Recall that welfare rises with the budget shares of groups of taxed commodities. Thus, it declines when i is withdrawn from $G_2 \cup \{i\}$, but increases when i is included into $G_1 \cup \{j\}$. Nevertheless, because welfare changes are inversely proportional to the price elasticity of groups, the fact that $m(G_1 \cup \{j\}) < m(G_2 \cup \{i\})$ makes the reform improving from the budget share perspective (this would be actually the case for any reform that transfers commodities from high to low price elasticity groups).

In the same time, this reform also induces intricate changes in price elasticities that affect welfare. As an example, if the price elasticity of i is low enough, $m(G_1 \cup \{i,j\})$ becomes less than $m(G_1 \cup \{j\})$, so that introducing i into $G_1 \cup \{j\}$ is welfare improving. Still, by the same argument, removing i from $G_2 \cup \{i\}$ induces a loss in social welfare (since $m(G_2 \cup \{i\}) < m(G_2)$) which opposes to both previous effects. In fact, Proposition 3 shows that this loss is always dominated.

Proposition 3 is established for a low amount of tax. In order to collect higher taxes, the social planner has to choose between a tax base enlargement and a rise in the tax rates. When elasticities of demand are not too sensitive to consumer prices, it seems reasonable to conjecture that taxation does rather concern a few samples of (heavily taxed) low price elasticity commodities. Indeed, an exemption has a direct welfare improving effect, but also forces the social planner to increase the available rates, which causes an indirect welfare loss. For high tax rates, the fiscal revenue levied from commodities with high price elasticity becomes negligible. Therefore, releasing such commodities from any tax should not require any significant increase in tax rates, which makes the indirect (negative) effect dominated.

Otherwise, when price elasticities are linked to tax rates, it may be more difficult to characterize the optimal grouping of commodities and the whole tax structure; see Belan et al. (2005) for some insights.

7. The case of interdependent demands

So far we have considered only goods with independent demands. Independence is a strong assumption. Suppose, as a simple example, that one can only tax two out of three goods of which good 1 and good 2 are close substitutes. Leaving one of the close substitutes untaxed will then potentially cause a serious distortion, which may constitute a strong case for taxing both close substitutes. This suggests that substitution effects may govern the whole grouping of commodities, as confirmed by Proposition 4, at least in the single rate case.

Proposition 4. Assume that there is only one available tax rate ($K=1$) and let the preferences of the consumer be represented by $U(X_1,\ldots,X_n)-L$. For a small amount R of tax collection, a tax structure $(\Gamma^1, \tau(R;\Gamma^1))$ welfare dominates a tax structure $(\Gamma^2,\tau(R;\Gamma^2))$ if and only if

$$\frac{X(G_1)}{m(G_1)} > \frac{X(G_2)}{m(G_1)},$$

with

$$m(G_k) = \sum_{i\in G_k} \frac{X_i}{X(G_k)}\varepsilon_{ii} - \sum_{i\in G_k}\sum_{\substack{j\in G_k\\ j\neq i}} \frac{X_i c}{X(G_k)}\varepsilon_{ij}$$

where ε_{ii} stands for the absolute value of the own price elasticity of good i and ε_{ij} is the cross price elasticity of good i with respect to the consumer price of good j evaluated at laissez-faire.

The optimal grouping architecture only depends on price elasticities of taxed commodities; in particular, cross price elasticities of taxed goods with respect to the price of untaxed goods are not relevant in the grouping criterion. As expected, the social planner should try to group highly substitute commodities ($\varepsilon_{ij}>0$) whose own price elasticity is low enough (ε_{ii} close to 0). As a result, with one tax rate, goods that are complementary will be exempted, as long as they are not too substitute with taxed goods.

Proposition 4 also validates the fact that budget share and price elasticity play a distinct role in the decision to tax. Nevertheless, the (unconstrained) Ramsey tax structure can still be of some guidance for low levels of tax collection, as Proposition 5 shows.

Proposition 5. Assume that there is only one available tax rate ($K=1$) and two consumption goods ($n=2$). Let the preferences of the consumer be represented by $U(X_1, X_2)-L$. For a small amount R of tax collection, if commodity 1 is more heavily taxed than commodity 2 in the unconstrained Ramsey framework ($K=2$), then the optimal group of taxed commodities should never be reduced to commodity 2 in the constrained framework ($K=1$).

Hence, the good that would be the less heavily taxed if fiscal authorities could set tax rates freely should never be taxed alone in the presence of a limited number of rates. This provides a simple interpretation of Proposition 3. That is, for low levels of tax collection, the optimal ordering of tax rates is the same whatever the number of tax rates is.

Moreover, Propositions 4 and 5 give a more precise information about the optimal size of the fiscal base: the most heavily taxed at Ramsey optimum, good 1, is always taxed and good 2 will be included into the base whenever (1) it is substitute enough to good 1, (2) it has a low own price elasticity, and (3) it is highly consumed (relatively to good 1).[3]

8. Conclusion

In the presence of an ad hoc constraint on the number of available tax rates, this paper emphasizes that the optimal indirect tax structure is shaped by price elasticities, as is usual, but also by consumption spendings. Nevertheless, Ramsey's insights are saved, since a version of the inverse elasticity rule applies. In addition, there may be many untaxed commodities at social optimum. Namely, a commodity with high elasticity will be exempted, as far as it is not highly demanded. Indeed, taxing such commodities does not lead to any significant increase in the total amount of collected fiscal liabilities while, in the same time, this makes the demand for the whole group of taxed commodities more sensitive to tax rates, which is detrimental to welfare.

Although this property seems fairly intuitive, our analysis entails simplifying assumptions, e.g., a low level of tax collection, a linear technology in the production side or the absence of equity considerations. Such extensions would provide interesting information about the design of the optimal indirect fiscal schemes.

Appendix A.

Proof of Lemma 1. By Proposition 1, it is welfare improving to tax a commodity j at the same rate as commodities of some group G_k if and only if

$$\frac{X(G_k \cup \{j\})}{m(G_k \cup \{j\})} > \frac{X(G_k)}{m(G_k)}.$$

Assume that the group G^* of all exempted commodities in Γ^1 whose price elasticity is less than ε_j is not empty. Since $X(G_k \cup G^* \cup \{j\}) > X(G_k \cup \{j\})$ and $m(G_k \cup G^* \cup \{j\}) < m(G_k \cup \{j\})$, we have

$$\frac{X(G_k \cup G^* \cup \{j\})}{m(G_k \cup G^* \cup \{j\})} > \frac{X(G_k \cup \{j\})}{m(G_k \cup \{j\})},$$

which shows that every commodity of G^* should be also taxed. □

Proof of Lemma 2. Suppose that the number of taxed commodities is greater than K, so that there is one group G_k which comprises several commodities. Let $j \in G_k$ be such that $\varepsilon_j = \sup\{\varepsilon_s; s \in G_k\}$. Let also $i \in G_k$ be such that $\varepsilon_i = \sup\{\varepsilon_s; s \in G_k\ \{j\}\}$. If

$$\frac{\varepsilon_j}{\varepsilon_i} > 2 + \frac{X_j}{X_i}, \tag{14}$$

[3] More precisely, using the relation $X_1\varepsilon_{12} = X_2\varepsilon_{21}$, it directly follows from Proposition 4 that it is better to group goods 1 and 2 if and only if

$$\left(2 + \frac{X_2}{X_1}\right)\varepsilon_{11} > \varepsilon_{22} - 2\varepsilon_{21}.$$

This condition may be compared with the one given in Lemma 2.

then

$$\varepsilon_j > \left(2+\frac{X_j}{X_i}\right)\varepsilon_i \geq \left(2+\frac{X_j}{X(G_k\diagdown\{j\})}\right)m(G_k\diagdown\{j\}).$$

It follows that

$$\frac{X(G_k\diagdown\{j\})}{m(G_k\diagdown\{j\})} > \frac{X(G_k)}{m(G_k)}. \tag{14}$$

By Proposition 1, it is consequently welfare improving to exempt j. Since Eq. (14) holds true whenever $\varepsilon_j/\varepsilon_i > 2+X_j/X_i$ for any $j>i$, the proof is complete. □

Proof of Proposition 3. Let Eqs. (11) and (12) be satisfied. In order to prove that it is then welfare improving to introduce j into $G_2 \cup \{i\}$, or i into $G_1 \cup \{j\}$, one must distinguish three different cases, depending on how ε_i and ε_j are ordered with respect to $m(G_1 \cup \{j\})$ and $m(G_2 \cup \{i\})$.

Case 1. If $\varepsilon_i = m(G_1 \cup \{j\})$, it is welfare improving to introduce i into $G_1 \cup \{j\}$. Indeed, the function

$$F_1(X) = \frac{\left(X(G_1)+X+X_j\right)^2}{X(G_1)m(G_1)+X\varepsilon_i+X_j\varepsilon_j} + \frac{\left(X(G_2)+X_i-X\right)^2}{X(G_2)m(G_2)+(X_i-X)\varepsilon_i}$$

is increasing with respect to X, $X \in [0, X_i]$. Thus, $F_1(0) < F_1(X_i)$, and so the assertion directly follows from Proposition 1.

Case 2. If $m(G_2 \cup \{i\}) \leq \varepsilon_j$, it is welfare improving to introduce j into $G_2 \cup \{i\}$ since

$$F_2(X) = \frac{\left(X(G_1)+X_i+X\right)}{X(G_1)m(G_1)+\left(X_j-X\right)\varepsilon_j} + \frac{\left(X(G_2)+X_i+X\right)^2}{X(G_2)m(G_2)+X_i\varepsilon_i+X\varepsilon_j}$$

is increasing with respect to X, $X \in [0, X_j]$, which implies that $F_2(0) < F_2(X_j)$.

Case 3. If $m(G_1 \cup \{j\}) < \varepsilon_i < j < m(G_2 \cup \{i\})$, we have

$$sup\left\{\frac{X(G_1\cup\{i,j\})}{m(G_1\cup\{i,j\})}+\frac{X(G_2)}{m(G_2)};\frac{X(G_1)}{m(G_1)}+\frac{X(G_2\cup\{i,j\})}{m(G_2\cup\{i,j\})}\right\} > \frac{X(G_1\cup\{j\})}{m(G_1\cup\{j\})}+\frac{X(G_2\cup\{i\})}{m(G_2\cup\{i\})} \tag{15}$$

for any admissible $\varepsilon_i < \varepsilon_j$. To see this point, first observe that Eq. (15) will be satisfied for any admissible $\varepsilon_i < \varepsilon_j$ if it is satisfied for $\varepsilon_i < \varepsilon_j$. Indeed, both functions

$$\frac{X(G_1\cup\{i,j\})}{m(G_1\cup\{i,j\})}+\frac{X(G_2)}{m(G_2)} - \left(\frac{X(G_1\cup\{j\})}{m(G_1\cup\{j\})}+\frac{X(G_2\cup\{i\})}{m(G_2\cup\{i\})}\right)$$

and

$$\frac{X(G_1)}{m(G_1)}+\frac{X(G_2\cup\{i,j\})}{m(G_2\cup\{i,j\})} - \left(\frac{X(G_1\cup\{j\})}{m(G_1\cup\{j\})}+\frac{X(G_2\cup\{i\})}{m(G_2\cup\{i\})}\right)$$

are decreasing (resp. increasing) with respect to ε_i (resp. ε_j) because, by Eqs. (11) and (12), $m(G_1 \cup \{i,\ j\}) < m(G_2 \cup \{i\})$ and $m(G_1 \cup \{j\}) < m(G_2 \cup \{i,\ j\})$. With $\varepsilon \equiv \varepsilon_i = \varepsilon_j$ and $X \equiv X_i + X_j$, Eq. (15) rewrites

$$\left\{ \frac{[X(G_1)+X]^2}{[X(G_1)m(G_1)+X\varepsilon]} + \frac{X(G_2)}{m(G_2)}; \frac{X(G_1)}{m(G_1)} + \frac{[X(G_2)+X]^2}{[X(G_2)m(G_2)+X\varepsilon]} \right\} > \frac{[X(G_1)+X_i]^2}{[X(G_1)m(G_1)+X_i\varepsilon]}$$
$$+ \frac{[X(G_2)+(X-X_i)]^2}{[X(G_2)m(G_2)+(X-X_i)\varepsilon]} \equiv F_3(X_i).$$

Hence, Proposition 3 will be proven if $\sup\{F_3(X); F_3(0)\} > F_3(X_i)$, which is satisfied if F_3 is strictly convex for every $X_i \in]0, X[$. But, the second derivative of F_3 has the same sign as

$$\frac{X(G_1)[m(G_1)-\varepsilon]}{[X(G_1)m(G_1)+X_i\varepsilon]^2} \left(1 - \frac{X(G_1)\varepsilon + X_i\varepsilon}{X(G_1)m(G_1)+X_i\varepsilon}\right) - \frac{X(G_2)[\varepsilon - m(G_2)]}{[X(G_2)m(G_2)+(X-X_i)\varepsilon]^2}$$
$$\times \left(1 - \frac{X(G_2)\varepsilon + (X-X_i)\varepsilon}{X(G_2)m(G_2)+(X-X_i)\varepsilon}\right),$$

which is positive. This concludes the proof. □

Proof of Proposition 4. Two tax structures are equivalent at first-order for R close enough to 0. Therefore, one should tax commodities of G_1 instead of commodities of G_2 if and only if

$$\frac{d^2V}{dR^2}\left(\mathbf{t}\left(0;\Gamma^1\right)\right) > \frac{d^2V}{dR^2}\left(\mathbf{t}\left(0;\Gamma^2\right)\right),$$

where

$$\frac{d^2V}{dR^2}(\mathbf{t}(0;\Gamma)) = -\sum_{i\in G}\sum_{j\in G}\frac{\partial X_i}{\partial q_j}(\mathbf{0})\left(\frac{d\tau(0)}{dR}\right)^2 - \sum_{i\in G}X_i(\mathbf{0})\frac{d^2\tau(0)}{dR^2}.$$

The second-order differentiation of the government budget constraint with respect to R, evaluated at $R=0$, is written

$$2\sum_{i\in G}\sum_{j\in G}\frac{\partial X_i}{\partial q_j}(\mathbf{0})\left(\frac{d\tau(0)}{dR}\right)^2 + \sum_{i\in G}X_i(\mathbf{0})\frac{d^2\tau(0)}{dR^2} = 0.$$

Thus

$$\frac{d^2V}{dR^2}(\mathbf{t}(0;\Gamma)) = \frac{1}{X(G)}\sum_{i\in G}\left(-\varepsilon_{ii} + \sum_{j\in G, j\neq i}\frac{X_i}{X(G)}\varepsilon_{ij}\right),$$

where

$$\varepsilon_{ii} - \frac{\partial \log X_i}{\partial \log(1+t_i)}, \quad \varepsilon_{ij} = \frac{\partial \log X_i}{\partial \log\left(1+t_j\right)}.$$

This concludes the proof. □

Proof of Proposition 5. When $K=2$, Ramsey tax rates $t_1(R)$ and $t_2(R)$ satisfy

$$(\lambda-1)X_i + \lambda\left(X_i + \sum_{j=1}^{2} t_j\frac{\partial X_j}{\partial t_i}\right) = 0$$

for $i=1$, 2. Differentiating this equation with respect to R, at $R=0$, leads to

$$-\varepsilon_{11}t_1'(0)+\varepsilon_{12}t_2'(0)=-\lambda'(0)=\varepsilon_{21}t_1'(0)-\varepsilon_{22}t_2'(0). \tag{16}$$

Let commodity 1 be the most heavily taxed, i.e., $t_1'(0)>t_2'(0)$, so that $t_1'(0)>0$. From Eq. (16), $\lambda'(0)/t_1'(0)=(\varepsilon_{11}\varepsilon_{22}-\varepsilon_{21}\varepsilon_{12})/(\varepsilon_{22}+\varepsilon_{12})$. The property of negative semi-definiteness of the Slutsky matrix ($\varepsilon_{11}\varepsilon_{22}-\varepsilon_{12}\varepsilon_{21}\geq 0$) and $\lambda'(0)>0$ thus imply $\varepsilon_{22}+\varepsilon_{12}>0$.

We are going to show that, if good 1 is more heavily taxed than good 2 when $K=2$, then it is better to tax 1 and 2 than to tax 2 only when $K=1$, i.e., from Proposition 4,

$$\left(2+\frac{X_1}{X_2}\right)(\varepsilon_{22}+\varepsilon_{12})>(\varepsilon_{11}+\varepsilon_{21}). \tag{17}$$

If $\varepsilon_{11}+\varepsilon_{21}\leq 0$, Eq. (17) holds since $\varepsilon_{22}+\varepsilon_{12}>0$. If $\varepsilon_{11}+\varepsilon_{21}>0$, then Ramsey tax rates are positive, by Eq. (16). It follows that $t_1(R)>t_2(R)$ if and only if $\varepsilon_{22}+\varepsilon_{12}>\varepsilon_{11}+\varepsilon_{21}$. Hence, Eq. (17) is satisfied, which concludes the proof. □

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