Weak redistribution and certainty equivalent domination*

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Abstract

We assess optimal deterministic nonlinear income taxation in a Mirrlees economy with a continuum of risk-averse agents whose utilities are quasilinear in labor. A weak redistribution motive makes random taxes more likely socially dominated by the deterministic policy where after-tax income lotteries are replaced with their certainty equivalents.

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1 Introduction

Tax authorities sometimes organize lotteries or random drawings for individuals who have filed their taxes correctly and on time, with winners receiving a tax refund or other financial incentives. Randomness in taxes may also be due to administrative tax errors, uncertainties in the tax environment implied by political instability and frequent fiscal reforms, or arise as a response to tax evasion.[1] Such features are much in contrast with Adam Smith's classical economics recommendation of making taxes certain. This paper is interested into the choice between random versus deterministic policies.

The academic literature actually shows that, in the presence of asymmetric information, the optimal menu offered to risk averse agents can involve lotteries. In public finance, lotteries take the form of random taxes implying randomness in after-tax income while the before-tax income is certain, such as in Weiss (1976), Stiglitz (1981), Stiglitz (1982) or Brito, Hamilton, Slutsky, and Stiglitz (1995). The intuition is straightforward. Suppose that the government wants to redistribute income to low-skilled workers in a population of risk-averse workers. Redistribution is limited if the government observes neither skill nor the exact amount of labor, as high-skilled may reduce labor effort to enjoy higher transfers. Introducing random noise on the after-tax income designed for low-skilled is detrimental to their welfare, but this also expands the scope of possible redistribution by discouraging high-skilled from relaxing effort.

This argument suggests that deterministic taxation should be used if the welfare cost incurred by those facing noise overcomes the gain from expanded scope of redistribution, a situation more likely to happen if high-skilled do not suffer much from income risk compared to low-skilled. The analysis in Hellwig (2007) indeed shows that an unweighted utilitarian (Benthamite) government should rely on deterministic redistribution if risk aversion decreases with labor productivity, i.e., risk aversion is higher for low than high-skilled.

This paper explores the case of a weighted utilitarian government. The social valuation of the suffering of the less well-off part of the population that faces random noise is then magnified when the government assigns more

[1] See, e.g., Slemrod (2019) for an example of a tax lottery used in Montevideo, or Chimilila et al. (2023) for experimental evidence in Tanzania. In, e.g., the Nota Fiscal Paulista program implemented in 2007 by the government of São Paulo in Brazil, consumers who request and register invoices for their purchases can participate in a lottery drawing (see Naritomi (2019)).

importance to these individuals. However, on the other hand, greater redistribution implies more pressure from incentives, as the bundles designed for the poor become more desirable to the other agents. Therefore the potential gains in social welfare from discouraging the rich from mimicking the poor tend to be larger.

We analyze how this trade-off is resolved when the government replaces random after-tax incomes with their certainty equivalents. Our results suggest that maintaining incentive compatibility of certainty equivalents puts strong limits on the social welfare gains from switching to the deterministic policy. Eventually such a reform improves social welfare upon the menu of lotteries in the case of weak redistribution motives, with a social welfare function close enough to the Benthamite pattern. Instead sharper redistribution motives reduce the likelihood that one can find incentive compatible certainty equivalents improving upon random redistribution.

The paper proceeds as follows. The setup is described in Section 2. Section 3 provides conditions for incentive compatibility of menus of lotteries. Section 4 compares lotteries and the associated certainty equivalents. Incentive compatibility of the menu of certainty equivalents is analyzed in Section 5. Section 6 concludes.

2 General framework

A government wants to redistribute income between a continuum of agents in a population of total unit size. Every agent is indexed by her type θ, a real parameter taking values in $\Theta = [\theta^{\inf}, \theta^{\sup}]$, with cdf F associated with positive probability density function f. The preferences of a type θ agent are represented by the quasilinear utility function

$$u(c, \theta) - y \tag{1}$$

when she earns before-tax income y and pays $y - c$ as tax. The after-tax income c is also her consumption. Earning y requires providing an effort, hence the disutility cost.

The function u is assumed to be increasing, differentiable everywhere in c and θ, and strictly concave in c. It also satisfies the Spence-Mirrlees condition that the second-order cross-derivative $u''_{c\theta}(c, \theta)$ is negative for all (c, θ). That is, higher types have lower marginal utility $u'_c(c, \theta)$ from consumption, which is the marginal rate of substitution between consumption and before-tax income.

Let us consider two specific cases where these assumptions are satisfied to grasp a better idea about the possible interpretations of the parameter θ. A first case is one where $u(c, \theta)$ is set to

$$u(c + \theta), \tag{2}$$

with θ some extra income that is not subject to the tax on y, e.g., from intra-family transfers, remittances or black market operations. Agents with high θ are better endowed with this extra income, and redistributive purposes require taxing them more heavily by lowering their after-tax income c. An alternative interpretation comes in the multiplicative formulation where $u(c, \theta)$ is set to

$$u(\theta c). \tag{3}$$

Then our assumptions on u are met if the (absolute value of the) coefficient of relative risk aversion associated with u is above 1, which falls within the range of 1 to 3 found in the literature, such as in Gertler and Gruber (2002). In this case, θ could be viewed as some individual ability to benefit from consumption. Again a redistributive government would likely aim to compensate low-ability types by giving them higher after-tax incomes.

The redistribution policy used by the government is represented by a menu $(\tilde{c}(\theta), \tilde{y}(\theta))_{\theta\in\Theta}$ of lotteries. The lottery $\tilde{c}(\theta)$ (resp., $\tilde{y}(\theta)$) is a random variable with realizations that are consumption (resp., before-tax income) levels designed for every type θ agent. The specific class of lotteries that we consider is spelled out in Assumption A1 in Section 3.

The menu is feasible if total consumption is less than total production,

$$\int_{\Theta} \mathbb{E}[\tilde{c}(\theta) - \tilde{y}(\theta)]\mathrm{d}F(\theta) \leq 0, \tag{4}$$

where, in all the paper, the expectation operator applies to the variables denoted with a tilde, e.g., the expected value $\mathbb{E}[\tilde{c}(\theta)]$ is to be taken with respect to the after-tax income random variable for a given type θ.

If θ is private information to the agent, the government must also ensure that every agent chooses the income pair designed for her. This is satisfied if the incentive constraints

$$\mathbb{E}[u(\tilde{c}(\theta), \theta) - \tilde{y}(\theta)] \geq \mathbb{E}[u(\tilde{c}(\tau), \theta) - \tilde{y}(\tau)] \tag{5}$$

hold for all (θ, τ) in $\Theta \times \Theta$.

The social welfare objective is

$$\int_{\Theta} V(\theta)\, \mathrm{d}G(\theta) \tag{6}$$

where $V(\theta) = \mathbb{E}[u(\tilde{c}(\theta), \theta) - \tilde{y}(\theta)]$ is the indirect utility of type θ. The social weights embodied in $G(\cdot)$ are non-negative and normalized so that they sum up to 1. Hellwig (2007) considers an unweighted utilitarian government where all agents are valued equally, $F(\theta) = G(\theta)$ for all θ. In (6) the utility of type θ can be assigned any given (non-negative) weight. In the redistributive case where the government would seek to compensate low types for their lower endowment with other sources of income or lower ability to enjoy consumption, higher weights should be given to these types, implying $F(\theta) \leq G(\theta)$ for all θ.

An optimal redistribution policy is a menu of lotteries that maximizes the social objective (6) subject to the feasibility constraint (4) and the incentive constraints (5). This policy must have the feasibility constraint (4) binding since otherwise a uniform reduction of before-tax income would be feasible, meet the incentive constraints (5) and yield a higher objective (6).

We are interested into circumstances where the optimal policy is deterministic. A deterministic policy consists of degenerated lotteries $(\tilde{c}(\theta), \tilde{y}(\theta))$ yielding a sure outcome $(c(\theta), y(\theta))$ for every type θ. In view of the quasilinear utility (1), replacing the before-tax income lottery $\tilde{y}(\theta)$ with the sure before-tax income $\mathbb{E}[\tilde{y}(\theta)]$ affects neither the constraints (4) and (5) nor the objective (6). Therefore, in the sequel, we consider that every type is assigned a sure before-tax income $y(\theta)$. Any social improvement compared to a redistribution policy that involves lotteries must come from ensuring certainty in the after-tax income.

The quasilinear assumption is employed in simple versions of optimal tax problems as it facilitates the treatment of incentive issues. Most of the literature relying on this simplification assumes quasilinearity in consumption, where all income effects fall on consumption. In (1) quasilinearity instead is applied to labor or before-tax income, as in Lollivier and Rochet (1983), Weymark (1986) or Weymark (1987). Then income effects only fall on before-tax income, which may be less empirically grounded. Specifically, for these preferences, the marginal rate of substitution between consumption and before-tax income is solely determined by the amount of consumption and the private information characteristic θ. It remains independent of before-tax income. Our analysis could actually accommodate both specifications. The formulation (1), where random noise on before-tax income is useless while randomness in after-tax income may be beneficial, is consistent with an interpretation of taxation as introducing random noise into the redistribution.

3 Dealing with incentives

The lottery $\tilde{c}(\theta)$ is such that a type θ agent receives an after-tax income smaller than c with probability $H(c, \theta)$. We denote by $h(c, \theta)$ the associated density. In the sequel, we consider menus of lotteries that satisfy Assumption A1.

Assumption A1. The after-tax income lotteries $(\tilde{c}(\theta))_{\theta \in \Theta}$ have common support $\mathcal{C} = [c^{\inf}, c^{\sup}]$ and their cdf $H(c, \theta)$ are continuously differentiable with respect to θ, $\theta \in \Theta$.

We denote by $H'_\theta(c, \theta)$ and $h'_\theta(c, \theta)$ the partial derivatives of $H(c, \theta)$ and $h(c, \theta)$ in θ, respectively. The common support restriction appears to be mild, as density can be taken arbitrarily small at the lower and/or upper tails of the income distribution to approximate lotteries with supports varying across types. Instead, the differentiability assumption is demanding, as it implies that any given after-tax income is received by neighboring types with neighboring probabilities. It is made to ensure differentiability of the certainty equivalents (see footnote 3).

Lemma 1. *The incentive constraints (5) associated with a menu of lotteries* $(\tilde{c}(\theta), y(\theta))_{\theta \in \Theta}$ *are satisfied only if*

$$V'(\theta) = \mathbb{E}\left[u'_\theta(\tilde{c}(\tau), \theta)\right] \tag{7}$$

and

$$\frac{\partial}{\partial \tau}\mathbb{E}\left[u'_\theta(\tilde{c}(\tau), \theta)\right] \geq 0 \tag{8}$$

for all θ *and* $\tau = \theta$*. These conditions are sufficient for incentive compatibility if (8) holds true for all* θ *and* τ*.*

Proof. We reproduce standard arguments used in the deterministic case, e.g., in Section 2.3 of Salanié (2017). The incentive constraints (5) can be rewritten as

$$\theta = \arg\max_{\tau} \mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau) \tag{9}$$

for all θ. This requires that, for any given θ, the truthful report $\tau = \theta$ is a local extremum of the utility $\mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau)$, i.e.,

$$\frac{\partial}{\partial \tau} \mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau) = 0 \tag{10}$$

at $\tau = \theta$. Using the envelope theorem applied to the maximization problem (9), where θ is treated as a parameter, this is equivalent to (7).

Truthful reporting $\tau = \theta$ is a local maximizer of the utility if $\mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau)$ is locally concave in τ at the extremum $\tau = \theta$. If the necessary first-order condition (10) holds at $\tau = \theta$ for all θ, then we have by differentiation in θ

$$\begin{aligned}
\frac{\partial^2}{\partial \tau^2} (\mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau)) &= -\frac{\partial^2}{\partial \tau \partial \theta} (\mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau)) \\
&= -\frac{\partial}{\partial \tau} \mathbb{E}\left[u'_\theta(\tilde{c}(\tau), \theta)\right]
\end{aligned}$$

at $\tau = \theta$, for all θ. Local concavity thus reads as (8).

Conditions (7) and (8) are necessary and sufficient for $\mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau)$ to be lower than $\mathbb{E}[u(\tilde{c}(\theta), \theta)] - y(\theta)$ for all τ close to θ. They do not ensure that truthful reporting is a global maximum. If (10) with $\theta = \tau$,

$$\frac{\partial}{\partial \tau} \mathbb{E}[u(\tilde{c}(\tau), \tau)] - y(\tau) = 0,$$

is satisfied for all τ, then a sufficient condition for a global maximum obtains by observing that

$$\frac{\partial}{\partial \tau} (\mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau)) = \int_{\tau}^{\theta} \frac{\partial}{\partial \tau} \mathbb{E}\left[u'_\theta(\tilde{c}(\tau), z)\right] \mathrm{d}z.$$

To prove the last statement of Lemma 1, suppose that (8) holds true for all τ and θ. Then the sum in the right-hand side of the above equality has the same sign as $\theta - \tau$, which implies that $\mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau)$ is single-peaked in τ, with a global maximum attained at $\tau = \theta$. $\square$

Random noise makes the second-order condition (8) for incentive compatibility related to both agents preferences and properties of consumption lotteries. This differs from the deterministic case where, thanks to the Spence-

Mirrlees condition, the second-order condition for local incentive compatibility only requires that consumption decreases with type, and implies global incentive compatibility.[2]

Lemma 2 shows that for a special class of menus incentive compatibility obtains if and only if (7) is met.

Lemma 2. *Suppose that $\tilde{c}\,(\theta_1)$ first-order stochastically dominates $\tilde{c}\,(\theta_2)$ for any two types θ_1 and θ_2, with $\theta_1 < \theta_2$. Then inequality (8) holds true for all τ and θ. Therefore incentive compatibility obtains if and only if (7) is met.*

Proof. Assumption A1 implies

$$\frac{\partial}{\partial \tau}\mathbb{E}\left[u'_\theta(\tilde{c}(\tau),\theta)\right] = \int\limits_{\mathcal{C}} u'_\theta(c,\theta)\,\mathrm{d}H'_\theta(c,\tau).$$

Using the integration by parts formula yields

$$\int\limits_{\mathcal{C}} u'_\theta(c,\theta)\mathrm{d}H'_\theta(c,\tau) = \left[u'_\theta(c,\theta)H'_\theta(c,\tau)\right]_{\mathcal{C}} - \int\limits_{\mathcal{C}} u''_{\theta c}(c,\theta)H'_\theta(c,\tau)\mathrm{d}c.$$

The common support in Assumption A1 gives $H(c^{\inf},\tau) = 0$ and $H(c^{\sup},\tau) = 1$ for all τ. Thus, by differentiating in τ, we have $H'_\theta(c^{\inf},\tau)$ and $H'_\theta(c^{\sup},\tau)$ both equal to 0 for all τ. It follows that

$$\int\limits_{\mathcal{C}} u'_\theta(c,\theta)\mathrm{d}H'_\theta(c,\tau) = -\int\limits_{\mathcal{C}} u''_{\theta c}(c,\theta)H'_\theta(c,\tau)\mathrm{d}c.$$

The Spence-Mirrlees condition $u''_{\theta c}(c,\theta) < 0$ for all (c,θ) implies that (8) holds true for all τ and θ if $H'_\theta(c,\theta) \geq 0$ for all (c,θ), i.e., $H(c,\theta_1) \leq H\,(c,\theta_2)$ for all c and $\theta_1 \leq \theta_2$. This corresponds to the case where $\tilde{c}\,(\theta_1)$ first-order stochastically dominates $\tilde{c}\,(\theta_2)$. The result then follows from Lemma 1. □

Lemma 2 provides us with a natural generalization of the familiar monotonicity condition for incentive compatibility in a deterministic environment. The monotonicity of the deterministic consumption is merely replaced with a stochastic dominance ordering of lotteries. Namely, higher types face higher probabilities of getting low consumption.

[2]Let $c(\tau)$ denote a degenerate lottery that delivers the amount $c(\tau)$ with probability 1 to a type τ agent. The inequality (8) reduces to $c'(\tau) \leq 0$ at any point of differentiability, and it implies that $u(c(\tau),\theta) - y(\tau)$ is a single peaked function of τ reaching its global maximum when $\tau = \theta$.

4 Certainty equivalent domination

Suppose now that we switch from a feasible and incentive compatible menu of lotteries $(\tilde{c}(\theta), y(\theta))_{\theta\in\Theta}$ to the deterministic menu where type θ instead gets the after-tax income certainty equivalent $\mathbb{C}(\tilde{c}(\theta), \theta)$ with probability 1. The certainty equivalent $\mathbb{C}(\tilde{c}, \theta)$ of type θ when facing some lottery $\tilde{c}$ is the sure consumption such that

$$u(\mathbb{C}(\tilde{c}, \theta), \theta) = \mathbb{E}[u(\tilde{c}, \theta)]. \tag{11}$$

The certainty equivalent is $\mathbb{C}(\tilde{c}(\tau), \theta)$ for type θ when facing the lottery $\tilde{c}(\tau)$ designed for type τ. The corresponding risk premium is $\pi(\tilde{c}(\tau), \theta) = \mathbb{E}[\tilde{c}(\tau)] - \mathbb{C}(\tilde{c}(\tau), \theta)$. Both are differentiable in θ. By Assumption A1, they are also continuously differentiable in τ.[3]

The reform to a sure after-tax income modifies incentives among risk-averse agents. To keep with incentive requirements, the before-tax income is adjusted for a deterministic amount $\delta(\theta)$ so that every type θ produces $y(\theta) - \delta(\theta)$. Intuitively, allocating the certainty equivalent rather than the lottery provides an insurance to agents, which allows the government to meet incentives while extracting extra before-tax income resources that can be used to increase social welfare.

Proposition 1. *Consider a feasible and incentive compatible menu of lotteries $(\tilde{c}(\theta), y(\theta))_{\theta\in\Theta}$. There is a deterministic menu where every type θ gets the certainty equivalent $\mathbb{C}(\tilde{c}(\theta), \theta)$ that is feasible, incentive compatible, and yields a higher social welfare (6) than the lotteries if and only if the following two conditions are satisfied:*

[3]From (11) the certainty equivalent of the lottery $\tilde{c}(\tau)$ for type θ is such that

$$u(\mathbb{C}(\tilde{c}(\tau), \theta), \theta) = \int_{\mathcal{C}} u(z, \theta) h(z, \tau) \,\mathrm{d}z.$$

By Assumption A1, the density $h(z, \theta)$ can be differentiated in θ. When evaluated for the lottery designed for type τ,

$$\mathbb{C}'_\tau(\tilde{c}(\tau), \theta) u'_c(\mathbb{C}(\tilde{c}(\tau), \theta), \theta) = \int_{\mathcal{C}} u(z, \theta) h'_\theta(z, \tau) \,\mathrm{d}z.$$

1. *the inequality*

$$\int\limits_{\Theta} \left[\pi(\tilde{c}(\theta),\theta) - \frac{G(\theta)-F(\theta)}{f(\theta)}\pi'_\theta(\tilde{c}(\theta),\theta)u'_c(\mathbb{C}(\tilde{c}(\theta),\theta),\theta)\right]\mathrm{d}F(\theta) > 0 \tag{12}$$

   is met.

2. $\mathbb{C}(\tilde{c}(\theta),\theta)$ *is non-increasing in* θ*.*

A proof is in Appendix A. A heuristic derivation of the inequality (12) obtains by considering the simpler reform that replaces $\tilde{c}(\theta)$ with $\mathbb{C}(\tilde{c}(\theta),\theta)$ only for types between some $\underline{\theta}$ and $\bar{\theta} = \underline{\theta} + \mathrm{d}\theta, \mathrm{d}\theta > 0$ small.

For such types, the sub-utility derived from consumption by type θ when mimicking τ was $u(\mathbb{C}(\tilde{c}(\tau),\theta),\theta)$ before the reform. It becomes $u(\mathbb{C}(\tilde{c}(\tau),\tau),\theta)$ after the reform. Type θ is therefore discouraged from mimicking type τ if $\mathbb{C}(\tilde{c}(\tau),\theta) > \mathbb{C}(\tilde{c}(\tau),\tau)$, which is $(\theta-\tau)\mathbb{C}'_\theta(\tilde{c}(\tau),\tau) > 0$ for θ close to τ. As a result, incentive compatibility can be achieved by relying on adjustments of the before-tax income such that

$$\delta'(\theta) = -\mathbb{C}'_\theta(\tilde{c}(\theta),\theta)u'_c(\mathbb{C}(\tilde{c}(\theta),\theta),\theta)$$

for all these types.

For types outside $[\underline{\theta},\bar{\theta}]$, the after-tax income remains unchanged after the reform. Only the before-tax income can change. Incentive compatibility thus requires that $\delta(\theta)$ is a uniform amount $\underline{\delta}$ for all $\theta \le \underline{\theta}$, and $\bar{\delta}$ for all $\theta \ge \bar{\theta}$. Relying on the approximation $\bar{\delta} \simeq \underline{\delta} + \delta'(\underline{\theta})\mathrm{d}\theta$, we have

$$\bar{\delta} \simeq \underline{\delta} - \mathbb{C}'_\theta(\tilde{c}(\underline{\theta}),\underline{\theta})u'_c(\mathbb{C}(\tilde{c}(\underline{\theta}),\underline{\theta}),\underline{\theta})\mathrm{d}\theta. \tag{13}$$

The adjustments $\underline{\delta}$ and $\bar{\delta}$ follow from (13) and the feasibility constraint that, after the reform, total resources

$$\int\limits_{\Theta} [y(\theta) - \delta(\theta)]\mathrm{d}F(\theta)$$

must finance consumption

$$\int\limits_{\theta^{\inf}}^{\underline{\theta}} \mathbb{E}[\tilde{c}(\theta)]\mathrm{d}F(\theta) + \int\limits_{\underline{\theta}}^{\underline{\theta}+\mathrm{d}\theta} \mathbb{C}(\tilde{c}(\theta),\theta)\mathrm{d}F(\theta) + \int\limits_{\underline{\theta}+\mathrm{d}\theta}^{\theta^{\sup}} \mathbb{E}[\tilde{c}(\theta)]\mathrm{d}F(\theta).$$

Replacing $\mathbb{C}(\tilde{c}(\theta),\theta)$ with $\mathbb{E}[\tilde{c}(\theta)] - \pi(\tilde{c}(\theta),\theta)$ and using (4) at equality, we get

$$\int\limits_{\Theta} \delta(\theta)\mathrm{d}F(\theta) = \int\limits_{\underline{\theta}}^{\underline{\theta}+\mathrm{d}\theta} \pi(\tilde{c}(\theta),\theta)\mathrm{d}F(\theta).$$

This shows that the before-tax income resources created by the reform equal the aggregate risk premium of agents who no longer face income risk. For $\mathrm{d}\theta$ close to 0, this equality can be written as

$$\underline{\delta}F(\underline{\theta}) + \bar{\delta}(1 - F(\underline{\theta})) \simeq \pi(\tilde{c}(\underline{\theta}),\underline{\theta})f(\underline{\theta})\mathrm{d}\theta, \tag{14}$$

where the left-hand side uses $F(\bar{\theta}) \simeq F(\underline{\theta}) + f(\underline{\theta})\mathrm{d}\theta$ and neglects the second-order term $(\bar{\delta} - \underline{\delta})f(\underline{\theta})\mathrm{d}\theta$.

The system formed by (13) and (14) gives the adjustments of before-tax income consistent with feasibility and incentive compatibility at the outcome of the reform,

$$\underline{\delta} \simeq \left[\pi(\tilde{c}(\underline{\theta}),\underline{\theta})f(\underline{\theta}) + (1 - F(\underline{\theta}))\mathbb{C}'_{\theta}(\tilde{c}(\underline{\theta}),\underline{\theta})u'_c(\mathbb{C}(\tilde{c}(\underline{\theta}),\underline{\theta}),\underline{\theta})\right]\mathrm{d}\theta,$$

and

$$\bar{\delta} \simeq \left[\pi(\tilde{c}(\underline{\theta}),\underline{\theta})f(\underline{\theta}) - F(\underline{\theta})\mathbb{C}'_{\theta}(\tilde{c}(\underline{\theta}),\underline{\theta})u'_c(\mathbb{C}(\tilde{c}(\underline{\theta}),\underline{\theta}),\underline{\theta})\right]\mathrm{d}\theta.$$

By definition of the certainty equivalent, every agent derives the same level of sub-utility from consumption before and after the reform. Therefore, in the absence of redistributive concerns, $F(\theta) = G(\theta)$ for all θ, the change in social welfare implied by the reform coincides with the total change in before-tax income resources. The change in these resources, which appears in the right-hand side of (14), is positive. Hence, in this case, the deterministic menu improves upon the menu of lotteries.

Instead, if redistributive concerns matter, the reform improves social welfare if the social value of the total change in before-tax income resources is positive, $\underline{\delta}G(\underline{\theta}) + \bar{\delta}(1 - G(\underline{\theta})) > 0$. Reintroducing the expressions of $\underline{\delta}$ and $\bar{\delta}$, this is

$$\pi(\tilde{c}(\underline{\theta}),\underline{\theta})f(\underline{\theta}) + (G(\underline{\theta}) - F(\underline{\theta}))\mathbb{C}'_{\theta}(\tilde{c}(\underline{\theta}),\underline{\theta})u'_c(\mathbb{C}(\tilde{c}(\underline{\theta}),\underline{\theta}),\underline{\theta}) > 0.$$

Using $\mathbb{C}'_{\theta}(\tilde{c}(\theta),\theta) = -\pi'_{\theta}(\tilde{c}(\theta),\theta)$ yields the expression that appears in (12).

The inequality (12) is consistent with the intuitive idea that there is no socially beneficial income tax randomization in an economy consisting of agents who display high risk aversions ($\pi(\tilde{c}(\theta), \theta)$ is high). This inequality can however be satisfied if the risk premia, even small, are identical across agents, $\pi'_\theta(\tilde{c}, \theta) = 0$ for all $\tilde{c}$ and θ. This happens if taxpayers have the same preferences, $u(c, \theta)$ does not depend on θ, but this can also accommodate preference heterogeneity. In, e.g., the formulation $u(c, \theta) = v(c)/\theta$ similar to the one used by Lollivier and Rochet (1983), the certainty equivalent of lottery $\tilde{c}$ is defined by $v(\mathbb{C}(\tilde{c}, \theta)) = \mathbb{E}[v(\tilde{c})]$, and so it does not depend on θ.

To address the role played by redistribution tastes, observe first that the inequality (12) is satisfied independently of risk aversions for unweighted utilitarian social preferences ($G(\theta) = F(\theta)$ for all θ). In this sense, weak redistribution motives favor deterministic taxation. The inequality also holds for weighted utilitarian social preferences if $[G(\theta) - F(\theta)]\pi'_\theta(\tilde{c}, \theta)$ remains non-positive for all types, i.e., if the socially favored agents display a higher risk aversion (captured by a higher risk premium), a property much in line with Hellwig (2007).

The empirical literature usually finds a risk aversion decreasing with income. If the poor are indeed low types, i.e., those agents with high marginal utility of consumption, then the Spence-Mirrlees condition suggests to keep with the case where $\pi'_\theta(\tilde{c}, \theta) \leq 0$. Then the inequality (12) is met if redistribution also puts higher weights on low types, $G(\theta) \geq F(\theta)$ for all θ. The deterministic menu, if incentive compatible, performs better than the lotteries and redistribution should be made deterministically.

Note that (12) relies on the partial derivative $\pi'_\theta(\tilde{c}, \theta)$ evaluated at $\tilde{c}(\theta)$, i.e., the behavior of the risk premium when the type varies while the after-tax income remains fixed. This does not preclude some form of over-compensation where low types would eventually receive a higher total income $\theta + \mathbb{C}(\tilde{c}(\theta), \theta)$ or a higher efficient income $\theta\mathbb{C}(\tilde{c}(\theta), \theta)$, even though incentive compatibility requires that the after-tax income $\mathbb{C}(\tilde{c}(\theta), \theta)$ is non-increasing in θ.

CARA-Gaussian example. In the CARA-Gaussian case, (12) is satisfied for a high enough level of risk aversion independently of the shape of $G(\theta)$ $F(\theta)$, i.e., whatever the social desires for redistribution. Type θ agents have CARA preferences

$$u(c, \theta) = -\frac{1}{\theta} \exp(-\theta c),$$

with θ her absolute risk aversion coefficient. Utility is increasing in c and θ, concave in c, and it meets the Spence-Mirrlees assumption, $u''_{c\theta}(c, \theta) = -\theta \exp(-\theta c) < 0$. Agents face Gaussian after-tax income lotteries ($\tilde{c}(\theta)$)

with mean $m(\tilde{c}(\theta))$ and variance $v(\tilde{c}(\theta)) > 0$. Since

$$\mathbb{E}[u(\tilde{c}(\tau), \theta)] = -\frac{1}{\theta} \exp\left[-\theta\left(m(\tilde{c}(\tau)) - \frac{\theta}{2} v(\tilde{c}(\tau))\right)\right],$$

the certainty equivalent of $\tilde{c}(\tau)$ for a type θ agent is

$$\mathbb{C}(\tilde{c}(\tau), \theta) = m(\tilde{c}(\tau)) - \frac{\theta}{2} v(\tilde{c}(\tau)),$$

which is assumed to be positive, and the associated risk premium is

$$\pi(\tilde{c}(\tau), \theta) = \frac{\theta}{2} v(\tilde{c}(\tau)).$$

Hence inequality (12) is equivalent to

$$\int_{\Theta} v(\theta)[\theta - (G(\theta) - F(\theta)) \exp(-\theta \mathbb{C}(\tilde{c}(\theta), \theta))] \mathrm{d}\theta > 0.$$

Since both $G(\theta) - F(\theta) \leq 1$ and $\exp(-\theta \mathbb{C}(\tilde{c}(\theta), \theta)) \leq 1$ for all θ, we have $(G(\theta) - F(\theta)) \exp(-\theta \mathbb{C}(\tilde{c}(\theta), \theta)) \leq 1$ for all θ. Hence, for $\theta^{\inf} \geq 1$, every term in the sum in (12) is non-negative. ■

In practice, as we argued above, one may expect $\pi'_\theta(\tilde{c}, \theta) \leq 0$ and $G(\theta) \geq F(\theta)$ in (12), which suggests that the certainty equivalents should perform better than the lotteries. We must nevertheless be careful when drawing such a conclusion since the lotteries must also have lower certainty equivalents for higher types, by Item 2 of Proposition 1. It is not clear whether this monotonicity condition is implied by incentive compatibility of the initial menu of lotteries. Actually Section 5 suggests that the relevant case for (12) instead is one where $[G(\theta) - F(\theta)]\pi'_\theta(\tilde{c}, \theta)$ is positive, as incentives in the menu of lotteries point to $\pi'_\theta(\tilde{c}, \theta) \geq 0$. The conflict between redistribution to the poor, captured by $G(\theta) - F(\theta) > 0$, and the preservation of incentives when switching to certainty equivalents weakens the case for deterministic redistribution.[4]

[4] As suggested by the associate editor, (12) may relate to the direction in which incentive constraints are binding. Indeed downward incentive constraints should be binding whenever $G(\theta) - F(\theta) \geq 0$ as the government cares about low types, presumably poor agents. The empirically plausible case where $\pi'_\theta(\tilde{c}, \theta) \leq 0$ then makes randomization non-desirable. But if the government instead wants to favor high types, $G(\theta) - F(\theta) \geq 0$, it becomes more likely that low types envy high types, which fits a pattern where upward incentive constraints are binding. Then, the case where $\pi'_\theta(\tilde{c}, \theta) \leq 0$ tends to make the initial random menu superior to the certainty equivalents.

5 Monotone certainty equivalents

We now examine the monotonicity of the certainty equivalents required in Item 2 of Proposition 1. Following Lemma 2, if $\tilde{c}(\theta_1)$ first-order stochastically dominates $\tilde{c}(\theta_2)$, then any given risk averse agent θ prefers $\tilde{c}(\theta_1)$ to $\tilde{c}(\theta_2)$. So $\mathbb{C}(\tilde{c}(\theta_1), \theta) \geq \mathbb{C}(\tilde{c}(\theta_2), \theta)$ for all θ. This ordering does not imply a certainty equivalent decreasing in θ, which involves the more demanding comparison between $\mathbb{C}(\tilde{c}(\theta_1), \theta_1)$ and $\mathbb{C}(\tilde{c}(\theta_2), \theta_2)$, i.e., how different types value the two lotteries.

To delineate circumstances where the menu of certainty equivalents meets the incentive constraints in Item 2, observe that, by Assumption A1, total differentiation in θ of (11) evaluated at $\tilde{c} = \tilde{c}(\theta)$ leads to

$$\frac{\mathrm{d}\mathbb{C}(\tilde{c}(\theta), \theta)}{\mathrm{d}\theta} \leq 0$$

$$\Leftrightarrow \mathbb{E}\left[u'_\theta(\tilde{c}(\theta), \theta)\right] - u'_\theta(\mathbb{C}(\tilde{c}(\theta), \theta), \theta) + \int_{\mathcal{C}} u(c, \theta) h'_\theta(c, \theta) \mathrm{d}c \leq 0. \qquad (15)$$

We now provide simple sufficient conditions for both a negative difference between the first two terms, and a negative last sum in (15).

We start with the difference between $\mathbb{E}\left[u'_\theta(\tilde{c}(\theta), \theta)\right]$ and $u'_\theta(\mathbb{C}(\tilde{c}(\theta), \theta), \theta)$. Recall that $\mathbb{C}(\tilde{c}(\theta), \theta) = \mathbb{E}[\tilde{c}(\theta)] - \pi(\tilde{c}(\theta), \theta) < \mathbb{E}[\tilde{c}(\theta)]$. Therefore the Spence-Mirrlees condition $u''_{c\theta}(c, \theta) < 0$ yields

$$u'_\theta(\mathbb{C}(\tilde{c}(\theta), \theta), \theta) > u'_\theta(\mathbb{E}[\tilde{c}(\theta)], \theta),$$

or equivalently,

$$\mathbb{E}\left[u'_\theta(\tilde{c}(\theta), \theta)\right] - u'_\theta(\mathbb{C}(\tilde{c}(\theta), \theta), \theta) < \mathbb{E}\left[u'_\theta(\tilde{c}(\theta), \theta)\right] - u'_\theta(\mathbb{E}[\tilde{c}(\theta)], \theta).$$

It follows that $\mathbb{E}\left[u'_\theta(\tilde{c}(\theta), \theta)\right] - u'_\theta(\mathbb{C}(\tilde{c}(\theta), \theta), \theta) \leq 0$ if

$$\mathbb{E}\left[u'_\theta(\tilde{c}(\theta), \theta)\right] \leq u'_\theta(\mathbb{E}[\tilde{c}(\theta)], \theta),$$

a condition that is satisfied if $u'_\theta(c, \theta)$ is concave in c.

We now turn to the sign of the last sum in (15). Following the insights in Lemma 2, we exploit stochastic dominance properties. Suppose that the lotteries designed for lower types first-order dominate those designed for higher types, $H(c,\theta+\mathrm{d}\theta) \leq H(c,\theta)$ for all c, θ and $\mathrm{d}\theta < 0$. For neighboring types, $\mathrm{d}\theta \simeq 0$, this inequality reads

$$H'_\theta(c,\theta) \geq 0.$$

Using the integration by parts formula, one can write

$$\int_{\mathcal{C}} u(c,\theta)h'_\theta(c,\theta)\mathrm{d}c = -\int_{\mathcal{C}} u'_c(c,\theta)H'_\theta(c,\theta)\mathrm{d}c.$$

Utility is increasing with consumption, so that this sum is negative.

We have consequently shown that:

Lemma 3. *The certainty equivalent $\mathbb{C}(\tilde{c}(\theta),\theta)$ is non-increasing in θ if the following two conditions are satisfied:*

1. *$\tilde{c}\,(\theta_1)$ first-order stochastically dominates $\tilde{c}\,(\theta_2)$ for any two types θ_1 and θ_2, with $\theta_1 < \theta_2$.*

2. *$u'_\theta(c,\theta)$ is concave in c.*

Item 1 relates to the initial menu of lotteries. It implies that any given agent prefers the lotteries designed for low types to those designed for high types. In view of Lemma 2, the conditions for incentive compatibility of the lotteries align with those of the certainty equivalents. Keeping with the interpretation of low types as being the poor, incentives tend to be preserved if the preferred bundles are designed for the less well-off part of the population.

Item 2 instead is on individual preferences. If, from Pratt's theorem, one refers to concavity of the utility function u in c as an equivalent measure of risk aversion, then the monotonicity properties of the certainty equivalent needed for implementing the deterministic menu obtain if higher types display a higher risk aversion. That is, the second derivative of the function u with respect to consumption $u''_{cc}(c,\theta)$, which takes negative values for risk averse agents, is decreasing with θ. Item 2 thus fails to be satisfied in the empirically plausible case where the poor, rather than the rich, display the greatest risk aversions. Incentive compatibility of the menu of lotteries tends to be inconsistent with incentive compatibility of the menu of the certainty equivalents.

We conclude that (12) is to be considered with $G(\theta) - F(\theta) \geq 0$, if the poor are socially favored, and $\pi'_\theta(\tilde{c},\theta) \geq 0$ to maintain incentive compatibility of certainty equivalents. Then, incentive-compatible menus of certainty equivalents dominate random redistribution provided that $G(\theta) - F(\theta)$, if positive, is not too large, a configuration of weak redistribution social tastes.

6 Conclusion

We have provided necessary and sufficient conditions for incentive compatibility of a menu of consumption or after-tax income lotteries. If the marginal utility of consumption is decreasing with type, the first-order approach can be applied if the lotteries designed for low types first-order stochastically dominate those designed for high types. Low types give more importance to consumption, and as such they may be considered as the less well-off part of the population. If the government puts a higher valuation on these agents, our results suggest that incentive compatibility of income lotteries may not be preserved when they are replaced with their certainty equivalents. Eventually, the additional resources coming from confiscated risk premia when switching to the deterministic policy allows for a social gain for weak enough redistribution motives in high enough risk-averse populations.

Certainty equivalents provide us with a natural benchmark for deterministic alternatives, but other deterministic menus could obviously improve social welfare upon lotteries. Hence it may be that redistribution should be made deterministically while the certainty equivalents are dominated or fail to meet incentive compatibility. Characterization of such alternative menus could be the subject of further work.

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A Proof of Proposition 1

The switch to the menu of certainty equivalent incomes leads to a change in social welfare (6) equal to

$$\int_{\Theta} [u(\mathbb{C}(\tilde{c}(\theta), \theta), \theta) + \delta(\theta))] \mathrm{d}G(\theta) - \int_{\Theta} \mathbb{E}[u(\tilde{c}(\theta), \theta)] \mathrm{d}G(\theta). \tag{16}$$

Using (11), this change reduces to

$$\int_{\Theta} \delta(\theta) \mathrm{d}G(\theta). \tag{17}$$

Social welfare improves if agents with high social valuations enjoy a reduction in their before-tax income.

The before-tax income adjustments $(\delta(\theta))$ must meet feasibility (4) and incentive compatibility (5). The incentive constraints associated with the final deterministic schedule are

$$\theta = \arg\max_{\tau} u(\mathbb{C}(\tilde{c}(\tau), \tau), \theta) - y(\tau) + \delta(\tau)$$

for all θ. The differentiability properties of $\mathbb{C}(\tilde{c}(\tau), \theta)$ and the fact that type $\tau = \theta$ solves the maximization program in (16) imply that the before-tax income $y(\tau) + \delta(\tau)$ is continuously differentiable (see Guesnerie and Laffont (1984), Theorem 1). Incentive compatibility thus requires

$$[\mathbb{C}'_{\tau}(\tilde{c}(\tau), \tau) + \mathbb{C}'_{\theta}(\tilde{c}(\tau), \tau)]\, u'_c(\mathbb{C}(\tilde{c}(\tau), \tau), \theta) - y'(\tau) + \delta'(\tau) = 0 \tag{18}$$

for all θ and $\tau, \tau = \theta$. The incentive constraints for the initial random menu $(\tilde{c}(\theta), y(\theta))$,

$$\theta = \arg\max_{\tau} \mathbb{E}[u(\tilde{c}(\tau), \theta)] - y(\tau) = \arg\max_{\tau} u(\mathbb{C}(\tilde{c}(\tau), \theta), \theta) - y(\tau)$$

for all θ, require $\mathbb{C}'_{\tau}(\tilde{c}(\tau), \theta) u'_c(\mathbb{C}(\tilde{c}(\tau), \theta), \theta) - y'(\tau) = 0$ for all θ and $\tau, \tau = \theta$. Hence (18) simplifies to

$$\delta'(\theta) = -\mathbb{C}'_{\theta}(\tilde{c}(\theta), \theta) u'_c(\mathbb{C}(\tilde{c}(\theta), \theta), \theta)$$

for all θ. By summation over types we obtain

$$\delta(\theta) = \delta\left(\theta^{\inf}\right) - \int_{\theta^{\inf}}^{\theta} \mathbb{C}'_{\theta}(\tilde{c}(z), z) u'_c(\mathbb{C}(\tilde{c}(z), z), z) \mathrm{d}z \tag{19}$$

for all θ.

The feasibility constraint (4) at equality gives the value of $\delta(\theta^{\inf})$. The derivation is as follows. After replacing in (4) the sure income $\mathbb{C}(\tilde{c}(\theta), \theta)$ with the difference $\mathbb{E}[\tilde{c}(\theta)] - \pi(\tilde{c}(\theta), \theta)$, the feasibility constraint takes the form

$$\int_{\Theta} [y(\theta) - \delta(\theta) - \mathbb{E}[\tilde{c}(\theta)] + \pi(\tilde{c}(\theta), \theta)] \mathrm{d}F(\theta) = 0.$$

Since the initial random menu $(\tilde{c}(\theta), y(\theta))$ also meets (4), this equality simplifies to

$$\int_{\Theta} [\pi(\tilde{c}(\theta), \theta) - \delta(\theta)] \mathrm{d}F(\theta) = 0. \tag{20}$$

Using (19), one can express $\delta(\theta)$ as a function of $\delta\left(\theta^{\inf}\right)$. Finally the identity $\mathbb{C}'_\theta(\tilde{c}(\theta), \theta) = -\pi'_\theta(\tilde{c}(\theta), \theta)$ gives

$$\delta\left(\theta^{\inf}\right) = \int_{\Theta} \left[\pi(\tilde{c}(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)} \pi'_\theta(\tilde{c}(\theta), \theta) u'_c(\mathbb{C}(\tilde{c}(\theta), \theta), \theta)\right] \mathrm{d}F(\theta).$$

We are now in a position to write the change in social welfare from a reform that replaces the lotteries with certainty equivalents. Reintroducing the expression of $\delta(\theta)$ found in (19) into (17) and using the integration by parts formula, the change in social welfare (17) rewrites

$$\delta\left(\theta^{\inf}\right) - \int_{\Theta} \frac{1 - G(\theta)}{f(\theta)} \mathbb{C}'_\theta(\tilde{c}(\theta), \theta) u'_c(\mathbb{C}(\tilde{c}(\theta), \theta), \theta) \mathrm{d}F(\theta).$$

The expression of $\delta\left(\theta^{\inf}\right)$ derived above yields the inequality stated in Item 1. Indeed there is a social welfare improvement if and only if (17) is positive.

To prove the statement in Item 2, observe that (8) in Lemma 1 specialized to the deterministic case reduces to the monotonicity condition on $\mathbb{C}(\tilde{c}(\theta), \theta)$ in Item 2 as necessary for incentive compatibility. Under the Spence-Mirrlees condition, it also ensures that the incentive constraints hold for every type θ and every report τ. This concludes the proof.