Coordinating Coordination Failures in Keynesian Models
Author(s): Russell Cooper and Andrew John
Reviewed work(s):
Source: *The Quarterly Journal of Economics*, Vol. 103, No. 3 (Aug., 1988), pp. 441-463
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/1885539
Accessed: 19/04/2012 07:00

THE

QUARTERLY JOURNAL
OF ECONOMICS

Vol. CIII | August 1988 | Issue 3

COORDINATING COORDINATION FAILURES IN KEYNESIAN MODELS*

RUSSELL COOPER AND ANDREW JOHN

This paper focuses on the importance of strategic complementarities in agents' payoff functions as a basis for macroeconomic coordination failures. Strategic complementarities arise when the optimal strategy of an agent depends positively upon the strategies of the other agents. We first analyze an abstract game and find that multiple equilibria and a multiplier process may arise when strategic complementarities are present. Often these equilibria can be Pareto ranked. We then place additional economic content on the analysis of this game by considering strategic complementarities arising from production functions, matching technologies, and commodity demand functions in a multisector, imperfectly competitive economy.

I. INTRODUCTION

There are three types of papers in the macroeconomic literature on unemployment theory. First, those of the new classical macroeconomics have sought to argue that underemployment-unemployment arises from intertemporal substitution of leisure or misperceptions of prices due to an inability to distinguish perfectly between changes in relative prices and changes in the general level of prices. Second, articles in the Keynesian tradition suggest that unemployment arises from nonrational expectations or wage and price rigidities; many insights of these theories have been formalized in the fix-price literature. And, third, there is a group of papers that start with the observation that there are two theories of unemployment—new classical and Keynesian—and then offer an

*Financial assistance from the Cowles Foundation at Yale University for both authors and from the National Science Foundation (Grant No. SES 8605302) to the first author is gratefully acknowledged. We received helpful comments and advice from seminar participants at Yale, New York University, the Board of Governors of the Federal Reserve System, the Conference on Unemployment at York University, Olivier Blanchard, Nobuhiro Kiyotaki, David Romer, and an anonymous referee.

alternative model. The discussion here is concerned with papers in this third category.

More specifically, a number of authors have recently constructed examples of economies that exhibit underemployment equilibria, but where the results do not derive from the usual Keynesian assumptions; see, for example, Bryant [1983], Diamond [1982], Hart [1982], and Weitzman [1982]. The models in these papers instead generate their results from the inability of agents to coordinate their actions successfully in a many-person, decentralized economy. These examples of macroeconomic inefficiencies are superficially very dissimilar: for example, Diamond's model is grounded in search theory; Bryant emphasizes imperfect information; and Weitzman stresses increasing returns. As a result, the similarities of these models have been obscured, and their crucial elements have not been fully identified.

One of the aims of this paper, therefore, is to provide a general framework that we can use to analyze these different models and explain how they relate to one another. We achieve this by isolating the crucial common elements of these models and indicating their role in generating Keynesian results. Our more ambitious goal is to use this framework to yield further insights into coordination failures and to relate this literature to other models in the Keynesian tradition, including those with fixed prices.

The two features highlighted in this paper are *spillovers* and *strategic complementarities*. The former refers to the interactions between agents at the level of payoffs, while the latter refers to interactions at the level of strategies. Suppose that in a game there are two players who select single dimensional strategies. Spillovers arise if an increase in one player's strategy affects the *payoffs* of the other players. Following Bulow, Geanakoplos, and Klemperer [1985], we say that strategic complementarities arise if an increase in one player's strategy increases the *optimal strategy* of the other player.

Our analysis has two components. We first construct a simple game, essentially devoid of economic structure, and show that strategic complementarities are associated with the presence of "Keynesian features" such as multiple equilibria and a multiplier process. Spillovers imply that these equilibria generally will be inefficient and can be Pareto-ranked. When this occurs, a *coordination failure* is present: mutual gains from an all-around change in strategies may not be realized because no individual player has an incentive to deviate from the initial equilibrium. We then analyze a number of examples in which spillovers and strategic complementarities naturally emerge.

In Section II, therefore, we develop a game in which players' optimal strategies depend upon the strategies of other agents. Because we wish to analyze situations where agents may fail to coordinate their actions, the natural equilibrium concept to utilize is that of Nash equilibrium, which is by its nature noncooperative. Given strategic complementarity, there may be multiple symmetric Nash equilibria; we examine the welfare properties of these and show that, given spillovers, these equilibria will be inefficient. In addition, we show that strategic complementarity is necessary and sufficient for multiplier effects in our game.

The analysis in Section II allows us to conclude that any economy which can be represented in our game and displays strategic complementarity will also possess certain Keynesian features. It can be conjectured that a wide class of models might admit such representation and hence exhibit multiple equilibria and multiplier effects. To illustrate this, we present some extended examples in Section III which show that complementarities can arise from the production technology, from the matching technology, and from agents' demands. These examples draw upon, and develop, other models of coordination failures, but we emphasize that their message, and that of this paper, goes beyond the particular models discussed. In Section IV we summarize our arguments and discuss possible extensions and questions for future research.

II. Symmetric Nash Equilibria and Strategic Complementarity

To begin the analysis, we consider a relatively abstract game exhibiting a multiplicity of symmetric Nash equilibria (SNE). Assume that there are I agents indexed $i = 1,2,\ldots,I$ where agent i's strategy variable (or "action") is $e_i \in [0,E]$, where E is finite and bounds an agent's action. The nature of this strategy variable will depend upon the economic context, as discussed in Section III. While we refer to "agents" throughout, "I" can equivalently be interpreted as the number of coalitions in the economy. The important assumption is that there are (groups of) agents whose actions have nonnegligible effects on the payoffs of others and who behave strategically with respect to one another. Denote the payoff of agent i by $\sigma(e_i, e_{-i}; \theta_i)$, where e_{-i} is the vector of strategies chosen by the other players and θ_i is a parameter of i's payoff function.[1] Assume that the payoff functions are continuously differentiable, that $\partial^2\sigma()/\partial e_i^2 < 0$, and that $\partial^2\sigma()/\partial e_i \partial\theta_i > 0$.

Denote the payoff to agent i of action e_i when all other agents take action $\bar{e}$ by $V(e_i, \bar{e})$, and let $e_i^*(\bar{e})$ be the optimal response of agent i when $e_j = \bar{e}$ for all $j \neq i$.[2] At a SNE, if all other agents are choosing e, it is in the interest of the remaining agent to select e as well (that is, at a SNE, $e_i^*(e) = e$). Hence the SNE of the game are defined by

$$S = \{e \in [0,E] \mid V_1(e,e) = 0\},$$

where the subscript denotes a partial derivative in the usual manner. To ensure the existence of an interior solution, assume that

$$\lim_{e \to 0} V_1(e,e) > 0 \qquad \text{and} \qquad \lim_{e \to E} V_1(e,e) < 0.$$

Given our continuity assumptions on $\sigma()$, there will exist an $e \in S$.

To carry out welfare analyses, it is useful also to consider symmetric cooperative equilibria (SCE), defined by an action for all agents that represents a local solution to the problem of maximizing the welfare of a representative agent. Exploiting symmetry, the set of SCE can be characterized by

$$\tilde{S} = \{e \in [0,E] \mid V_1(e,e) + V_2(e,e) = 0; \\ V_{11}(e,e) + 2V_{12}(e,e) + V_{22}(e,e) < 0\}.$$

We assume that

$$\lim_{e \to 0} V_1(e,e) + V_2(e,e) > 0 \qquad \text{and} \qquad \lim_{e \to E} V_1(e,e) + V_2(e,e) < 0,$$

ensuring the existence of an interior SCE. Note that the number of SCE may differ from the number of SNE.

Before presenting our analysis of this game, we introduce the following definitions:

(i) if $V_2(e_i, \bar{e}) > 0$, the game exhibits *positive spillovers*;
(ii) if $V_2(e_i, \bar{e}) < 0$, the game exhibits *negative spillovers*;

1. Assume that when θ_i is equal for all i, payoff functions are identical.
2. We can also interpret $\bar{e}$ as some aggregate index of other agents' strategies; many of the examples that we investigate in Section III lend themselves to such an interpretation. This idea that an individual's payoff may depend upon economy-wide aggregates also often seems to be a feature of Keynesian models. The function $V()$ is used because of our emphasis on symmetric equilibria.

(iii) if $V_{12}(e_i,\bar{e}) > 0$, the game exhibits *strategic complementarity*;
(iv) if $V_{12}(e_i,\bar{e}) < 0$, the game exhibits *strategic substitutability*;
(v) if $d\Sigma\, e_j^*/d\theta_i > de_i^*/d\theta_i > \partial e_i^*/\partial\theta_i$, the game exhibits *multiplier effects.*

Definitions (i) and (ii) are straightforward; they simply characterize the externality implicit in the game formulation. Most of our examples exhibit positive spillovers, so an increase in the strategy of all but one agent bestows an external benefit upon the remaining agent.

Strategic complementarity (definition (iii)) is the feature that we emphasize in our analysis. It implies that an increase in the action of all agents except agent i increases the marginal return to agent i's action. Hence, e_i will be an increasing function of $\bar{e}$. Definition (iv) is just the converse of strategic complementarity.

Definition (v) requires a little more explanation. We consider a variation in θ_i in the neighborhood of a stable SNE and contrast the initial response of agent i with the equilibrium response of that agent and of all other agents. Multiplier effects are present when the aggregate response exceeds the individual response. (We could equally consider the case where all agents face identical shocks.)

Using these definitions, we now establish the following propositions in this game. We first state our results, then discuss them below.

PROPOSITION 1. Strategic complementarity is necessary for multiple SNE.

Proof of Proposition 1. This is evident from inspection of Figure I, which shows agent i's reaction function for symmetric changes in the action of all other players, $(e_i^*(\bar{e}))$. Given our assumption of continuity, multiple intersections of this function with the 45 degree line are only possible if the reaction function is upward-sloping in some range. The slope of this reaction function is $\rho = -V_{12}/V_{11}$. Strategic complementarity thus is equivalent to $\rho > 0$ and hence is necessary for multiple equilibria.[3]

PROPOSITION 2. If the game exhibits spillovers at $e \in S$, then e is inefficient.

Proof of Proposition 2. This follows trivially from the defini-

3. Note that if $\rho > 1$ at a SNE, the equilibrium is locally unstable.

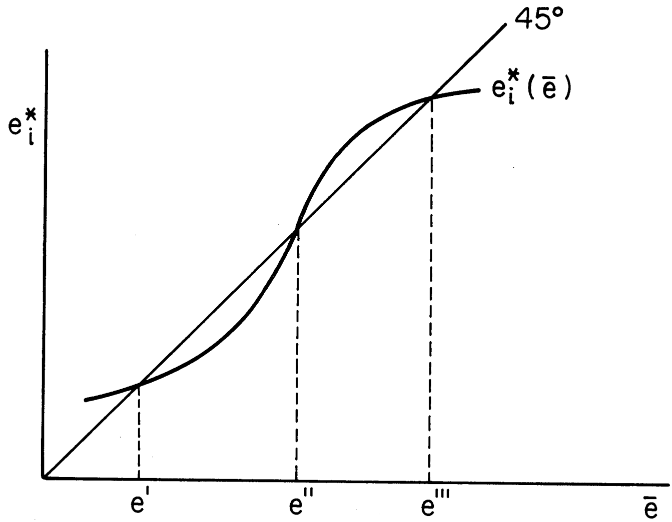

FIGURE I

tions of S and $\tilde{S}$ and from the fact that any efficient equilibrium is a SCE.

PROPOSITION 3. Given positive spillovers at $e \in S$, there exists $e' \in \tilde{S}$ with $e' > e$.

Proof of Proposition 3. This follows from continuity and the definitions of S and $\tilde{S}$.

PROPOSITION 4. Given multiple SNE and positive spillovers globally, then the SNE can be Pareto ranked by the equilibrium action; i.e., higher action equilibria are preferred.

Proof of Proposition 4. $d[V(e_i^*(\bar{e}),\bar{e})]/d\bar{e} = V_2(e_i,\bar{e})$ by the envelope theorem. Hence agent i's payoff, evaluated along that agent's reaction function, increases as all other agents increase their action.[4]

PROPOSITION 5. If $e_i^*(\bar{e}) = \bar{e}$ over some interval and there are positive spillovers, then there is a continuum of equilibria with welfare increasing in e over that interval.

Proof of Proposition 5. By definition of $e_i^*(\bar{e})$ and Proposition 4.

4. We thank Nobuhiro Kiyotaki for this observation.

PROPOSITION 6. Strategic complementarity is necessary and sufficient for multipliers in this game.

Proof of Proposition 6. Let $s = \rho/(I - 1)$; s can be interpreted as the response of agent i to a change in the strategy of one other agent. As before, let $\bar{e}$ be the action adopted by all agents except i, and consider a shock to agent i's payoff function. Then, using the first-order conditions for a SNE, it can be shown that[5]

$$\frac{de_i^*}{d\theta_i} = \left[\frac{1 - \rho + s}{(1 + s)(1 - \rho)}\right] \frac{\partial e_i^*}{\partial \theta_i}; \qquad \text{and} \qquad \frac{d\bar{e}}{d\theta_i} = \left[\frac{s}{(1 + s)(1 - \rho)}\right] \frac{\partial e_i^*}{\partial \theta_i}.$$

Since

$$\frac{d\Sigma\, e_j^*}{d\theta_i} = \frac{de_i}{d\theta_i} + (I - 1)\frac{d\bar{e}}{d\theta_i},$$

it follows that

$$\frac{d\Sigma\, e_j^*}{d\theta_i} = \left[\frac{1 + s}{1 - \rho + s}\right] \frac{de_i^*}{d\theta_i} = \left[\frac{1}{1 - \rho}\right] \frac{\partial e_i^*}{\partial \theta_i}.$$

As we are restricting attention to stable SNE, $\rho < 1$. It is easily verified that the equilibrium response of agent i will exceed the partial response whenever $\rho \neq 0$. However, the aggregate equilibrium response will only exceed the response of agent i when the multiplier $(1/1 - \rho)$ exceeds 1; i.e., when the game exhibits strategic complementarity.

The first proposition highlights the connection between strategic complementarity and multiplicity of equilibria. It should be emphasized that strategic complementarity, while necessary for multiple equilibria, is not sufficient. Indeed, the much stronger condition that $\rho \geq 1$ is also necessary (and still not sufficient). That is, the reaction function must somewhere have slope of at least unity, implying that agent i's action increases at least one-for-one with other agents' actions. A sufficient condition for multiplicity is $\rho > 1$ at a SNE.

5. In general, calculating these multipliers involves total differentiation and application of Kramer's rule to a system of I first-order conditions. At a SNE, and allowing only θ_i to vary, this is equivalent to solving the system:

$$\begin{bmatrix} 1 & -\rho \\ -s & (1 - \rho + s) \end{bmatrix} \begin{bmatrix} de_i \\ d\bar{e} \end{bmatrix} = \begin{bmatrix} \frac{\partial e_i}{\partial \theta_i} \\ 0 \end{bmatrix} d\theta_i.$$

Note that $\partial e_i/\partial \theta_i > 0$ from our earlier assumptions.

The next three propositions set out the basic welfare properties of this game. Proposition 2 simply notes the familiar inefficiency due to the externalities in the payoff functions. Agents, in choosing their strategy, do not take account of their influence on the payoffs of others; hence SNE are not efficient relative to the set of feasible allocations. Proposition 3 shows that in the presence of positive spillovers, there will be a tendency to insufficient action in Nash equilibrium. That is, a coordinated increase in the strategies of all agents would be welfare improving.

Proposition 4 considers the welfare properties of the set of SNE when there are multiple equilibria. In the presence of positive spillovers, equilibria with higher levels of action are preferred by all agents. The economy can get stuck at an inefficient equilibrium with a low level of "economic activity," even though a better equilibrium exists. This is a *coordination failure*: if there was a mechanism for agents to coordinate their activities, they could achieve a better (cooperative) equilibrium. As we discuss in Section III, many examples of models with Keynesian features involve such coordination failures.

Some of the models we consider below exhibit a continuum of equilibria. These have the virtue of providing clear cases of welfare orderings dependent on the equilibrium strategy. Proposition 5 sets out the condition for a continuum of equilibria over some interval. Along this continuum, symmetric increases in strategies are welfare improving when positive spillovers are present.

Propositions analogous to 3, 4, and 5 can be proved in the presence of negative spillovers. That is, among symmetric Nash equilibria, agents prefer those with *lower* actions, and there will exist some symmetric action, below that in any SNE, which agents prefer to all the Nash equilibria. Other rankings of equilibria are naturally possible if spillovers are positive in some ranges and negative in others.

Proposition 6 focuses on another feature of Keynesian models that is of interest here, namely the multiplier. The proposition shows that, given strategic complementarity, the aggregate response to a shock exceeds the individual response. In the presence of strategic complementarities, changes in agent i's strategy will induce changes in the actions of other agents in the same direction, which will in turn lead to a further change in agent i's action, much as in the standard multiplier story. Proposition 6 implies that, in the presence of strategic complementarities, small shocks may result in large changes in economic variables.[6] Furthermore, the aggregate welfare implications of a shock are greater than the initial welfare effect.

The key results of this section are thus that strategic complementarity is necessary and sufficient for multiplier effects and necessary for the existence of multiple equilibria. Further, under the assumption of positive spillovers, agents will prefer equilibria with higher levels of activity, and even the best Nash equilibrium is dominated by some higher (symmetric) strategy. In the following section we present some examples of these propositions.

III. Some Economic Examples

This section of the paper brings some economic life to the game discussed in Section II. We discuss both market and nonmarket games, and focus on a number of sources of strategic complementarity. Following Scitovsky [1954], we consider externalities in technologies as well as those arising from transactions and discuss how these sources of externalities can play a role in generating multiple, inefficient equilibria, and multipliers.

A. Input Games

First, consider the problem of coordination among input suppliers to a shared production process. Let e_i be the effort (input) of player i in the production of a public good, c. The production function is $c = f(e_i,\bar{e})$ with $f_1 > 0$ and $f_2 > 0$. Agents have identical utility functions defined over consumption and effort $U(c,e)$, with $U_1 > 0$, $U_2 < 0$, and $U()$ quasi-concave. These preferences, together with the production function, generate agent i's payoff function:

$$V(e_i,\bar{e}) = U(f(e_i,\bar{e}),e_i. \tag{1}$$

Note that $V_2\ () = U_1 f_2 > 0$, so the model exhibits positive spillovers.

Differentiating the payoff function with respect to e_i and $\bar{e}$ yields

$$V_{12} = U_1 f_{12} + U_{11} f_1 f_2 + U_{12} f_2. \tag{2}$$

6. Jovanovic [1987] investigates the aggregate implications of microeconomic shocks in large economies. Haltiwanger and Waldman [1985] focus on the disproportionate effect of inertial behavior in the presence of strategic complementarity (in their terminology, synergism).

In order to focus attention on technological interactions, assume that preferences between consumption and effort are separable. A necessary condition for strategic complementarity is then that inputs be complementary within the production process ($f_{12} > 0$). Increases in $\bar{e}$ will induce increases in e_i if f_{12} is large enough to offset the reduction in the marginal utility of consumption brought about by $f_2 > 0$. If $U()$ is linear, $f_{12} > 0$ is sufficient for strategic complementarities.

Note further that

$$\rho = \frac{-V_{12}}{V_{11}} = \frac{U_1 f_{12} + U_{11} f_1 f_2}{-[U_{11}(f_1)^2 + U_1 f_{11} + U_{22}]}. \tag{3}$$

The necessary condition for multiplicity ($\rho \geq 1$) is

$$U_1(f_{12} + f_{11}) + U_{11}[(f_1)^2 + f_1 f_2] \geq -U_{22}. \tag{4}$$

Multiple equilibria are thus more likely when the utility function is not very concave with respect to consumption and effort; when inputs are highly complementary in production; and when the production function is not very concave with respect to own effort. (Note, though, that there must be sufficient concavity to ensure that the second-order conditions are satisfied.) We now consider two examples of this model.

Example 1. Suppose that $f(e_i,\bar{e}) = g(e_i + (I-1)\bar{e})$, with $g' > 0$ and $g'' > 0$. Under this technology, the marginal product of an individual's effort depends positively on the aggregate level of effort in the economy: the increasing returns to scale thus generates a complementarity in the production process. Assume again that preferences are separable. Then

$$\rho = \frac{(I-1)[g''/g' + g'U_{11}/U_1]}{-[g''/g' + g'U_{11}/U_1 + U_{22}/g'U_1]}, \tag{5}$$

so strategic complementarities are present when the convexity of the production function exceeds the concavity of the utility function. If $U()$ is nearly linear in consumption, then $\rho > 0$. Variations in the effort supplied by an individual agent (say due to an individual shock to the disutility of effort) will result in correlated and magnified variations in effort by other agents.

Example 2. Bryant [1983] assumed that $f(e_i,\bar{e}) = \min(e_i,\bar{e})$ and interpreted $f()$ as a per capita production function. (This technology implies discontinuities in marginal products, so the analysis in Section II does not directly apply.) The input choice of

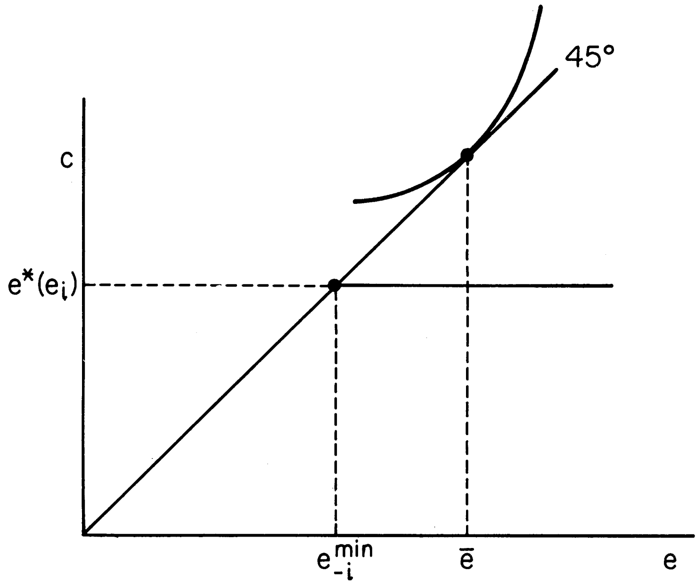

FIGURE II

agent i is shown in Figure II. Let $\hat{e}$ solve $U_1(\hat{e},\hat{e}) = -U_2(\hat{e},\hat{e})$. If $\bar{e} < \hat{e}$, then the optimal response of agent i is to set $e_i = \bar{e}$. Since agents are identical, their reaction curves coincide with the 45 degree line, and there is a continuum of equilibria in the interval $[0,\hat{e}]$. These equilibria are Pareto-ranked (welfare increases in e) with $\hat{e}$ Pareto efficient. Due to coordination failures, the economy can get stuck at a low level of output.[7]

These examples illustrate that coordination failures may emerge from technological complementarities within a shared production process. Our interpretation of $f()$ as either a production function for a public good or per capita consumption leaves open the question of whether or not there exist mechanisms that can overcome these coordination failures. Holmstrom's [1982] work on team incentives is an attempt to answer this question from the perspective of the internal organization of a firm.

7. A market version of Bryant's model (drawing on Cooper [1983]) was provided in an earlier version of this paper. There we showed that the continuum of equilibria could be supported as market outcomes in an economy with suppliers of inputs and producers of final goods, *if* input suppliers are monopolists and final goods producers are competitive. The importance of imperfect competition in generating coordination failures is a theme to which we return in Section III C.

B. Trading Externalities

Another important model exhibiting strategic complementarity is that of Diamond [1982]. In this economy individuals face production decisions that arrive stochastically and have varying costs. Having made a decision to produce, agents then seek trading partners, who also arrive stochastically. Individuals trade on a one-for-one basis and consume the good so obtained; utility depends negatively upon the cost of production and positively upon consumption. While Diamond's model is set in continuous time, the essential point, for our purposes, can be illustrated in a static model, where agents face a single production opportunity with an uncertain cost and then face a given probability of finding a trading partner. If a trading partner is not found, the produced good perishes.

In terms of the game of Section II, an agent's payoff is an expected utility; if an individual elects to produce, then he or she faces a certain cost, and a return conditional on finding a trading partner. We define the payoff as that expected *prior* to the arrival of the production opportunity. An individual's strategic decision is thus that of whether or not to accept a given production opportunity, or—equivalently—it is the choice, ex ante, of a cutoff cost of production (e_i) below which the individual will choose to produce. This latter interpretation yields a continuous strategy variable.

Strategic complementarity arises in Diamond's model if the probability of finding a trading partner is an increasing function of the number of individuals seeking to trade. Then the expected payoff to agent i from producing increases as more individuals produce (positive spillovers), and more production opportunities are expected to be profitable, so agent i will increase e_i (strategic complementarity).

Let U be the utility obtained from consuming (with zero utility from zero consumption), and let $(\tilde{e}_i - \theta_i)$ be the realized cost of production in utility terms. We consider the game after the realization of θ_i and before the realization of $\tilde{e}_i$. Assume that $\pi(\bar{e})$ is the probability that agent i finds a trading partner if others select $\bar{e}$ as a cutoff production cost, and let $G(\tilde{e})$ be the distribution of production costs. Then it follows that

$$V(e_i,\bar{e}) = \int_0^{e_i} [\pi(\bar{e})U - (\tilde{e} - \theta_i)]g(\tilde{e})\, d\tilde{e}, \tag{6}$$

where $g(\tilde{e})$ is the probability density function associated with $G(\tilde{e})$. Hence $e_i^* = \pi(\bar{e})U + \theta_i$, and $\rho = \pi'(\bar{e})U$.

In our version of Diamond's economy strategic complementarity arises through the $\pi(\bar{e})$ function, which in turn depends upon the distribution $G(\tilde{e})$. Suppose that each agent randomly meets one and only one potential trading partner after production has taken place. Then, $\pi(\bar{e}) \equiv G(\bar{e})$. If we assume further that $\theta_i = 0$ and that costs are distributed on the interval $[0,U]$, then there will be at least two symmetric Nash equilibria, since $e_i^*(0) = G(0)U = 0$ and $e_i^*(U) = G(U)U = U$.[8] The existence of other equilibria will depend upon the probability density function; if it has a single, interior peak, there will be one other SNE. These equilibria will be Pareto-ranked, with welfare increasing in the volume of trade.

It is easy to show that a special case of this model will generate a continuum of equilibria, as in the Bryant model. If production costs are uniformly distributed over $[0,U]$, then $\pi(\bar{e}) = G(\bar{e}) = \bar{e}/U$. Hence $\pi'() = 1/U$, $\rho = 1$, and $e_i^*(\bar{e}) = \bar{e}$.

In this model a positive θ_i shock simply shifts agent i's reaction function up uniformly, so $\partial e_i/\partial \theta_i = 1$. We can carry out a multiplier analysis in the usual way: the multiplier equals $1/(1 - \pi'()U)$. This is intuitive: an increase in $\bar{e}$ increases the probability of finding a trading partner. The greater the change in probability, adjusted for its effect on utility, the more production opportunities will agent i be willing to accept.

One may interpret Diamond's model as that of a participation externality in which the willingness of agents to participate (i.e., produce) in the market depends on the number of active agents. Chatterjee [1987] shows that a participation externality may also arise in uncertain environments in which the entry of agents into the market reduces the uncertainty associated with participation by others. Chatterjee shows that multiple, Pareto-ranked equilibria are possible in this environment.

Alternatively, Diamond's externality can be viewed as a complementarity in production or demand. Noting that this trading technology is outside the agents' control, one could argue that it is simply part of the production process. Conversely, the number of potential traders can be taken as an index of demand for an individual's output, calling to mind a demand externality. This similarity—or ambiguity—is not surprising, for it reflects the common intuition in all of these examples. In the next subsection we clarify this further by examining complementarities in demand.

8. Note that the high-level SNE in this case is also a SCE, since the spillover goes to zero at $e = U$.

C. Demand Externalities

As a final example, we consider complementarities created by demand linkages between agents in a multisector economy. This is perhaps the most compelling of our models, since it captures the intuition that economies may get stuck at low levels of activity when agents are constrained in their sales. It is also perhaps the most "Keynesian" of our examples in that demand constraints play a crucial role. There is a coordination problem in such economies if low-level equilibria could be avoided by a simultaneous increase in the output of all firms. However, in a decentralized system there may be no incentive for a single firm to increase production because this agent takes the actions of others as given. Hence, the "externality" is brought about by demand linkages that individual firms do not internalize.

Coordination problems of this type are impossible in a Walrasian economy, where agents can sell any amount they choose at a given price. A demand externality may arise, though, in market structures where agents require information on both prices and quantities in making choices: this includes economies with imperfect competition or price rigidities. In both cases, quantities matter to individual decision makers, and prices do not completely decentralize allocations.

The key aspect of these models is positive demand linkages across sectors of an economy. These linkages are in turn a consequence of the normality of consumption goods in individual demand functions. This normality assumption, combined with an assumption that agents consume products other than those they produce (i.e. specialization in production relative to consumption), leads to models of imperfect competition with coordination failures and multipliers.

Perhaps because of the Keynesian flavor of these models, it is not surprising that there are numerous examples of them currently in the literature. Recent papers by Cooper [1986], Drazen [1985], Hart [1982], Heller [1986], Kiyotaki [1985], Roberts [1984, 1986], Shleifer [1986], Startz [1986], and Weitzman [1982] all exploit imperfect competition as a means of understanding underemployment equilibria and multipliers.

Because of the variety of these models, we do not attempt to relate them all directly to the game discussed in Section II. Instead, we first outline a model similar to that explored by Hart [1982] and Heller [1986], to highlight the sources of complementarities in a multisector, imperfectly competitive economy. We then investigate a specific example.

Consider an economy composed of many sectors with single, identical firms producing in each. There is also a nonproduced *numeraire* good in the economy that is endowed to another group of agents termed outsiders. These agents do not behave strategically, so that their presence represents a difference between this analysis and the game in Section II.

The firm in sector i produces output q at a cost $C(q)$. This function represents the cost of producing q units of output at current factor prices (which are omitted from the notation) and is assumed to be increasing and convex. The sector i firm (i.e., shareholders) as well as factor suppliers (workers) spend income generated by activities in sector i on the goods produced in the other sectors and on the nonproduced good. This is where the assumption of specialization in production relative to consumption is imposed. If producers and workers consumed only their own output, these demand externalities would be completely internalized, and coordination failures would not emerge. This corresponds to the requirement in the Diamond [1982] model that individuals must trade their output for the otherwise identical output of other agents.

To begin the discussion, we consider the behavior of the monopolist in sector 1 given the behavior of others in the economy. Denote by $P_1(q_1,Y,P)$ the inverse demand curve for sector 1. The price in sector 1 depends on the amount of sector 1 output produced, q_1, the overall level of income in the economy, Y, and some measure of the aggregate price level, P. Y is an aggregate measure of the profits of firms and factor income in other sectors and the outsider's endowment of the nonproduced good.

Taking (Y,P) as given, the firm selects its output level to maximize profits of $R(q_1,Y,P) - C(q_1)$, where $R(q_1,Y,P) \equiv q_1P_1(q_1,Y,P)$. Price is then determined by the auctioneer to clear the market. The result of this optimization is the usual condition of equality between marginal revenue and marginal cost:

$$MR(q_1,Y,P) = P_1()[1 - 1/\epsilon()] = C'(q_1), \tag{7}$$

where $\epsilon()$ is the elasticity of the demand curve. Condition (7) together with market clearing implicitly determines (q_1,P_1) as a function of the aggregate variables (Y,P). There is a similar equation linking the price and quantities in other sectors to these aggregate variables. The simultaneous solution of this equation system then determines a Nash equilibrium for the economy as a whole.

The linkages across the sectors of the economy, which provide the basis for the strategic complementarities and multiple Nash equilibria, can be illustrated through (7). Suppose that other firms in the economy increase their output. The direct result of this is an increase in the overall level of income, Y. This shifts out the demand curve in the remaining sector, inducing an expansion in that sector's output provided that demand does not become too price inelastic as income increases.

There are also "price linkages" across sectors. As output in other sectors is increased, prices have to fall to clear markets. This may induce a reduction in factor prices so that firms in other sectors, finding that their marginal costs have fallen, produce more output. The fall in output prices, however, may also reduce incomes, thus shifting in the demand curves of the remaining sector.

Heller [1986] analyzes the conditions on demand functions necessary to generate multiple Nash equilibria. These are essentially restrictions on the marginal rate of substitution (between the produced good and leisure in Heller's model and between the produced and nonproduced goods in our model ($P_1()$)) and the elasticity of demand. Heller presents a method for constructing examples of multiple equilibria by independently setting the first and second partials of the utility function at a particular point of the commodity space. It is also possible to generate multiple equilibria by allowing richer cost structures—see Kiyotaki [1985].

When strategic complementarities are present across sectors, our earlier analysis on the multiplicity of equilibria and multiplier effects will then apply to this economy. In addition, these economies exhibit positive spillovers to the extent that increases in sector i output benefit consumers of that product. Multiple equilibria may emerge which can be indexed by the level of economic activity and welfare. Movements from an equilibrium with a low output level to one of high output may be desirable for all agents, but no single agent has an incentive to undertake this expansion unilaterally. Because expansions in one sector will induce expansions in others, empirically one would observe correlated movements in output and employment across the sectors of this economy. Furthermore, aggregate movements need not be the consequence of aggregate shocks but may instead be the result of sector-specific shocks coupled with demand spillovers.

There are a number of extensions and alterations to this basic model. One version, following Hart [1982], admits multiple firms in each sector of the economy. Within a sector, firms each select output levels taking as given the output choices of all other firms in their sector and in other sectors. Under the usual assumptions about the curvature of the inverse demand curve, the reaction of a firm to an increase in output of another firm producing an identical product is to reduce output. That is, the nature of the interaction between firms in the same sector is *strategic substitutability.* However, the reaction *across* sectors is still one of complementarity as long as the conditions on demand curves discussed above continue to hold. This is also illustrated in the example below, which allows multiple firms in the same sector.

As an alternative to adopting the Cournot-Nash approach, Blanchard and Kiyotaki [1985], Startz [1986], Weitzman [1982], and others consider multisector models of monopolistic competition. The substantive difference between these approaches is the strategy variable of the firms. In the Cournot-Nash model, firms select output, taking as given the *outputs* chosen by other firms and recognizing that an auctioneer will set prices to clear markets. In the monopolistic competition model, firms each set a price taking as given the *prices* set by other firms. Quantities are determined by consumer demands at these prices. These models of monopolistic competition generate inefficient equilibria [Blanchard and Kiyotaki, 1985] and multiplier effects [Startz, 1986], and may exhibit strategic complementarities in prices [Ball and Romer, 1987].[9]

Example. We now turn to an illustrative example that should clarify the nature of the complementarities in these models. Suppose that there are $F > 1$ firms in each of two sectors of an economy. Firms within a sector produce identical products. There are outsiders endowed with $\overline{m}$ units of a numeraire commodity who spend their endowment equally on the two produced goods.

Firms in sector 1 have direct utility functions given by

$$c_2^{1/2} m^{1/2} - kq_1. \tag{8}$$

So, they consume the good produced in the other sector (c_2) and the nonproduced good (m). The disutility of production, k, is less than one.

9. Roberts [1984, 1986], and Kreps and Scheinkman [1983] consider models in which prices and quantities are both strategic variables for players.

Given this structure, we can solve for a firm's indirect utility level for an arbitrary level of output. Firm f of sector 1 then selects its output level q_1^f, to

$$\text{maximize } (z_1 P_1() - k) q_1^f,$$

where $z_1 = \frac{1}{2}(1/P_2)^{1/2}$ and is independent of this firm's actions. This z_1 is the analogue of the price linkage from our previous discussion. Here it arises because the marginal utility of income to the firm depends on the price of the good produced in the other sector.

In solving this problem, the firm takes as given the output decisions of other firms and the inverse demand curve. As in the Cournot-Nash structure, prices are determined by an auctioneer to clear markets given the quantities selected by the firms. The price in sector 1 is given by $P_1 = E_1/Q_1$, where E_1 equals $\overline{m} + \frac{1}{2}(P_2 O_2)$ and is the expenditure on sector 1. Q_i denotes total sector i output. Because of the Cobb-Douglas preferences, total expenditures on sector 1 are independent of P_1.

There are positive spillovers in this model, since an increase in Q_2, and a consequent reduction in P_2, increases z_1 while leaving P_1 unchanged. The presence of the outsiders does not alter the welfare results reported in Propositions 2–5, since they benefit from these price reductions as well.

The solution to the firm's optimization problem is given by

$$z_1 E_1/Q_1 \, [1 - q_1^f/Q_1] = k. \tag{9}$$

This expression equates the marginal revenues and costs to firm f in sector 1 of producing another unit of output. The symmetric Nash equilibrium in sector 1, given E_1, is thus

$$Q_1 = z_1 E_1 \eta / k \qquad \text{and} \qquad P_1 = k/\eta z_1, \tag{10}$$

where $\eta = 1 - (1/F)$ and is a measure of the competitiveness of this sector.

Invoking symmetry and using the price equation in (10), we can derive the non-autarkic equilibrium price (P^*);

$$P^* = [2k/\eta]^{1/2}. \tag{11}$$

There also exists an autarkic solution in which firms produce zero output and prices are infinite.

Substituting for E_1 and z_1 in (10) and using (11), we can calculate the symmetric Nash equilibrium level of output in sector 1 as a function of the output in sector 2 when firms in sector 1 take as given both Q_2 and $P_2 = P^*$:

$$Q_1 = \overline{M}/P^* + \tfrac{1}{2}Q_2. \tag{12}$$

This can be given the interpretation of a sector "reaction curve" in that it expresses the level of output in sector 1 *at a symmetric Nash equilibrium in that sector* as a function of the output level in sector 2 (given $P_2 = P^*$). This equation thus captures the complementarities across the sectors of a multisector economy with imperfect competition. The intercept is a measure of the autonomous expenditure on sector 1, while its slope reflects the extent to which income earned by sector 2 is returned to the system as expenditures on sector 1. For the Cobb-Douglas preferences, this "marginal propensity to consume" is simply ½.

The simultaneous solution to equation (12) and its symmetric analogue determines the overall equilibrium of the system, as indicated in Figure III. This representation of the system has close parallels with the Keynesian cross diagram. The equilibrium level of (per sector) output and employment is given by $Q^* = 2\overline{m}/P^*$. With $\eta < 1$, P^* exceeds the competitive price so that Q^* is less than the competitive level of output.

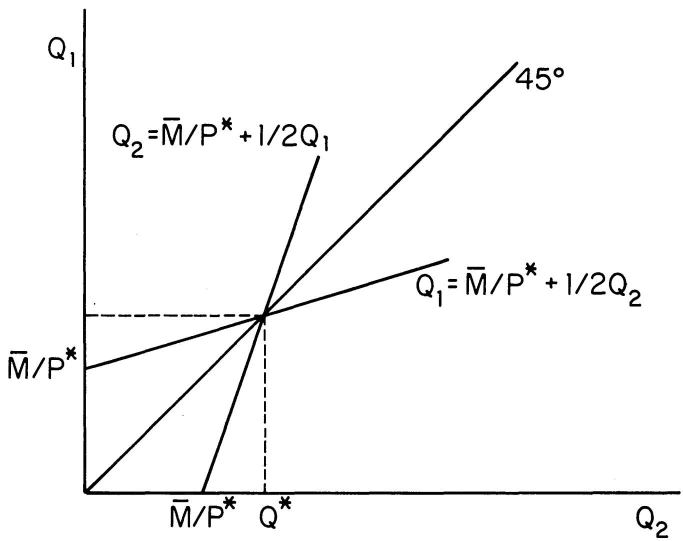

FIGURE III

An alternative representation of this example is derived from a two-stage game in which prices clear markets in the second period. From the market clearing condition in the second stage of the game, $P_2 = 2\overline{m}/Q_2$. Substituting this into (10) yields

$$Q_1 = (2\overline{m}/P^*)^{1/2} \qquad (Q_2)^{1/2} = (Q^* Q_2)^{1/2}. \tag{13}$$

The symmetric solutions to (13) are identical to those from our earlier specification of the game. In fact, (12) is just the linearization of (13) around Q^*.

The multiplier effects can be seen by considering an increase in the numeraire. From (12) or (13), $\partial Q_1/\partial\overline{m} = 1/P^*$, while $dQ^*/d\overline{m} = 2/P^*$. As P^* is independent of $\overline{m}$, the multiplier equals 2. This is illustrated in Figure III. The multiplier effect arises due to the presence of demand spillovers across sectors of this economy. Moreover, it is easy to see that sector-specific shocks that influence the position of one reaction curve are spread to other sectors through these spillovers.

As in our earlier examples, it is possible to generate a continuum of equilibria in this model. To do so, we eliminate the nonproduced good from the model so that firms' preferences are simply $c_2 - kq_1$. With this change, the sectoral reaction curve becomes $Q_1 = (k/\eta)Q_2$. When $k = \eta$, there will be a continuum of Nash equilibria. These equilibria are Pareto ranked, as in the cases of both the Diamond and Bryant models.

Fix-Price Models

Finally, we relate our model to those in the fix-price literature. Consider a simple fix-price specification of our model, where firms now take all prices as given and face a quantity constraint on the amount of output they can sell. Assume a symmetric rationing scheme, so each firm in a given sector can sell $1/F$ of the output demanded in that sector at the fixed price. Following Benassy [1975, 1982], suppose that agents express demands on each market, taking as given their constraints on other markets. For simplicity, assume also that the market for the nonproduced good always clears (or, more precisely, that the constraints on this market are just binding, but not strictly binding).

Specifically, suppose that $P_1 = P_2 = \overline{P} > 4k^2$, so that prices in each sector are identical and exceed the competitive equilibrium price (obtained by setting $\eta = 1$ in (11)). This implies that $z_i\overline{P} > k$ so firms wish to sell as much output as they can at the given prices; hence the quantity constraints are strictly binding on firms. It then follows that, for any $\bar{P}$, there exists a symmetric Nash equilibrium with $Q_1 = Q_2 = 2\bar{m}/\bar{P}$. To see this, suppose that firms in sector 2 are producing this level of output. From the Cobb-Douglas preferences, each firm in sector 2 spends $\bar{P}Q_2/2 = \bar{m}$ on sector 1. Total expenditure on sector 1 is thus $2\bar{m}$, implying a quantity-ration of $2\bar{m}/\bar{P}$ on the firms in that sector. A symmetric argument holds for sector 2, and the market for the nonproduced good clears, which completes the proof.

This result is of interest because it demonstrates that fix-price economies may also exhibit strategic complementarities (as is again seen by noting the multiplier effect associated with a change in $\bar{m}$). They arise in this case because an increase in the output of firms in sector 1 leads to a relaxation of the quantity constraint facing firms in sector 2, and hence to an increase in the output of firms in sector 2. This also perhaps provides strong circumstantial evidence for our intuition that strategic complementarity is a distinguishing element of models with Keynesian features.

IV. Conclusion

Our principal finding in this paper concerns the importance of strategic complementarity in agents' payoff functions as a condition for model economies to display Keynesian features. We have shown that strategic complementarities and spillovers can together generate both coordination failures (multiple, Pareto-ranked equilibria) and a multiplier process associated with changes in exogenous variables. The inefficiencies are driven by the presence of externalities in payoff functions, while the multiplicity of equilibria and the multiplier derive explicitly from positive interactions at the level of strategic choices; i.e., positively sloped reaction curves.

We placed our more general analysis into an economic context by drawing on a number of models in the literature displaying Keynesian features. Our analysis highlights the fact that these spillovers and strategic complementarities can arise at the levels of preferences and technology (as in the Bryant example) or in the manner in which agents organize their transactions (as in the Diamond model and the models of imperfect competition).

One can view this approach as arguing for the importance of macroeconomic quantities in microeconomic choice functions. In many of our examples, an individual's optimal strategy depended on an aggregate measure of the actions selected by others in the economy. This is the intuition behind the congestion problems found in search models and is extended in our examples to other settings.

While our analysis captures some important elements of Keynesian models, it does not address all Keynesian issues. For example, we have not directly considered unemployment (as opposed to underemployment). In addition, our models are real and hence shed no light on money nonneutralities. Recent related work on this topic includes Blanchard and Kiyotaki [1985], who obtain nonneutralities by introducing menu costs into a model of monopolistic competition (drawing on Akerlof-Yellen [1985] and Mankiw [1985]).

We plan to extend our analysis in a number of directions. First, it would be useful to consider dynamic, stochastic versions of these examples to shed more light on intertemporal macroeconomic coordination problems.[10] This would allow us, for example, to focus more explicitly on the role of expectations in coordination failures. Given these failures, our second goal would be to understand the role of the government in coordinating economic activity, particularly in an intertemporal context.

UNIVERSITY OF IOWA
MICHIGAN STATE UNIVERSITY

REFERENCES

Akerlof, G., and J. Yellen, "A Near Rational Model of the Business Cycle with Wage and Price Inertia," this *Journal,* C (1985), 823–38.

Ball, L., and D. Romer, "Sticky Prices as Coordination Failure," New York University, mimeo, 1987.

Benassy, J. P., "Neo-Keynesian Disequilibrium Theory in a Monetary Economy," *Review of Economic Studies,* XLII (1975), 503–23.

——, *The Economics of Market Disequilibrium* (New York: Academic Press, 1982).

Blanchard, O., and N. Kiyotaki, "Monopolistic Competition, Aggregate Demand Externalities and the Real Effects of Nominal Money," NBER Working Paper #1770, 1985.

Bryant, J., "A Simple Rational Expectations Keynes-Type Model," this *Journal,* XCVIII (1983), 525–29.

Bulow, J., J. Geanakoplos, and P. Klemperer, "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy,* XCIII (1985), 488–511.

Chamley, C., "A General Equilibrium Expression of the Paradox of Thrift," Cowles Foundation Discussion Paper No. 700, 1984.

Chatterjee, S., "Coordination Failure in a Competitive Model," University of Iowa, mimeo, 1987.

Cooper, R., "A Comment on Bryant's 'A Simple Rational Expectations Keynes-Type Model,' " Cowles Foundation, Yale University, mimeo, 1983.

——, "Sharing Some Thoughts on Weitzman's *The Share Economy,*" NBER Working Paper #1734, 1986.

10. See Chamley [1984] and John [1987] for work in that direction.

Diamond, P., "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy,* XC (1982), 881–94.
Drazen, A., "A Rational Expectations Keynesian Model with Price Flexibility," Working Paper #2-85, Foerder Institute for Economic Research, Tel-Aviv University, 1985.
Haltiwanger, J., and M. Waldman, "Rational Expectations and the Limits of Rationality: An Analysis of Heterogeneity," *American Economic Review,* LXXV (1985), 326–40.
Hart, O., "A Model of Imperfect Competition with Keynesian Features," this *Journal,* XCVII (1982), 109–38.
Heller, W., "Coordination Failure under Complete Markets with Applications to Effective Demand," in *Equilibrium Analysis: Essays in Honor of Kenneth J. Arrow,* Volume II, Walter P. Heller, Ross M. Starr, and David A. Starrett, eds. (Cambridge: Cambridge University Press, 1986).
Holmstrom, B., "Moral Hazard in Teams," *Bell Journal of Economics,* XIII (1982), 324–40.
John, A., "A Model of Intertemporal Coordination Failure," Yale University, mimeo, 1987.
Jovanovic, B., "Micro Shocks and Aggregate Risk," this *Journal,* CII (1987), 395–409.
Kiyotaki, N., "Implications of Multiple Expectational Equilibria Under Monopolistic Competition," Harvard University, mimeo, 1985.
Kreps, D., and J. Scheinkman, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics,* XIV (1983), 326–37.
Mankiw, N. G., "Small Menu Costs and Large Business Cycles," this *Journal,* C (1985), 529–37.
Roberts, J., "General Equilibrium Analysis of Imperfect Competition: An Illustrative Example," Stanford University, mimeo, 1984.
——, "Involuntary Unemployment and Imperfect Competition, A Game Theoretic Macro Model," Stanford University, mimeo, 1986.
Scitovsky, T., "Two Concepts of External Economies," *Journal of Political Economy,* LXII (1954), 70–82.
Shleifer, A., "Implementation Cycles," *Journal of Political Economy,* XCIV (1986), 1163–90.
Startz, R., "Monopolistic Competition as a Foundation for Keynesian Macroeconomic Models," University of Washington, mimeo, 1986.
Weitzman, M., "Increasing Returns and the Foundations of Unemployment Theory," *Economic Journal,* XCII (1982), 787–804.