DELTA

DÉPARTEMENT ET LABORATOIRE D'ÉCONOMIE THÉORIQUE ET APPLIQUÉE

48, BD JOURDAN - E.N.S. - 75014 PARIS

TÉL. : 33 (0) 1 43 13 63 00 - FAX : 33 (0) 1 43 13 63 10

www.delta.ens.fr

WORKING PAPER N° 2004-23

Eductively Stable Transmission of Information through Prices : A Brief Review of Results

Gabriel Desgranges

Roger Guesnerie

Mots clés :

Codes JEL :

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE - ÉCOLE DES HAUTES ÉTUDES EN SCIENCES SOCIALES - ÉCOLE NORMALE SUPÉRIEURE

Eductively Stable Transmission of Information through Prices : A Brief Review of Results.

Gabriel Desgranges*and Roger Guesnerie.

September 21, 2004

Abstract

The paper provides a brief review of existing results on the transmission of information through prices, when the revealing equilibria have to fit a criterion of "eductive stability". The work under review often suggests that, at odds with the "efficient market hypothesis", the plausibililty of equilibria, according to the criterion, decreases when equilibria transmit too much information

To Be Published *in "Assessing Rational Expectations :"eductive" stability in economics", MIT Press, 2005*

Résumé : Ce texte comporte une revue de la littérature sur la transmission d'information par les prix, quand cette transmission est contrainte à satisfaire les conditions de "stabilité divinatoire". Les travaux passés en revue suggèrent, en fort contraste à l'hypothèse d"'efficience des marchés", que la plausibilité des équilibres, pour le critère considéré, décroit quand ils transmettent beaucoup d'information

*THEMA, University of Cergy-Pontoise, France. gabriel.desgranges@eco.u-cergy.fr

1 Introduction

The transmission of information through prices has generated one highly visible literature ((Grossman (1976), Radner (1979)), at the end of the seventies. Some commentators argued, at least initially, that theorems on the existence of "fully revealing equilibria" were confirming the celebrated views of Hayek on the efficiency of markets as information processors. The enthusiasm for the results however declined, in part because refinements of the basic message (when the dimension of the signals was different from the dimension of prices) were less spectacular. But more basically, "full revelation" made the market so powerful in transmitting information that some paradoxes arose (Grossman-Stiglitz). Noise traders were introduced that led to focus on "partially" rather than "fully" revealing equilibria. And, subsequently, the focus of the literature switched from purely competitive situations, in which agents are both price takers and "informationally" small, to situations with monopolistic features (the informed monopolist and the single market maker of Kyle (1985)). However, the question of the plausibility of the equilibrium concept used, or if one prefers, of the conditions of implementation of the equilibrium outcomes in a market context, the acuteness of which was particularly visible in the definition of "fully revealing equilibria", more or less disappeared from the agenda of researchers. At least, the effectiveness of market procedures for the implementation of the equilibrium,[1] of which some of the earlier literature had been clearly aware, was only addressed only indirectly by studies that put emphasis on learning processes converging, (in real time but generally very slowly), to partially revealing equilibria (see in particular Vives (1993)).

The purpose of the present paper is to review studies that revisit, from an "eductive" rather than "evolutive" viewpoint, what we have just called the implementation problems raised by the concept of revealing price equilibria. In a sense, these studies try to see whether the revealing equilibria can be guessed through mental processes that rely upon the assumption that the model is Common Knowledge (see Guesnerie (2002) for a synthetical assessment of this method). Viewed form a different angle, they attempt to test the expectational robustness of equilibria.

The present paper will focus attention on "eductive" stability in finance-like models in which there are noisy traders. It will then successively review the models and results obtained by Desgranges (1999), Heinemann (2001), Gordon (1999) and compare them to the results obtained in Desgranges, Geoffard and Guesnerie (2003).

We proceed as follows :

- We first present the set of models under scrutiny,
- We describe the equilibrium concepts and recall existence results,
- We finally review and compare the stability results obtained.

[1] The more general question of implementation of the equilibrium by means of an arbitrary mechanism has received much attention (no matter whether this mechanism is a sensible description of a market or not). Early references are Laffont (1985) and Postlewaite and Schmeidler (1986).We focus attention here, as stated in Desgranges-Geoffard-Guesnerie (2003), on the related but different question of "market-like implementation".

2 The models

The different results have been developed in various static partial equilibrium models. Although they can receive several interpretations, all these models can be seen through "financial glasses" as the market for a risky asset. We briefly present these models, most of which are well-known.

The CARA/Gaussian models. The framework of Constant Absolute Risk Aversion (CARA) Gaussian models has been adopted, with variants, in the seminal contributions of Grossman (1976), Grossman and Stiglitz (1980) and Kyle (1989). All these models consider a market in which agents trade a risky asset whose future value is unknown. Agents set their demand by maximizing the expected Constant Absolute Risk Aversion (CARA) utility of their final wealth. All stochastic variables are Gaussian (hence the name CARA/Gaussian). We present here the variant introduced in Desgranges (1999). There is a set $[0,1]$ of infinitesimal agents i whose demand for a risky asset is denoted x_i. The liquidation value of the risky asset is denoted θ, its current price is p, and there is a safe asset with price and return normalized to 1. Every agent maximizes an expected CARA utility of his or her final wealth $(\theta - p)\, x_i$, with absolute risk aversion a. Agents have no initial endowment of any asset (this is without loss of generality in this CARA setting).

There is a stochastic supply ε of the risky asset that is observed by no agent, ε is normally distributed with mean 0 and variance σ^2. The market clearing equation for the risky asset is then written :

$$\int_I x_i di = \varepsilon.$$

The information structure is "differential": every agent receives his own private signal $s_i = \theta + \beta_c + \beta_i$. Each agent being differentially informed, the total available information consists here of the whole collection of signals s_i, $i \in I$. Still, all the stochastic variables θ, β_c, β_i ($i \in I$) are independent and normally distributed. All their means are taken equal to 0 (for the sake of simplicity) and their variances are t^2, and $b^2 - b_c^2$ respectively. It follows that the average $\bar{s}$ of the s_i is a sufficient statistic for the signals s_i. A law of large numbers (namely, $\int_I \beta_i di = 0$ with probability 1) implies that $\bar{s} = \theta + \beta_c$. The total available information is then contained in the signal $\bar{s}$.

The timing of the game is as follows : every agent observes his or her private signal and then submits a demand schedule $x_i\,(s_i, p)$. The price is then set as a market clearing price. To solve the possible multiplicity of market-clearing prices, a selection rule is defined (for example, choose the smallest market-clearing price). This selection rule is, however, not necessary as long as the demand is linear in the price.

As expected by any reader familiar with this literature, this model admits a unique linear Rational Expectations Equilibria (REE) (that is a REE in which the price is a linear function of $\bar{s}$ and ε). The next section will describe this statement more explicitly.

Two other variants. Desgranges (1999 a,b) also considers a variant of this model with a "hierarchical" information structure: a proportion α of agents privately observes the same noisy signal $\bar{s} = \theta + \beta_c$, and the remaining proportion receives no private signal. The total available information is then reduced to the signal $\bar{s}$ (and the notations are consistent with the "differential" information structure above).

Heinemann (2001) considers an (approximately) identical model with a differential information structure. The differences between it and the above model are twofold: there are finitely many agents $i = 1, ..., I$, and the private signal observed by every agent i is simply $s_i = \theta + \beta_i$ (*i.e.*, there is no noise common to all the signals, *i.e.*, $\beta_c = 0$ almost surely in the above expression $s_i = \theta + \beta_c + \beta_i$).

Unsurprisingly, these two model admit a unique linear REE.

The Guesnerie and Rochet (1993) modified model. This the reduced form model considered in Desgranges, Geoffard and Guesnerie (2003). The (few) differences between it and the above Desgranges (1999) model are as follows :

- The final value θ can take two possible values B and G only ($B < G$) with probabilities π and $1 - \pi$ respectively.

- The information structure is hierarchical again, but the proportion α of informed agents exactly observes θ (the remaining proportion receives no private signal).

- Every agent's utility is quadratic, namely every agent maximizes:

$$[E(\theta|\text{Information}) - p]\, x_i - a x_i^2,$$

where the information set of an informed agent is θ and the information set of an uninformed agent is the price.

- The stochastic supply is not necessarily normally distributed. It admits a C^2 symmetric density function μ.

The version of this model found in Desgranges, Geoffard and Guesnerie (2003) and reproduced in this book, has more general utility functions.

Compared to the CARA/Gaussian case, the problem of existence and uniqueness of equilibrium has, in this model, a distinct flavor.

The Kyle (1985) model. In the notorious Kyle (985) model, an uninformed market maker faces an order flow aggregating the order of an informed monopolist and the activity of so-called noise traders. The monopolist "hides" his information about the future value of the traded asset behind the noise traders' orders. Precisely, the monopolist i first observes the future value θ of the risky asset. Then, at every period $n = 1, ..., N$, he sends an orderto a "competitive" market maker to buy x_i^n shares of the asset. The market maker observes the aggregate order flow only that is $Q^n = x_i^n + \varepsilon^n$, where ε^n is a stochastic term accounting for the trading activity of so-called "noise" traders at period n. Every ε^n is a Gaussian centered variable with variance σ^2/N, independent from any other variable. The market maker then sets the price equal to the expected future value, namely:

$$p^n = E\left(\theta | Q^1, ..., Q^n\right).$$

At period n, the market maker extracts from the current and past order flows Q^1, ..., Q^n some information about θ as the order flows are noisy signals of the orders x_i^1, ..., x_i^n, which are noisy signals of θ. On the other hand, the decision made at n by the monopolist is a buy order $x_i^n\left(\theta, p^1, ..., p^{n-1}\right)$. A strategy of the monopolist then consists in a collection of the orders sent at every period: $x_i^1(\theta), x_i^2\left(\theta, p^1\right) ..., x_i^N\left(\theta, p^1, ..., p^{N-1}\right)$.

A particular case of this model is the static case where $N = 1$.

3 Rational Expectations Equilibria :Existence

Under incomplete and asymmetric information, the "natural" equilibrium concept is a Rational Expectations/Bayesian-Nash equilibrium in which the price conveys partially or totally the information possessed by the agents. Although the first definitions of the Rational Expectations Equilibrium (Grossman, Radner) lack strategic foundations, the equilibria that we consider here do coincide with the Bayesian-Nash equilibria of the market games described in the above section (with the exception of Heinemann (2001)).

Definition. In any of the above models (except the Kyle (1985) model), an equilibrium consists of:

- Equilibrium strategies: For every agent i, it consists of a demand function, that is $x_i(s_i, p)$.

- A market clearing rule: The equilibrium price clears the market and, if needed a selection rule is used, namely, for every (s, ε), p is the smallest price satisfying:[2]

$$\int_I x_i(s_i, p) = \varepsilon,$$

- Bayesian beliefs: Individual expected utility maximization takes into account the information revealed by the price, and the agent's beliefs are based on the *knowledge* of the joint *equilibrium* distribution θ, s_i, p, (the distribution of θ being then taken to be conditional on (s_i, p)).

Some remarks on this equilibrium concept:

- At an equilibrium, the price partially (because of ε) reveals the information possessed by agents.

- Every agent, assumed to know the joint distribution (θ, p) determines his demand by using the full informational content of the price.

- However, the coordination issue is clear: the price is an *endogenous* public signal, and the joint distribution (θ, p) depends on the strategies chosen by agents. Assuming that an agent correctly extracts information from the price amounts to assuming that an agent correctly predicts the aggregate behavior of the economy as a function of the information received at the individual level. This standard issue, associated with the Rational Expectations Hypothesis, however, takes here a special form.

Lastly, a REE is said to be linear when equilibrium individual demands are linear in the two conditioning variables s_i and p. In the CARA/Gaussian case, the price is then a linear function of the s_i and ε at a linear REE.

Existence results. As previously noted, the question of REE existence and uniqueness in the CARA/Gaussian framework has been extensively studied. Existence of a linear REE is a well-known result, and uniqueness of the linear REE obtains as well under mild assumptions.[3] In particular, both results obtain in the different variants of the CARA/Gaussian model considered here. However, uniqueness outside the linear case is not guaranteed.

As far as existence is concerned, Desgranges, Geoffard and Guesnerie (2003) encounters a problem, first taken into consideration by Green (1977) : when the

[2] Write $\sum_{i=1}^{I} x_i(s_i, p) = \varepsilon$ when there are a finite numberof agents.

[3] For uniqueness, see Nielsen (1996) and deMarzo and Skiadas (1998) among others.

distribution of noise does not have a monotone likelihood property, equilibrium may not exist. However, they have shown that this difficulty disappears when μ, the density function of the noise is log-concave (*i.e.*, $\log(\mu)$ is concave), a fact that holds true with the normal distribution. Perhaps more surprisingly, they have shown that under this condition, there exists a unique partially revealing equilibrium, with the property that aggregate excess demand is strictly decreasing in prices.

The Kyle (1985) model. The Kyle (1985) model is quite different from the other ones, and so is the definition of the REE: A linear REE is defined in the Kyle model, as a couple of strategies satisfying the following : (i) every $x_i^n\left(\theta, p^1, ..., p^{n-1}\right)$ is linear in θ and prices and the strategy of the informed monopolist maximizes the expected value of the portfolio $\sum_{n=1}^N \left(\theta - p^n\right) x_i^n$, given the strategy of the market maker, and (ii) every p^n is linear in $Q^1, ..., Q^n$ and satisfies $p^n = E\left(\theta | Q^1, ..., Q^n\right)$, given that $Q^n = x_i^n\left(\theta, p^1, ..., p^{n-1}\right) + \varepsilon^n$. Still, the REE in this model raises the same coordination issue as the one presented previously : a correct interpretation of the informational content of the order flow requires a correct understanding of the strategy pursued by the informed monopolist, and a correct guessing of the price requires a correct understanding of the pricing rule chosen by the market maker.

The result of Kyle (1985) is that there is a unique linear REE in this model.

4 Eductive stability: Definition and Results

The "eductive" construction of the revealing equilibrium, described in Desgranges, Geoffard and Guesnerie (2003) can be transposed to all the models under consideration. It always consists of a mental learning process, based on guessing and second guessing and that takes place in virtual time, providing to the agent an additional rationale for playing his equilibrium strategy: the equilibrium is then said to be "Dominant-Solvable", "Strongly Rational" or "Eductively" Stable. At some basic level, the just-evoked mental process relies on Common Knowledge (from now on CK) assumptions : we assume CK of individual rationality and CK of the model. The assumption of rationality means that every agent submits a demand function that is optimal at every price, given beliefs. CK of the model means CK of the structure of preferences, market clearing rule and every detail presented previously. Subsequent analysis will refer to these CK assumptions while being rather informal in terms of the pure game-theoretical standards.

4.1 The CARA/Gaussian case

Desgranges (1999). The "eductive" process in Desgranges (1999a,b) is not truly global : it relies on an initial assumption about agents' behavior. We define such an initial CK restriction as follows:

Definition 1 *Let $x^*(s_i, p)$ be the individual demand at the linear equilibrium (x^* is independent of the identity i of the agent). An initial CK restriction is defined by three positive real parameters $\left(\bar{\eta}, \eta_s, \eta_p\right) \in I\!R_{++}^3$ and is such that* *every individual demand functions* $x_i : I\!R^2 \rightarrow I\!R$ *is linear and satisfies, for every* s_i *and* p*:*

$$|x_i(s_i, p) - x^*(s_i, p)| \leq \bar{\eta} + \eta_s |s_i| + \eta_p |p| \,. \tag{1}$$

In other words, it is hypothetically *a priori* CK, that demand is linear and that its parameters $(\bar{\eta}, \eta_s, \eta_p)$ meet the restriction associated with Condition (1) (hence the term "initial CK restriction").

We can now proceed to a brief description of the eductive process:
- Assume first an initial CK restriction,
- Then, it is CK that every agent pursues a strategy that is a best response to some strategy profile of others lying in the initial CK restriction. This defines a new set of admissible strategies for every agent.
- Then, it is CK that every agent pursues a strategy that is a best response to some strategy profile of others lying in this new set of admissible strategies. This defines a third set of admissible strategies for every agent.
- Every further step is analogously defined... Stability obtains when this process converges to the REE strategies.

It should be clear that the eductive process characterizes the "rationalizable" solutions of a market game. This statement has a game-theoretical precise game-theoretical counterpart in some of the models described here. In others, it refers more loosely to the game-theoretical theories of iterated elimination.

The following proposition then gives a striking necessary and sufficient condition for "Eductive stability" :

Proposition 1 *Consider the above initial CK restriction. The linear equilibrium is "eductively stable" for this initial CK restriction if and only if:*

$$Var(\theta|p) > Var(\theta|s_i) \,, \tag{2}$$

where $Var(\theta|s_i)$ *is the variance of return* θ *conditional to the private signal* s_i*, and* $Var(\theta|p)$ *is the variance of return* θ *conditional to the price* p *and computed with the equilibrium price distribution.*

As, at equilibrium, the random variables (θ, s_i, p) are jointly normally distributed, the conditional variances $Var(\theta|p)$ and $Var(\theta|s_i)$ are independent of the values of p and s_i and they provide then an index for the precision of the information revealed by a signal (either p or s_i). Hence, the proposition can be restated in a more evocative way: *the equilibrium is stable if and only if price reveals less information than any private signal.* In other words, there is a dilemma in regard to the informational efficiency of the market *at equilibrium* and the relevance of the equilibrium concept (seen from the stability viewpoint): if the equilibrium prices are "too much" efficient, the equilibrium becomes less plausible.

The fact that informational efficiency can drive instability may be surprising at first sight. Still, the intuitive explanation of this result is quite clear: *informational efficiency creates incentives for every agent to learn "aggressively" from the price and to make his demand very sensitive to the information that (the agent thinks) is contained in the price, and then to discard partly his own* *information; but if everybody does that, the actual correlation between price and information is very sensitive to agents' beliefs, and therefore it is hard to guess.*

A somewhat equivalent explanation of the Proposition is as follows. Above a threshold of informativeness of the private signal, it is not very important for an agent i to make an accurate prediction of the price/information correlation, as the price cannot add much information to the private signal. Hence, the coordination issue is not really a big one: a mistake of i in forecasting others' forecasts has a small impact on i' s forecast of the price/information correlation. Below the threshold, the same argument leads to the opposite conclusion: it is important for i not to make an (even small) forecast error because (i) the price adds much information to the private signal, and (ii) even a small mistake of i in forecasting others' forecasts can have a significant impact on i' s forecast of the price/information correlation (as i knows that others' demand is very sensitive to their own forecasts). This uncertainty regarding others' beliefs is a cumulative phenomenon, which creates instability of the equilibrium.

The preceding result is now restated in terms of the exogenous parameters of the economy. The condition $Var\left(\theta|s_i\right) < Var\left(\theta|p\right)$ can be rewritten as follows :

Corollary 2 *If $a^2\sigma^2 b_c^2 > 1/8$, then the linear REE is stable for any initial CK restriction. Otherwise, consider an initial CK restriction. Then, there exist two real values $\underline{B}^2\left(a,\sigma^2,b_c^2\right)$ and $\bar{B}^2\left(a,\sigma^2,b_c^2\right)$ such that the linear equilibrium is stable for this restriction if and only if:*

$$b^2 \notin \left[\underline{B}^2\left(a,\sigma^2,b_c^2\right), \bar{B}^2\left(a,\sigma^2,b_c^2\right)\right]. \tag{3}$$

$\underline{B}^2$ (resp. $\bar{B}^2$) is continuously increasing (resp. decreasing) with respect to a, σ^2 and b_c^2. In particular, $\underline{B}^2\left(a,\sigma^2,0\right) = 0$ and $\bar{B}^2\left(a,\sigma^2,0\right) = 1/a^2\sigma^2$. Furthermore, $\underline{B}^2$ and $\bar{B}^2$ do not depend on the restriction.

The conditions stated in this corollary are rather intricate. Still, they illuminate the role played by the exogenous parameters:

- An increase in the risk aversion a or the variance σ^2 of the noisy supply is good for stability, as one could have expected. Both factors contribute to inertia of agents' behavior and favor prediction of aggregate demand. Also, interpreting this result with the stability condition $Var(\theta|p) > Var(\theta|s_i)$ is easy: $Var(\theta|p)$ increases whereas $Var(\theta|s_i)$ is not affected. Analogously, a decrease in the precision of the pooled information $\bar{s}$ (an increase in b_c^2), while private signal's precision remains unchanged, enhances stability. Namely, the equilibrium price becomes less informative and the demand is then less sensitive to agents' beliefs regarding the price/information correlation, implying that the information extracted from the price is more reliable.

- More surprising are the effects of the variations of the precisions of the private signal. Increasing the precision of private information (decreasing b^2) while the precision of pooled information is kept constant has two effects: *a first direct effect of increasing the quality of private information favors stability* ($Var(\theta|s_i)$ decreases) *and a second indirect effect of increasing the amount of information revealed by prices tends to make the equilibrium unstable* ($Var(\theta|p)$ decreases). The stabilizing effect is the usual one (high-quality information makes the equilibrium stable) whereas the destabilizing effect is more surprising (too much private information is destabilizing). The resulting total effect is *a priori* ambiguous and the corollary shows that either one effect or the other can be dominant. If pooled information is bad (large b_c^2), the equilibrium is stable whatever the quality of private signal. If pooled information is good but not complete (small $b_c^2 > 0$), the equilibrium is stable either when private information is bad ($b^2 > \bar{B}^2$) or when private information is good ($b^2 < \underline{B}^2$). Furthermore, as $b^2 - b_c^2$ measures the informational asymmetries among agents, the corollary states that, whenever agents are very reactive (small aversion a and much to learn from the price: small σ^2 and b_c^2), both very small asymmetries and large ones favor stability. Then, *increasing the precision of private information (decreasing b^2) can well stabilize or destabilize the equilibrium, depending of the initial precision $1/Var\,(\theta|s_i)$.*

Remark. It follows that publicly revealing some information that was known to some agents only (decreasing b^2 while keeping b_c^2 constant) does not always favor stability. Still, the public revelation of a new piece of information is always good for stability.[4] That is, a further examination of Condition (3) shows that a decrease in b^2 and b_c^2, while $b^2 - b_c^2$ remains constant, always favor stability. In terms of the stability condition $Var(\theta|p) > Var(\theta|s_i)$, this means that the stabilizing effect of decreasing $Var(\theta|s_i)$ always dominates the destabilizing effect of decreasing $Var(\theta|p)$. This fact obtains because the price is not able to incorporate all the new information (because of the existence of a noisy supply), which makes the decrease in $Var(\theta|p)$ smaller than that in $Var(\theta|s_i)$. Thus, concerning the role played by public information, two cases should be distinguished: reducing existing asymmetries of information can sometimes destabilize the equilibrium, whereas bringing new information to the (common) knowledge of everyone stabilizes the equilibrium.

A hierarchical information structure. In the variant with two groups of agents (a proportion α of perfectly informed agents and the remaining proportion of uninformed agents), Desgranges (1999a,b) shows that the linear REE is stable if and only if:

$$\alpha \left[\frac{1}{var\,(\theta|\bar{s})} - \frac{1}{var\,(\theta)} \right] > (1-\alpha) \left[\frac{1}{var\,(\theta|p)} - \frac{1}{var\,(\theta)} \right].$$

This condition can be interpreted as the preceding one: stability requires that the REE price is not too revealing in comparison with the private information available to agents. Here, the precision of a signal (either $\left[\frac{1}{var(\theta|\bar{s})} - \frac{1}{var(\theta)} \right]$ for the private signal, or $\left[\frac{1}{var(\theta|p)} - \frac{1}{var(\theta)} \right]$ for the public signal, *i.e.*, the price) has to be combined with the proportion of agents holding this signal.

Rewriting this condition in terms of the exogenous parameters gives:

$$\alpha - 2\alpha^2 < a^2 b^2 \sigma^2.$$

The role played by a and σ^2 remains the same. The influence of the information, represented here by the proportion of informed agents, is not monotonic again. Lastly, notice that increasing b^2 is always good for stability.

[4] Formally, consider that there is a publicly observed signal of the form $s_{pub} = \theta + \beta_{pub}$, where β_{pub} is a Gaussian white noise that is not correlated with any of the other variables.

Heinemann (2001). Heinemann (2001) considers a variant of the above eductive process: At every step of the process, an agent considers the set of demand schedules that any agent (including himself or herself) may play. He or she computes the resulting possible price/private signals distributions. Then, for every demand schedule that he may play himself, he checks whether this demand schedule is optimal with respect to. one of the possible price/signals distributions, *considering that price/signals distribution as given.* If it is not, he eliminates this demand schedule. This ends a step of the process.

This process is slightly different from the one in the previous subsection. It relies on the idea (usually associated with the REE)[5] that agents are "schizophrenic". Indeed, an agent first takes account of the influence of the demand on the price when computing the possible price/signals distributions. Then, he forgets this influence (considers the price/signals distribution as given) when checking the optimality of his demand schedule. Still, in the limiting case in which all the agents are infinitesimal, this difference is irrelevant.

Heinemann (2001) obtains the following result:

Proposition 3 *The Rational Expectations Equilibrium is stable if and only if, at equilibrium,*

$$\frac{\partial x_i}{\partial s_i}(s_i, p) \leq \sqrt{\frac{\sigma^2}{(I-1)(I-2)\,Var(\beta_i)}},$$

where I is the (finite) number of agents, $x_i(s_i, p)$ is the (linear) demand schedule, and $Var(\beta_i)$ is the variance of the noise in the private signal ($s_i = \theta + \beta_i$). Furthermore, a necessary condition for stability is that $Var(\theta|p) > Var(\theta|s_i)$.

Again, this result suggests two intuitions for stability: low sensitivity of demand to private information, and low precision of the information conveyed by prices relatively to the precision of the private signal. Both intuitions also arose from the preceding results.

4.2 Desgranges, Geoffard and Guesnerie (2003) :

In this model, the eductive process is based on a point by point inspection of excess demand, the principle of which is rather simple. It starts from the following considerations :

Assume that an uninformed agent happens to know for some reason that total excess demand at some price p is z^e if B; then he knows that the probability of the good state G, if he observes this p has some value $\Pi(z^e, \alpha\Delta)$ that depends on z^e and on $\alpha\Delta$, the difference between excess demand in the good state and excess demand in the bad state. This probability Π is simply (according to Bayes'law) $\mu(z^e + \alpha\Delta)/\{\mu(z^e + \alpha\Delta) + \mu(z^e)\}$, where μ is the noise density function.

Indeed, the likelihood function Π plays a key role in the stability analysis: Desgranges, Geoffard and Guesnerie (2003) show that if the slope of the likelihood function Π'_z is such that:

$$(1-\alpha)\Delta\,|\Pi'_z| < 1.$$

[5] At an REE, agents are price-takers while they use the correlation between private signals and price, thereby acknowledging that their signals, and then their demands, influcence the price. These two aspects of agents' behavior are not consistent (when agents are not infinitesimal), and Hellwig (1980) therefore calls the agents "schizophrenic".

then the equilibrium is "eductively stable".

A detailed comment of this condition can be found in Desgranges, Geoffard and Guesnerie (2003). The bottom line is the following:

- $|\Pi'_z|$ represents the *sensitivity* of Bayesian beliefs of uninformed agents to expected aggregate demand,

- $(1-\alpha)\Delta$ measures the size of the response of non informed agents to the beliefs concerning the occurrence of the good event G: this factor describes the *amplification effect* of the beliefs on θ by the aggregate demand of uninformed agents.

- The condition can then be then restated in a more intuitive and evocative way: *Stability obtains if the product of the sensitivity effect and of the amplification effect is smaller than one.*

>From that analysis, the authors show that if one returns to the basic parameters of the model and supposing that ε follows a normal distribution with mean 0 and variance σ^2, the unique partially revealing equilibrium is "eductively stable" when:

$$\alpha(1-\alpha)\Delta^2 \leq 4\sigma^2.$$

This Proposition allows us to enter into some interesting comparative statics. An increase in σ, that is an increase in the exogenous noise that decreases the informational content of prices, favors stability. An increase in Δ, which, in some sense, measures the influence on individual demand of the information held by the informed agents, is detrimental to stability. The intuition somewhat relates to the one obtained in the CARA/Gaussian case: not too much useful information should be transmitted.

Lastly, as in the CARA/Gaussian case, the proportion of informed agents α has an ambiguous effect: many informed agents as well as few favor stability, when an intermediate number is less favorable. This ambiguous role played by α results from the existence of two opposite effects of α, that exactly correspond to the two effects discussed previously. On the one hand, a large α has a positive effect on stability, because it reduces the amplification effect (bounded by $(1-\alpha)\Delta$), which decreases the influence on aggregate demand of the beliefs of non informed agents. On the other hand, a small α has a positive effect on stability as well, because it reduces the sensitivity effect (bounded by $\alpha\Delta/4\sigma^2$ in the case under consideration), which makes the beliefs of non-informed agents not too much sensitive to their expectations regarding aggregate demand.

It is worthwhile restating the second result of Chapter E3 :

Proposition 4 *1. The equilibrium is stable if and only if the aggregate excess demand $Z(B,p)$ (or $Z(G,p)$) satisfies for every price p:*

$$\frac{dZ}{dp}(B,p) \leq -\frac{1}{2},$$

and strict inequality holds except for a set of zero Lebesgue measure.

2. If there are few informed agents ($\alpha < 1/2$), a necessary condition for eductive stability of an equilibrium is that the demand of non informed agents be a decreasing function of the price.

This proposition relates with the equilibrium features of excess demand and is less directly comparable with the CARA/Gaussian results. However, a smaller slope of excess demand signals an aggressive extraction of information by non-informed agents (a fact that is particularly obvious when they have an upward-sloping excess demand). This fact is detrimental to stability, and this destabilizing effect increases in the number of uninformed agents. This is also in line with previous intuitions.

4.3 The Kyle (1985) model

Gordon (1999) studies the eductive stability of the linear REE in the Kyle (1985) model. The eductive process is here defined along the same lines of the rationalizability theory as above:

- Assume first an initial CK-restriction (stating that the strategies actually pursued lie in a neighborhood of the REE strategies),

- Then, it is CK that an agent pursues a strategy that is a best response to some strategy of the other agent lying in the initial CK restriction. This defines a new set of admissible strategies.

- Then, it is CK that an agent plays a strategy that is a best response to some strategy of the other agent lying in this new set of admissible strategies. This defines a third set of admissible strategies.

- Every further step is analogously defined... Stability obtains when this process converges to the REE strategies.

The result is the following:

Proposition 5 *The REE is locally stable if there are one or two trading periods. When the number of trading periods is larger than two, then the REE is not stable.*

The underlying intuition of that proposition is that increasing the horizon of agents makes coordination at the first trading periods more difficult, therefore destabilizing the whole REE path. The fact that the result is independent of the features of the model (variance of θ and noise trading) is largely an artifact of the model. Comparisons with the other results are therefore not easy. One only notes the role played by the heterogeneity of agents (one informed trader, one market maker) that is the main difference between this model and the other ones under consideration here. Namely, up to some extent, heterogeneity contributes to making the REE stability largely insensitive to the values of the parameters. In particular, a more volatile noise trading (an increase in σ^2) decreases the informational content of the price, making the pricing rule of the market maker less sensitive to his expectations of others' decision and therefore favoring stability. Still, the same increase in σ^2 favors instability; and this less beliefs-sensitive behavior of the market maker gives the informed monopolist incentives to trade more aggressively as well.

5 Conclusion and Further Issues

In this concluding section, we briefly present various issues that should be investigated in further research. First, comparing the "eductive" results with results of "evolutive" learning suggests that they are complementary approaches; second, studying the non noisy case shows some continuity problem with the case with a noisy supply; third, taking account of more complex information structures raises the question of what information deserves to be called "sharp"; and finally, empirical research is evoked.

Comparison with Bayesian learning. The lessons drawn from the "eductive" analysis stress dramatically different aspects of a learning process than the ones emphasized by Bayesian learning, as developed in Vives (1993) for example. A precise comparison of these two kinds of "learning" is provided in Desgranges, Geoffard and Guesnerie (2003). Here, we only stress the main features of this comparison. Recall first that the Bayesian learning story consists in a repeated play of the static model of Chapter E3 : At the initial time, the true state θ is revealed to the informed agents only; then, at every date t, every uninformed agent observes the current price, allowing him to update his beliefs on θ. In the long run, non-informed agents always learn θ, but the speed of convergence depends on the parameters of the model. In the Gaussian case, this speed of convergence is exponential, at the rate $-(\alpha\Delta)^2/\sigma^2$. The difference between this speed of convergence and the result in the preceding section, concluding at eductive stability with a Gaussian noise is twofold:

- The factors favoring a high speed of convergence of Bayesian learning are a large difference $\alpha\Delta$ between aggregate demand in both states and a small variance σ^2. It is remarkable that these two factors are unfavorable to the success of "eductive" learning. Intuitively, Bayesian learning is made easier by the informativeness of the public signal, whereas eductive stability is made more difficult when this signal is too informative, as already emphasized.

- The amplification effect has no counterpart in Bayesian learning. Indeed, this effect corresponds to the impact on demand of mistakes by non informed agents when guessing others' behavior. With Bayesian learning, no agent can make such an unpredictable mistake that will prevent others from correctly extracting information from prices.

The non noisy case. Desgranges and Guesnerie (2000) considers a non noisy model that is a simple case of the Desgranges, Geoffard and Guesnerie (2003) model in which the noisy supply ε is taken equal to 0 with probability 1. In this context, equilibrium prices are fully revealing and always "eductively" stable. This result is in sharp contrast with the conditions of stability that have been detailed previously and that suggests instability when the noisy supply is "small" (the equilibrium is unstable when the noisy supply is Gaussian with a small variance σ^2). Still, this remark, which reveals some form of discontinuity at the limit, is misleading as there is no entirely compelling definition of a small noise. Indeed, stability obtains for some other kinds of "small" noise. In particular, this is the case when the noisy supply has support in $[-\sigma, \sigma]$ for a small enough σ. The properties of equilibrium are then similar to those of the non noisy case (*i.e.*, the case $\sigma = 0$ as studied in Desgranges-Guesnerie (2000)): Equilibrium prices are fully revealing and "eductively" stable. Additional remarks on this question are provided in Desgranges-Guesnerie-Geoffard (2003).

Other information structures. Although too simplistic, the model of Desgranges and Guesnerie (2000) and Desgranges, Geoffard and Guesnerie (2003) allows to a first assessment of Guesnerie's conjecture according to which "Strongly

Rational Expectations Equilibria" (those that can be "educed") transmit information only if information is "sharp". To sustain this conjecture, Desgranges and Guesnerie (2000) consider a simple case in which information is not sharp and the (fully revealing) equilibrium is never "educed". The example goes as follows: there are three groups of equal size; each group either observes R or V, but the signal may differ across groups. Hence there are four states of aggregate information: RRR, RRV, RVV and VVV. Note then that, in state VVR (for example) no agent, observing either observing private signal V or R, knows with certainty that state VRR has not occurred. Information may then be called "diffuse", rather than sharp. Along the same lines, Desgranges (2000) characterizes broader classes of sharp information structures and diffuse information structures. Intuitively, at least in the main cases, diffuseness of an information structure means that no agent is able to distinguish with certainty between two states.

Relationship with data. *Last, but not least,* is the issue of confronting these results with data. We are not aware of any attempt at tackling this issue. Still, the results presented earlier normally have testable implications, such as the following ones:

- there are two regimes of volatility (stability of the relationship between price and fundamentals), depending on the nature of the situation

- Heterogeneous beliefs (beliefs being observed on derivative markets for example) are more or less persistent.

References

[1] P. DeMarzo and C. Skiadas (1998) Aggregation, Determinacy, and Informational Efficiency for a Class of Economies with Asymmetric Information, Journal of Economic Theory, 80(1) 123-152.

[2] G. Desgranges (1999) Révélation d'information par les prix et connaissance commune de la rationalité, thesis, EHESS.

[3] G. Desgranges (2000) , CK-Equilibrium and Informational Efficiency in a Competitive Economy, mimeo, University of Cergy-Pontoise.

[4] Desgranges, G., P. Y. Geoffard, et al. (2003). "Do Prices Transmit Rationally Expected Information?" Journal of the European Economic Association 1(1): 124-153.

[5] Desgranges, G. and R. Guesnerie (1996). Common Knowledge and the Information Revealed through Prices : Some Conjectures. Price on Good and financial markets: new results in theory and empirical research. G. P. F. Gardes, Elgar

[6] Green, J. (1977). "The Non-Existence of Informational Equilibria." Review of Economic Studies 44: 451-463.

[7] Gordon, S. (1999). "Fréquence des transactions et coordination des anticipations sur un marché financier." mimeo, DELTA.

[8] Grossman, S. (1976). "On the Efficiency of Competitive Stock Markets where Traders have Diverse Information." Journal of Finance 31: 573-585.

[9] Grossman, S. and J. Stiglitz (1976). "Information and Competitive Price Systems." American Economic Review 66: 246-253.

[10] Guesnerie, R. (2002). "Anchoring Economic Predictions in Common Knowledge." Econometrica 70(2): 439-480.

[11] Guesnerie, R. and J.-C. Rochet (1993). "(De) Stabilizing Speculation: an Alternative Viewpoint." European Economic Review 37(5): 1043-1063.

[12] Heinemann, M. (2001). "Existence of Strongly Rational Expectations Equilibria on Asset Markets with Asymmetric Information." mimeo, University of Hannover.

[13] M. Hellwig (1980) On the Aggregation of Information in Competitive Markets, Journal of Economic Theory, 22, 477-498.

[14] A. Kyle (1985) Continuous Auctions and Insider Trading, Econometrica, 53(6), 1315-1335.

[15] Kyle, A. (1989). "Informed Speculation with imperfect competition." Review of Economic Studies 56: 317 - 356.

[16] Laffont, J. J. (1985). "On the Welfare Analysis of Rational Expectations Equilibria with Asymmetric Information." Econometrica 53(1): 1-29

[17] L. Nielsen (1996) Common knowledge: the case of linear regression, Journal of Mathematical Economics, 26, 285-304.

[18] A. Postlewaite and D. Schmeidler (1986) Implementation in differential information economies, Journal of Economic Theory 39, 14-33.

[19] Radner, R. (1979). "Rational Expectations Equilibrium : Existence and the Information Revealed by Prices." Econometrica 47: 655-678.

[20] Vives, X. (1993). "How fast do rational agents learn ?" Review of Economic Studies 60: 329-347.