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Fair Long-Term Care Insurance*

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May 9, 2019

Abstract

The study of optimal long-term care (LTC) social insurance is generally carried out under the utilitarian social criterion, which penalizes individuals who have a lower capacity to convert resources into well-being, such as dependent elderly individuals or prematurely dead individuals. This paper revisits the design of optimal LTC insurance while adopting the *ex post* egalitarian social criterion, which gives priority to the worst-off in realized terms (i.e. once the state of nature has been revealed). Using a lifecycle model with risk about the duration of life and risk about old-age dependence, it is shown that the optimal LTC social insurance is quite sensitive to the postulated social criterion. The optimal second-best social insurance under the *ex post* egalitarian criterion involves, in comparison to utilitarianism, higher LTC benefits, lower pension benefits, a higher tax rate on savings, as well as a lower tax rate on labor earnings.

Keywords: long-term care, social insurance, fairness, mortality, compensation, egalitarianism.

JEL codes: J14, I31, H55.

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1 Introduction

The rise of long-term care constitutes a major challenge for advanced economies. Due to ageing, an increasingly large proportion of the population falls into dependency, that is, is unable to carry out basic daily activities such as eating, washing, etc., and, hence, needs to receive long-term care (LTC). The provision of LTC, either formal (i.e. supplied by private firms on the market, or by a public transfer) or informal (i.e. provided by family and friends), is extremely costly, which raises the question of the funding of LTC.[1]

Given the high probability to fall into dependency and the high costs of LTC, one would expect, from the perspective of rational choice theory, that individuals purchase private LTC insurance, in such a way as to insure themselves and their family against the high costs of LTC. However, as shown by Brown and Finkelstein (2007, 2011), only a small fraction of the population at risk purchases private LTC insurance. Indeed, the private share of LTC spending in OECD countries is about 15% (OECD, 2011). This is the well-known LTC private insurance puzzle. This puzzle can be explained by various factors, on both the supply side (e.g. high loading factors, adverse selection) and the demand side (e.g. myopia or denial of dependency) of the private LTC insurance market.[2]

The rise of LTC needs and the low coverage rate provided by the LTC private insurance market motivate the involvement of another economic agent, i.e. the State, which could intervene and construct a social - instead of private - LTC insurance. In Europe, there is indeed a will to build progressively what could be regarded as the “fifth pillar" of the Welfare State: besides insuring individuals against unemployment, diseases, accidents and old-age, the State could also provide a social insurance against the risk of LTC.

The design of an optimal social LTC insurance raises lots of problems, since many dimensions are present, both in terms of efficiency and equity (see Jousten et al 2005, Pestieau and Sato 2008, Cremer and Roeder 2013, Cremer and Pestieau 2014). In particular, when examining the design of an optimal social LTC insurance, a key question that arises concerns the fairness of that social insurance. What would be a fair LTC social insurance?

The goal of this paper is to examine the construction of a fair social LTC insurance. That question is complex, and there are at least two traps to be avoided when considering the design of a fair social LTC insurance. This paper will pay a particular attention to avoiding those two traps.

The first trap concerns the treatment of heterogeneity in preferences within the population. Once a person falls into dependency, his preferences are affected, and do not remain the same as the preferences of a person who is healthy and autonomous. In particular, as shown by Finkelstein et al (2013), a deterioration of the health status contributes to reducing the marginal utility of consumption

[1] According to Norton (2000), about 2/3 of LTC is provided informally. However, the share of the informal care may decrease over time, and various scenarios exist regarding the extent of the decrease of informal care with respect to formal care.

[2] On the various possible explanations of that puzzle, see Brown and Finkelstein (2010), Pestieau and Ponthiere (2011), Boyer et al. (2017) and Ameriks et al. (2018).

in comparison with situations of good health. As a consequence, models based on state-invariant utility are only approximative representations of individual well-being, and should be replaced by models with state-dependent utility. But the presence of state-dependent utility raises difficulties at the normative level: once dependent individuals have, at the margin, a lower capacity to convert resources into well-being, how should we treat them in comparison with healthy individuals? The classical utilitarian social criterion, which sums up the utilities of all individuals, provides here a counterintuitive answer to that question: it assigns *fewer* resources to the dependent elderly than to the healthy elderly, on the grounds of the lower capacity of the former to produce well-being (i.e. marginal utility being lower under dependency). There is thus a first trap, which concerns the treatment of heterogeneity in a society where some, but not all, individuals fall into dependency at the old age.

A second trap raised by the construction of a fair social LTC insurance concerns the definition of the population under study. When considering the design of optimal social LTC insurance, it is tempting to focus only on the elderly population, since that population is the one that is subject to the risk of dependency. One then considers how social insurance could provide some compensation to the unlucky old individuals who fall into dependency. The problem with this approach is that it ignores a sizeable part of the population, which dies prematurely *before* being subject to the risk of dependency. In Western Europe, about 10 percent of the male population does not reach the age of 60, and, hence, can hardly be subject to any LTC risk. Whereas one may regard this fact as irrelevant for the design of a social LTC insurance, it matters, since the fairness or unfairness of a social LTC insurance must be evaluated while taking into account the *entire* population, and the associated distribution in terms of lifetime well-being. One cannot ignore the short-lived, since doing this may lead to implement regressive social insurance, i.e. transferring resources from worst off (short-lived) individuals to better off (long-lived) individuals.

In order to examine the design of a fair social LTC insurance, this paper develops a lifecycle model where a population differing in labor productivity faces risk about the duration of life, as well as risk of old-age dependency. Our analysis proceeds in three stages. First, we characterize the laissez-faire equilibrium, and compare it with the utilitarian social optimum. Second, given the limited capacity of the utilitarian criterion to take into account in an adequate way the heterogeneity of individuals in terms of longevity and health status, we consider another social criterion, i.e. the *ex post* egalitarian criterion, which gives priority to the worst-off in realized terms (i.e. once uncertainty about longevity and the health status has been revealed). In a third stage, we develop a second-best analysis, where a government has only four available (uniform) policy instruments: a linear tax on labor earnings, a linear tax on private aggregate savings (annuities and LTC private insurance), a pension benefit and a LTC allowance. We compare second-best policies under the utilitarian and the *ex post* egalitarian criteria.

Our motivation for comparing the standard utilitarian optimum with the *ex post* egalitarian one goes as follows. As stated above, utilitarianism faces major difficulties when the population is heterogeneous on fundamental dimensions such as the duration of life or the health status (autonomy or dependency).[3] Those difficulties come from the fact that utilitarianism leads to the equalization, at the margin, of the utility gains obtained from alternative uses of resources. This equalization of marginal utilities across rival uses of resources is questionable in the context of unequal lifetimes or unequal health status, where it leads to inequalities in *levels* of realized well-being. Those *ex post* well-being inequalities are hardly justifiable from an ethical perspective, since these are due, for a large part, to circumstances, and, in particular, to the genetic background.[4] Given that those inequalities are due to circumstances, the Principle of Compensation applies (Fleurbaey 2008), and governments should abolish those inequalities. This motivates our reliance on an alternative social criterion, the *ex post* egalitarian criterion proposed by Fleurbaey et al (2014), which does justice to compensating individuals for well-being inequalities due to circumstances.

Anticipating on our results, we first show that, whereas elderly dependent individuals receive, at the utilitarian optimum, fewer resources than healthy elderly ones, this is not the case under the *ex post* egalitarian optimum, in which the dependent elderly receives more resources than the healthy one. The intuition is that, while utilitarianism transfers more resources to the healthy elderly on the ground of their higher marginal utility of consumption, the *ex post* egalitarian solution aims at equalizing not marginal utilities, but the levels of utilities between the dependent elderly and the healthy elderly, which requires to give more resources to the former. Secondly, whereas, under utilitarianism, the optimal consumption profile is flat for healthy individuals and decreasing with the age for individuals who become dependent at the old age, the optimal consumption profile is, under the *ex post* egalitarian optimum, decreasing with the age, with a stronger decline for the healthy elderly, in such a way as to equalize lifetime well-being between the long-lived and the short-lived. Those fundamental differences between the utilitarian optimum and the *ex post* egalitarian optimum have major corollaries in terms of policy. Our theoretical analysis of the second-best problem - and the associated numerical simulations - point to several important differences between the optimal LTC insurance under the *ex post* egalitarian criterion and under the utilitarian one. In comparison to the utilitarian second-best, the *ex post* egalitarian second-best involves a higher LTC benefit, a lower pension benefit, a higher tax rate on (aggregate) savings, as well as a lower tax rate on labor earnings.

As such, this paper casts original light on a major policy challenge of our times - the design of an optimal LTC social insurance -, by highlighting how the postulated social criterion affects the design of the optimal social LTC insurance.

[3] Note that this limitation of utilitarianism is not specific to an economy with LTC needs. Actually, as stressed in Sen and Williams (1982), utilitarianism also faces problems in its treatment of handicaped persons, who have also a different capacity to convert resources into well-being in comparison with non-handicapped persons.

[4] Regarding the risk of old-age dependence, Farrer et al (1997) show that there exists a statistical association between apolipoprotein e genotype and the risk of alzheimer disease. As far as longevity is concerned, Christensen et al (2006) show that between 1/4 and 1/3 of longevity inequalities within a cohort are due to the genetic background.

Our analysis shows that not only does the level of LTC benefits varies with the underlying social criterion, but, also, the distribution of the financial burden of LTC along the life cycle. In the light of this, the design of the "fifth pillar" of the Welfare state could hardly avoid a discussion on the ethical foundations behind the construction of that pillar, and, in particular, about the ethical treatment of the interests of the dependent elderly and of the prematurely dead. Although the weight assigned to the interests of the latter may seem, at first glance, irrelevant for the design of a fair LTC social insurance, it matters actually quite a lot, especially concerning the distribution of the financial burden of LTC social insurance. Assigning a higher weight to the interests of the prematurely dead leads to a lower taxation of labor earnings during the active life.

This paper is related to several branches of the literature. First, it is linked to the literature on the design of the optimal LTC social insurance (see Jousten et al, 2005, Pestieau and Sato 2008, Cremer and Roeder 2013, Cremer and Pestieau 2014). In comparison to those papers, the specificity of this study is to lay a strong emphasis on the fairness dimension of LTC social insurance in the context of unequal lifetime and health status at the old age. Second, this paper is also related to other papers characterizing optimal policies under unequal longevity, such as Fleurbaey and Ponthiere (2013), Fleurbaey et al (2014), Fleurbaey et al (2016) and Leroux and Ponthiere (2018). The extra value of this paper with respect to that literature is that instead of focusing only on two individual outcomes - i.e. a short or a long life - we include here an intermediate life status - dependency at the old age -, which allows us to examine the sensitivity of the optimal LTC social insurance to the postulated social criterion.

The rest of the paper is organized as follows. Section 2 presents the model. The laissez-faire equilibrium is characterized in Section 3. Section 4 studies the utilitarian social optimum. The *ex post* egalitarian optimum is examined in Section 5. Second-best policy analysis is carried out in Section 6. Section 7 provides some numerical simulations. Conclusions are drawn in Section 8.

2 The model

Let us consider a two-period economy. Period 1 is young adulthood, during which individuals are healthy, supply labor, consume and save for their old days. Period 2 is old adulthood. That period is reached with a probability $0 < \pi < 1$. During the old age, individuals do not work and consume their savings. Moreover, individuals reaching the old age become dependent with a probability $0 < p < 1$. In case of dependency, agents bear additional exogenous LTC expenditures equal to an amount $S > 0$.

Heterogeneity *Ex ante* (i.e. before the duration of life and the old-age health status are revealed), individuals differ on a single dimension, which is their labor productivity w. For simplicity, we assume two productivity levels: type-H individuals have a high productivity, whereas type-L individuals have a low productivity, i.e. $w_H > w_L$. Individuals of type $i \in \{L, H\}$ are in proportion n_i, with $n_H + n_L = 1$.

Following the literature on the health/income gradient, we assume that individuals with higher productivity have also a higher probability to reach the old age, that is $\pi_H > \pi_L$ as well as a lower unconditional probability of old-age dependency, $\pi_H p_H < \pi_L p_L$.[5]

Preferences Preferences of individuals are represented by a standard expected utility function. As usual, it is assumed that lifetime welfare is time-additive, and the utility of being dead is normalized to 0. We also assume that temporal utility is state-dependent, in the sense that the transformation of resources into welfare is not the same when the individual is healthy and when he is dependent. Expected lifetime utility for an individual of type $i \in \{L, H\}$ is:

$$EU_i = u(c_i) + \pi_i(1 - p_i)u(d_i) + \pi_i p_i v(b_i) \tag{1}$$

where c_i is consumption at young age, d_i is old-age consumption in case of a healthy old age, while b_i denotes old-age consumption in case of old-age dependency (excluding LTC expenditure S). Hence b_i accounts for the net resources left for consumption in case of dependency, and $z_i = b_i + S$ for the gross resources (including LTC expenditure) needed in case of dependency.

Regarding the disutility of working at the young age, it is assumed, for simplicity, that working a duration ℓ_i creates a disutility, measured in consumption units, equal to $e(\ell_i)$ with $e(0) = 0$, $e'(\ell_i) > 0$ and $e''(\ell_i) > 0$.

As usual, it is assumed that $u'(\cdot) > 0$, $u''(\cdot) < 0$, and that there exists a critical consumption level $\bar{c} > 0$ that makes a person indifferent between, on the one hand, a healthy life with consumption $\bar{c}$, and, on the other hand, death (i.e. $u(\bar{c}) = 0$). That assumption is standard in the literature on life and death (see Becker et al 2005): assuming, on the contrary, that such a critical consumption level did *not* exist would amount to assume either that being alive (even with a zero consumption) is always better for individuals than being dead, or, alternatively, that being alive (even with a high consumption) is always worse for individuals than being dead, which are implausible assumptions.

Regarding individual utility under old-age dependency, it is assumed that $v'(\cdot) > 0$, $v''(\cdot) < 0$, and that there exists a critical consumption level $\tilde{c} > 0$ such that $v(\tilde{c}) = 0$. That critical level of consumption in case of dependency has the same justification as in the case of a healthy old-age. Without such a critical consumption level, being dead would always be either worse or better than being alive and dependent, which would be quite implausible.

Whereas the state-dependent temporal utility functions $u(\cdot)$ and $v(\cdot)$ share some properties - both are increasing, concave, with a negative intercept -, these differ nonetheless on two important aspects (see Figure 1).

[5] Data are obtained from SHARE (waves 2, 4 and 6). See Lefebvre et al. (2018). Note also that the above inequalities imply that $p_H < p_L$.

First, being dependent causes a utility loss with respect to being healthy, so that, for a given level of consumption, being dependent brings less welfare:

$$u(d_i) > v(d_i) \ \forall d_i \tag{2}$$

This inequality implies that the two critical consumption levels satisfy $\tilde{c} > \bar{c}$.

Second, as shown by Finkelstein et al (2013), being dependent reduces the marginal utility of consumption in comparison with being autonomous:

$$u'(d_i) > v'(d_i) \ \forall d_i \tag{3}$$

This is consistent with the LTC insurance puzzle, i.e. the stylized fact that a small fraction of population purchases private LTC insurance (see the Introduction).[6]

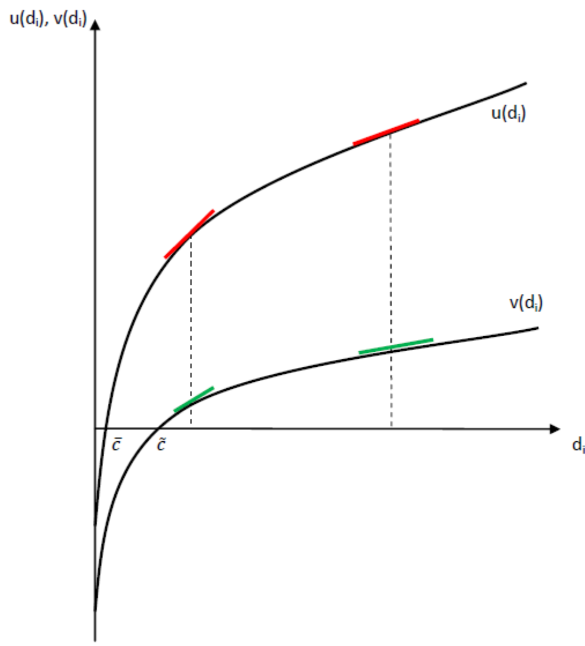

Figure 1: State-dependent utility functions $u(\cdot)$ and $v(\cdot)$.

Without additional assumptions, one could not exclude, in theory, that, in a poor economy (i.e. with a low labor productivity), individuals may prefer being

[6] It should be stressed that this assumption involves a simplification of reality. In real life, there are many consumption goods and services, and one cannot exclude that the marginal utility of consumption of *some* goods or services may be higher under dependency than under autonomy. However, our model is a one-good economy, and, in that simplified context, assuming that dependency reduces the marginal utility of consumption is a plausible proxy.

dead to being old and dependent (i.e. case where $b_i < \tilde{c}$), or, also, may prefer being dead to being old and healthy (i.e. case where $d_i < \bar{c}$). Indeed, if the economy is so poor that consumption levels are extremely low and lie at the left of the x axis on Figure 1, such results could emerge.

Given that we focus on the design of optimal LTC social insurance in advanced economies, we will, throughout this paper, exclude those extreme cases, and assume that the economy is productive enough, so that there are always enough resources so as to guarantee that old-age consumption is above the critical levels $\tilde{c}$ and $\bar{c}$, which implies that individuals are better off alive than dead. Concretely, we assume, in the rest of the paper, that the productivity level w_L is large enough that, both at the laissez-faire equilibrium and at the first-best and second-best optimum, consumption levels of low-productivity (and therefore of high-productivity) individuals, are always above the thresholds, $\bar{c}$ and $\tilde{c}$.

3 The laissez-faire equilibrium

Let us first characterize the equilibrium in the absence of governmental intervention. For that purpose, we assume that there exists a perfect annuity market, which yields actuarially fair returns on savings. Assuming that the interest rate is zero, the return on savings for an individual of type i in case of survival to the old age is thus $1/\pi_i$. It is also assumed that agents have access to an actuarially fair private LTC insurance market (yielding a return $1/\pi_i p_i$).

Individuals choose labor ℓ_i, savings s_i and private LTC insurance a_i in order to maximize their expected lifetime welfare subject to their budget constraint:

$$\max_{\ell_i, s_i, a_i} u\left(w_i \ell_i - e\left(\ell_i\right) - s_i - a_i\right) + \pi_i(1-p_i) u\left(\frac{s_i}{\pi_i}\right) + \pi_i p_i v\left(\frac{s_i}{\pi_i} + \frac{a_i}{\pi_i p_i} - S\right)$$

Focusing on an interior equilibrium, first-order conditions (FOCs) are:

$$w_i = e'\left(\ell_i\right) \quad (4)$$
$$u'(c_i) = (1-p_i) u'\left(d_i\right) + p_i v'\left(b_i\right) \quad (5)$$
$$u'(c_i) = v'\left(b_i\right) \quad (6)$$

The first FOC characterizes the optimal labor supply as the one equalizing, at the margin, the wage and the marginal disutility of labor. Note that, since $w_H > w_L$, we have here, given the convexity of $e(\cdot)$, that $\ell_H > \ell_L$, that is, that more productive individuals work more than less productive individuals.[7]

The second FOC characterizes the optimal level of savings, whereas the last FOC characterizes the optimal level of private LTC insurance. The optimal level of LTC insurance equalizes, at the margin, the utility loss from consumption reduction at the young age (due to premiums paid) and the utility gain from increased consumption at the old age in case of dependency.

[7] In other words, our structure on preferences implies that the substitution effect dominates the income effect.

Note that substituting the last FOC in the second one yields:

$$u'(c_i) = u'(d_i) = v'(b_i)$$

which implies, given the properties of $u(\cdot)$ and $v(\cdot)$, that $c_i = d_i > b_i \, \forall i$. These inequalities imply that $a_i/(\pi_i p_i) < S$ in equilibrium. Even though marginal utility under dependency is smaller than under autonomy, agents invest in a private annuity so as to partially cover for additional LTC expenditures S.[8]

Let us now study well-being inequalities prevailing at the laissez-faire equilibrium. For that purpose, we will, following Fleurbaey (2010), pay a particular attention not to inequalities in well-being *ex ante*, i.e. in terms of expected lifetime well-being, but, on the contrary, to inequalities in well-being *ex post*, i.e. in terms of realized lifetime well-being.

In a seminal paper on the social evaluation of risky situations, Fleurbaey (2010) argued that, from a normative perspective, one should give priority to the comparison of *ex post* well-being levels over the comparison of *ex ante* well-being levels. Fleurbaey's argument goes as follows. He distinguishes between, on the one hand, uninformed preferences (i.e. preferences before the state of the world is revealed), and, on the other hand, informed preferences (i.e. preferences after the state of the world is revealed). According to Fleurbaey, normative evaluation should give priority to informed preferences over uninformed preferences, since informed preferences are defined on what is really achieved by individuals. Hence, from that perspective, the social evaluation of situations should focus on the distribution of *ex post* well-being levels.

Fleurbaey's argument is most relevant for our framework. In our economy, individuals have well-defined preferences on lotteries of life, which drive their choices at the laissez-faire when they do not know what kind of life they will have. However, at the end of the day, i.e. *ex post*, all individuals will turn out to be either short-lived or long-lived and either dependent or autonomous at the old age. These are the outcomes that are achieved by all individuals, and the social evaluation of individual situations should concentrate on those outcomes, and on how individuals value these.[9] The social evaluation of situations should thus give priority to informed preferences, and concentrate on the distribution of *ex post* well-being levels.

Focusing on informed preferences leads us to compare not expected lifetime well-being across individuals, but *realized* lifetime well-being. In our framework, there are, *ex post*, not 2, but 6 types of agents, depending on the realized duration of life (short life or long life) and the realized health status at the old

[8] In the special case where $S = 0$, we would have to constrain $a_i = 0$ and $d_i = b_i \, \forall i$ as in such a case, $d_i > b_i$ would lead to $a_i < 0$, which is not possible.

[9] In some sense, adopting an *ex post* perspective amounts to consider that life expectancy or healthy life expectancy are not relevant pieces of information for social evaluation: only realized longevity and realized health status matter.

age (autonomous or dependent). The realized lifetime well-being levels are:

$$\begin{aligned} U^{H,SL} &= u(c_H) \\ U^{H,LL,D} &= u(c_H) + v(b_H) \\ U^{H,LL,A} &= u(c_H) + u(d_H) \\ U^{L,SL} &= u(c_L) \\ U^{L,LL,D} &= u(c_L) + v(b_L) \\ U^{L,LL,A} &= u(c_L) + u(d_L) \end{aligned}$$

where $\{A, D\}$ stand for being autonomous (A) or dependent (D), and $\{SL, LL\}$ stand for being short-lived (SL) or long-lived (LL).

Under the assumptions made in Section 2, consumption levels d_i and b_i satisfy $d_i > \bar{c}$ and $b_i > \tilde{c}$, so that the long-lived who is healthy at the old age is, *ex post*, better off than the short-lived of the same type. Moreover, the long-lived who is dependent is also, *ex post*, better off than the short-lived of the same type. We have thus the following inequalities in terms of realized lifetime well-being:

$$U^{H,SL} < U^{H,LL,D} < U^{H,LL,A} \text{ and } U^{L,SL} < U^{L,LL,D} < U^{L,LL,A}.$$

Concerning the impact of productivity differentials on realized lifetime well-being, one cannot, without additional assumptions, rank the realized well-being levels of individuals of types H and L. Indeed, in our economy, being highly productive does not only, through a higher hourly wage, extend consumption possibilities *ceteris paribus*, but it is also associated with a higher survival probability, which decreases the return of savings under perfect annuities, and, hence, reduces consumption possibilities *ceteris paribus*.[10] In the light of those opposite forces, a sufficient condition for having the intuitive case where type-H individuals are, *ceteris paribus*, better off *ex post* than type-L individuals, is that the gap in survival probabilities π_H and π_L is relatively small in comparison with the gap in w_H and w_L.[11] In that case, we have:

$$U^{H,SL} > U^{L,SL}, U^{H,LL,D} > U^{L,LL,D} \text{ and } U^{H,LL,A} > U^{L,LL,A}$$

that is, more productive individuals are, *ex post*, better off than low productivity individuals with the same longevity and old-age health status.

Note that assuming that the gap in survival probabilities is sufficiently small in comparison with the wage gap is not a strong assumption. However, that assumption is necessary to avoid counterintuitive cases where low productivity

[10] Note, however that a higher productivity is associated with a lower old-age unconditional dependency probability, which increases the return of private LTC insurance and thus consumption possibilities.

[11] Clearly, in the extreme case where π_L is much lower than π_H, and close to zero, the second effect may dominate the first one, and one cannot exclude that $U^{H,i} < U^{L,i}$, against intuition. Indeed in that case the few individuals of type L who are long-lived are like lottery winners, and obtain very high consumption levels, much higher than those of type H individuals.

individuals could turn out to be better off *ex post* than high productivity individuals with the same longevity and old-age health status. Proposition 1 summarizes our results.

Proposition 1 *At the laissez-faire equilibrium,*

- *Consumption is constant in the first two ages of life if the agent is autonomous, but falls in the old age if he becomes dependent.*
- *Individuals of type H work more than individuals of type L.*
- *The healthy long-lived is, ex post, better off than the unhealthy long-lived and the short-lived of the same type $i \in \{L, H\}$.*
- *The unhealthy long-lived is, ex post, better off than the short-lived of the same type $i \in \{L, H\}$.*
- *If the gap in survival probabilities is sufficiently small in comparison with the wage gap, individuals of type H are, for a given longevity and health status, better off ex post in comparison with individuals of type L.*

Proof. See above. ■

Proposition 1 states that the laissez-faire equilibrium involves, *ex post*, sizeable well-being inequalities across individuals, depending not only on their productivity type, but, also, on their realized longevity, and on whether these are, in case of survival to the old age, either healthy or dependent. Those inequalities in well-being are arbitrary, and hardly justifiable from an ethical perspective. In our model, being highly productive does not depend on past education investment or effort, but is purely exogenous. Moreover, being able to survive to the old age is also something purely exogenous in our economy. In a similar way, falling into dependency at the old age is, here again, a pure circumstance for individuals.

Those well-being inequalities being hardly justifiable from an ethical perspective, there is a strong support for policies aimed at correcting those inequalities. This is the major intuition supporting the construction of a social insurance system. However, as we will show, the particular form of the optimal social insurance is not insensitive to the underlying ethical foundations. To show this, the next two sections consider, as a preliminary step towards the design of optimal social insurance, the characterization of the social optimum under two distinct social criteria: on the one hand, the utilitarian criterion (Section 4), and, on the other hand, the *ex post* egalitarian criterion (Section 5).

4 The utilitarian optimum

When studying the optimal public policy, the normative benchmark in public economics is, in general, the utilitarian criterion, which consists in summing the (expected) well-being of all individuals in the population. In order to characterize the utilitarian social optimum in our economy, let us consider a benevolent social planner who chooses consumptions and labor times for types-H and -L individuals so as to maximize the sum of individual expected lifetime utilities.[12] The problem of the utilitarian social planner can be written as:

$$\begin{aligned} \max_{\ell_i, x_i, d_i, b_i} \quad & \sum n_i \left[u\left(c_i\right) + \pi_i (1-p_i) u\left(d_i\right) + \pi_i p_i v\left(b_i\right) \right] \\ \text{s.t.} \quad & \sum n_i \ell_i w_i = \sum n_i \left(x_i + \pi_i (1-p_i) d_i + \pi_i p_i (b_i + S) \right) \end{aligned}$$

where $c_i = x_i - e(\ell_i)$ and x_i is the goods component of consumption. That problem can be rewritten by means of the following Lagrangian:

$$\begin{aligned} \max_{\ell_i, x_i, d_i, b_i} \quad & \sum n_i \left[u\left(c_i\right) + \pi_i (1-p_i) u\left(d_i\right) + \pi_i p_i v\left(b_i\right) \right] \\ & + \mu \left[\sum n_i \ell_i w_i - \sum n_i \left(x_i + \pi_i (1-p_i) d_i + \pi_i p_i (b_i + S) \right) \right] \end{aligned}$$

where μ is the Lagrange multiplier associated with the resource constraint.

Focusing on an interior optimum, the FOCs yield:

$$e'(\ell_i) = w_i \tag{7}$$

$$u'(c_i) = u'\left(d_i\right) = v'(b_i) = \mu \tag{8}$$

The first FOC states a standard result: under utilitarianism, more productive individuals should work *more* than less productive individuals:

$$\ell_H^u > \ell_L^u.$$

The second expression states that resources should be allocated in such a way as to equalize, at the margin, the utility gain from the different uses of those resources, in terms of consumption at the young age and the old age. That condition has several corollaries.

A first corollary is that individuals of types H and L are treated equally, in terms of consumption, at the utilitarian optimum:

$$c_H = c_L = c^u, d_H = d_L = d^u, b_H = b_L = b^u.$$

Another, less intuitive, corollary is that the dependent elderly enjoy *less* consumption (equivalently, once LTC expenditure are paid, they are left with fewer resources) than the healthy elderly. Indeed, since $u'\left(d_i\right) > v'(d_i)$ for a given d_i, the equality $u'\left(d_i\right) = v'(b_i)$ can only be achieved provided:

$$c^u = d^u > b^u.$$

That result is counterintuitive. Dependent elderly persons are less lucky than healthy elderly persons, since the former enjoys, *ceteris paribus* (i.e. for a given

[12] Note that, due to the additivity of individual preferences across time and states of nature, this problem is formally similar to the choice of consumptions and labor times that maximize the average *ex post* lifetime well-being in the population.

amount of consumption), less well-being than the latter. Therefore it is not clear to see why one should penalize the dependent elderly further, and reduce their consumption below the one of the healthy elderly. The reason why this counterintuitive result arises is that the utilitarian criterion sums up individual utilities that take quite different forms (because of state-dependent preferences, based on the health status), such as the utility of the healthy elderly (given by $u(\cdot)$) and the utility of the dependent elderly (given by $v(\cdot)$), and favours those who are more able to transform, at the margin, resources into well-being, i.e. the healthy elderly. The lower capacity of the dependent elderly to convert resources into well-being explains why they are penalized by the utilitarian planner.

Regarding well-being comparisons, it should be stressed that utilitarianism, by equalizing consumptions for individuals of types H and L, abolishes *ex post* well-being inequalities between individuals who differ in productivity, but enjoy the same longevity and old-age health status. We thus have:

$$U^{H,SL} = U^{L,SL},\ U^{H,LL,D} = U^{L,LL,D} \text{ and } U^{H,LL,A} = U^{L,LL,A}$$

unlike what prevailed at the laissez-faire equilibrium.

Note, however, that utilitarianism does not make well-being inequalities due to unequal longevity or old-age health status vanish. Indeed, under the assumptions that $c, d \geq \bar{c}$ and $b \geq \tilde{c}$ (see Section 2), short-lived individuals are, at the utilitarian optimum, necessarily worse off than long-lived individuals (either in dependency or healthy). Moreover, the dependent elderly are, at the utilitarian optimum, also worse off than the healthy elderly. Hence *ex post* lifetime well-being levels satisfy, at the utilitarian optimum:

$$U^{H,SL} = U^{L,SL} < U^{H,LL,D} = U^{L,LL,D} < U^{H,LL,A} = U^{L,LL,A}.$$

The following proposition summarizes our results.

Proposition 2 *At the utilitarian optimum,*

- *Consumption is constant in the first two ages of life if the agent is autonomous, but falls in the old age if he becomes dependent.*
- *For a given longevity/health status, individuals of type H are, ex post, as well off as individuals of type L.*
- *The healthy long-lived is, ex post, better off than the unhealthy long-lived and the short-lived of the same type $i \in \{L, H\}$.*
- *The unhealthy long-lived is, ex post, better off than the short-lived of the same type $i \in \{L, H\}$.*

Proof. See above. ■

In comparison with the laissez-faire, the utilitarian optimum neutralizes one source of *ex post* well-being inequalities: inequalities in labor productivity across individuals. But labor productivity was only one source of inequalities, and two alternative sources of *ex post* well-being inequalities remain: on the one hand, the duration of life, and, on the other hand, the health status at the old age. Being long-lived or short-lived, or, alternatively, being healthy or dependent at the old age, these are, in our model, pure circumstances, dictated by Nature. Utilitarianism does not correct for those inequalities. At the utilitarian optimum, it remains that, like at the laissez-faire, short-lived individuals are worse-off than long-lived individuals. Moreover, it remains also that, like at the laissez-faire, dependent elderly individuals are, at the utilitarian optimum, worse off than healthy elderly individuals.[13]

5 The ex post egalitarian optimum

As shown in the previous section, the utilitarian optimum does not bring equality in *ex post* (realized) lifetime well-being, since it remains that, at the utilitarian optimum, the short-lived is worse-off than the long-lived, and the dependent elderly is worse off than the healthy one.

From an ethical perspective, those inequalities in *ex post* lifetime well-being due to unequal lifetime or old-age health status are hardly justifiable, since these are largely due to circumstances. For instance, the genetic background was shown to account for between 1/4 and 1/3 of longevity inequalities within a cohort (see Christensen et al 2006). Moreover, the probability to fall into dependency at the old age is also related to genetic background (see Farrer et al 1997). Given that the individual genetic background is a pure circumstance, individuals can hardly be regarded as responsible for inequalities in the length of life or in the old-age health status. Hence, from a normative perspective, the Principle of Compensation applies and so, well-being inequalities due to circumstances should be abolished by governments (see Fleurbaey and Maniquet 2004, Fleurbaey, 2008). This makes a lot of sense in our context, where individuals could hardly be regarded as responsible for their lifetime or for their health status at the old age.[14]

In the light of the Principle of Compensation, there is some ethical support for providing compensation to individuals who are disadvantaged either because of a short life, or because of a bad health status at the old age. As shown above, the utilitarian criterion cannot do justice to the idea of compensating individuals

[13] This result is driven by the fact that dependent individuals have, in our model, a lower capacity to convert resources into well-being. If one made, alternatively, the assumption that the dependent elderly enjoys a higher marginal utility of consumption (i.e. $v'(.) > u'(.)$), we would find that, at the utilitarian optimum, $b > d$, so that $u(d) \gtrless v(b)$ and $U^{LL,D} \gtrless U^{LL,A}$. But such assumption goes against the empirical evidence of Finkelstein et al (2013).

[14] Alternatively, if individuals were regarded as responsible for their situation, then the principle of Liberal Reward would apply (see Fleurbaey and Maniquet 2004, Fleurbaey 2008). That principle states that, when inequalities are due to efforts, governments should leave those inequalities unaffected. In many real-world situations, efforts and circumstances interact, which makes the principle of Compensation incompatible with the principle of Liberal Reward, so that an ethical dilemma arises. This section examines the corollaries of adopting the principle of Compensation for the design of LTC social insurance, leaving aside any responsibility issues.

for those damages. This motivates considering an alternative social criterion.

As proposed by Fleurbaey et al (2014) in the context of unequal lifetimes, the *ex post* egalitarian criterion does justice to the idea of compensating individuals for inequalities due to circumstances. Actually, the *ex post* egalitarian criterion gives priority to the worst off, whose situation is defined in realized terms, that is, *ex post* (i.e. once the state of nature has been revealed). The objective is thus to choose an allocation of resources that maximizes the minimum level of realized lifetime well-being in the population. The problem of the *ex post* egalitarian social planner can be written as:

$$\begin{array}{cc} \max_{\ell_i,x_i,d_i,b_i} & \min\left\{ \begin{array}{l} u\left(c_L\right), u(c_H), u\left(c_L\right)+v\left(b_L\right), u\left(c_H\right)+v\left(b_H\right), \\ u\left(c_L\right)+u\left(d_L\right), u\left(c_H\right)+u\left(d_H\right) \end{array} \right\} \\ \text{s.t.} & \sum n_i\ell_i w_i = \sum n_i\left(x_i+\pi_i(1-p_i)d_i+\pi_i p_i(b_i+S)\right) \end{array}$$

The objective function is not differentiable. However, it is possible to rewrite that problem as the maximization of the well-being of the short-lived of type L subject to the resource constraint of the economy and subject to several egalitarian constraints:

$$\begin{array}{cl} \max_{\ell_i,x_i,d_i,b_i} & u(x_L-e(\ell_L)) \\ \text{s.t.} & \sum n_i\ell_i w_i = \sum n_i\left(x_i+\pi_i(1-p_i)d_i+\pi_i p_i(b_i+S)\right) \\ \text{s.t.} & u(c_L)=u(c_H) \\ \text{s.t.} & u(c_i)+u(d_i)=u(c_i)\,\forall i \\ \text{s.t.} & u(c_i)+v(b_i)=u(c_i)\,\forall i \end{array}$$

The first egalitarian constraint states that individuals of type H who are short-lived should be exactly as well off as individuals of type L who are short-lived. The second egalitarian constraint states that, for a given type $i=H,L$, individuals who are healthy at the old age should be exactly as well off as individuals who are short-lived. The third egalitarian constraint states that, for a given type $i=H,L$, individuals who are dependent at the old age should be exactly as well off as individuals who are short-lived.

Focusing on an interior optimum, we obtain:

$$\begin{array}{rcl} e'(\ell_i) & = & w_i \implies \ell_H > \ell_L \\ c_L = x_L - e(\ell_L) & = & c_H = x_H - e(\ell_L) \implies x_H > x_L \\ d_L & = & d_H = \bar{c} \\ b_L & = & b_H = \tilde{c} > \bar{c} \end{array}$$

where the third and fourth lines are obtained by setting $u(d_i)=0\,\forall i$ and $v(b_i)=0\,\forall i$, so as to leave no inequality between short-lived and long-lived individuals. Like at the utilitarian optimum, it is also the case that highly productive individuals work more than less productive individuals. Moreover, like at the utilitarian optimum, consumptions are, for each age of life and each health status, equalized across types H and L: $c_L = c_H = c^e$, $d_L = d_H = d^e = \bar{c}$ and $b_L = b_H = b^e = \tilde{c}$.

However, a first, important, difference is that, unlike at the utilitarian optimum, consumptions at the young age and at the healthy old age are not equal. Actually, individuals who reach the old age and are healthy should here have a consumption equal to $\bar{c}$, i.e. the critical consumption level that makes a healthy person indifferent between life with that consumption and death. Under the assumptions made in Section 2, this "neutral" consumption level is smaller than consumption at the young age. The underlying intuition is that reducing old-age consumption allows to free resources, which are transferred to the young age. Transferring resources to the young age contributes to reducing the well-being of the surviving old, but allows also to increase the realized lifetime well-being of the unlucky young individuals who turn out to be short-lived, and, thus, who do not enjoy the old age. This rationale explains why the *ex post* egalitarian optimum involves a consumption profile *decreasing* with age.[15]

Moreover, another important difference with respect to the utilitarian optimum is that the *ex post* egalitarian optimum involves a higher consumption for the dependent elderly than for the healthy elderly, that is, we have $b^e = \tilde{c} > d^e = \bar{c}$. The underlying intuition goes as follows. From the egalitarian constraint, we have $u(d^e) = v(b^e) = 0$. Hence, given that, for a given d, $u(d) > v(d)$, the equalization of lifetime well-being across all agents implies that the dependent elderly should receive more resources than the healthy elderly, that is $b^e > d^e$. This result is different from what prevails under utilitarianism. Under the utilitarian criterion, dependent individuals receive fewer resources than the healthy elderly, because of a lower capacity to convert resources into well-being. On the contrary, from an *ex post* egalitarian perspective, having a lower capacity to convert resources into well-being justifies to receive, as a compensation, not less, but more resources. Regarding the comparison with young age consumption, it remains the case that old-age consumption in case of dependency is below young age consumption ($c^e > b^e = \tilde{c}$), so that, even in case of dependency, the consumption profile remains decreasing with the age.

In the light of the previous results, it appears that the consumption profile, which is here designed in such a way as to maximize the realized well-being of the worst-off, also equalizes *ex post* lifetime well-being levels across all individuals:

$$U^{H,SL} = U^{L,SL} = U^{H,LL,D} = U^{L,LL,D} = U^{H,LL,A} = U^{L,LL,A}.$$

The *ex post* egalitarian optimum allows to neutralize completely inequalities due to Nature, whatever these are caused by unequal productivity, lifetimes or health status at the old age. Proposition 3 summarizes our results.

Proposition 3 *At the ex post egalitarian optimum,*

- *Consumption is higher at the young age than at the old age (independently of the health status).*

[15] On this result, see also Fleurbaey et al (2014).

- *Old-age consumption is higher for dependent individuals than for healthy individuals.*
- *For a given longevity/health status, individuals of type H are, ex post, as well off as individuals of type L.*
- *The healthy long-lived is, ex post, as well off as the unhealthy long-lived, and as well off as the short-lived.*

Proof. See above. ■

The *ex post* egalitarian optimum involves a perfect equalization of realized lifetime well-being across all individuals, whatever these are long-lived or short-lived, and healthy or dependent in case of survival to the old age. This equalization of lifetime well-being across all types is achieved by designing a particular lifetime consumption profile, which is decreasing with the age, but less decreasing for those individuals who turn out to become dependent at the old age (since old-age consumption is larger under dependency than under good health).

Achieving such a perfect equalization of lifetime well-being *ex post* may seem surprising at first glance, since, *ex ante*, the social planner does not know who will reach the old age or not, and who will fall into dependency at the old age or will remain healthy. However, lifetime well-being is nonetheless equalized for all, by constructing a consumption profile that neutralizes the impact of longevity or health inequalities on well-being inequalities. Even if, *ex ante*, it is not possible to know who will be in each situation *ex post*, the fact that all individuals are subject to that consumption profile will necessarily lead to equalize lifetime well-being for all *ex post*. Thus, there is here perfect compensation of individuals for either premature death or for falling into dependency at the old age, in the sense that there is no welfare loss associated with these events.

At this stage, it is important to underline that this full equalization of lifetime well-being for all has a cost. Actually, for the healthy long-lived of a given type, lifetime well-being is higher at the laissez-faire equilibrium than at the *ex post* egalitarian optimum. From the perspective of a lucky healthy long-lived, a more smooth consumption profile is necessarily better. However, if one takes the position of the worst-off individual once the state of nature has been revealed, the *ex post* egalitarian optimum clearly provides the best allocation possible (while satisfying the egalitarian constraints).

6 Second-best analysis

Up to now, we considered a government who could have access to all possible policy instruments and had complete information on individuals' types: productivity and probability of dependency and survival. In that case, only differentiated lump-sum transfers at each period were needed to decentralize the utilitarian and the *ex post* egalitarian social optima.

Such a system of differentiated lump sum transfers being hardly available in real-world economies, this section considers a more realistic second-best setting, where the government can only set uniform policy instruments. We consider here four instruments: a flat tax on earnings τ, a flat tax on aggregate saving (i.e. private annuities and private LTC insurance) σ, a flat pension benefit ψ and a flat LTC benefit g. Taken together, those policy instruments $\{\tau, \sigma, \psi, g\}$ can be regarded as a reduced form - a 4-dimensional - social insurance system.

As before, there exist a perfect annuity market and a perfect LTC insurance market. From the individual's problem, we obtain that optimal saving, LTC insurance and working time are given by, respectively, $s_i^* = s(\psi, g, \tau, \sigma; w_i)$, $a_i^* = a(\psi, g, \tau, \sigma; w_i)$ and $\ell_i^*(\tau; w_i)$, and are determined by the FOCs:

$$-u'(c_i)(1+\sigma) + (1-p_i)u'(d_i) + p_i v'(b_i) = 0 \quad (9)$$

$$-u'(c_i)(1+\sigma) + v'(b_i) = 0 \quad (10)$$

$$u'(c_i)[w_i(1-\tau) - e'(\ell_i)] = 0 \quad (11)$$

with $c_i = w_i\ell_i^*(1-\tau) - (1+\sigma)(s_i^* + a_i^*) - e(\ell_i^*)$, $d_i = s_i^*/\pi + \psi$ and $b_i = s_i^*/\pi_i + a_i^*/\pi_i p_i + \psi + g - S$. Combining these conditions, we obtain that marginal utilities of consumption across time and health states must be equalized:

$$u'(c_i)(1+\sigma) = u'(d_i) = v'(b_i).$$

6.1 The utilitarian problem

The social planner chooses policy instruments τ, σ ψ and g in order to maximize the following Lagrangian:

$$\max_{\tau,\sigma,\psi,g} \mathcal{L} = \sum n_i \left[\begin{array}{l} u\left(\ell_i^* w_i(1-\tau) - (a_i^* + s_i^*)(1+\sigma) - e(\ell_i^*)\right) + \pi_i (1-p_i) u\left(\frac{s_i^*}{\pi_i} + \psi\right) \\ +\pi_i p_i v\left(\frac{s_i^*}{\pi_i} + \frac{a_i^*}{\pi_i p_i} + \psi + g - S\right) \\ +\mu\left[\tau w_i \ell_i^* + \sigma(a_i^* + s_i^*) - \pi_i\psi - \pi_i p_i g\right] \end{array} \right]$$

where μ is the Lagrange multiplier associated with the government's budget constraint and $s_i^* = s(\psi, g, \tau, \sigma; w_i), a_i^* = a(\psi, g, \tau, \sigma; w_i)$ and $\ell_i^* = \ell(\tau; w_i)$ are given by (9), (10) and (11).

Note that interior solutions for s_i^* and a_i^* will only be observed for low levels of public benefits, ψ and g. In the following, we assume that this is effectively the case.[16] Using the Envelop Theorem for s_i^* a_i^* and ℓ_i^* and replacing $\sum n_i$ by the expectation operator $E(\cdot)$, we obtain the following FOCs:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \tau} &= -Eu'(c)w\ell + \mu\left[Ew\ell + \tau Ew\frac{\partial \ell}{\partial \tau}\right] \\ \frac{\partial \mathcal{L}}{\partial \sigma} &= -Eu'(c)(s+a) + \mu\left[E(s+a) + \sigma E\frac{\partial(a+s)}{\partial \sigma}\right] \\ \frac{\partial \mathcal{L}}{\partial \psi} &= E\pi u'(d) - \mu\left[\bar{\pi} - \sigma E\frac{\partial(a+s)}{\partial \psi}\right] \\ \frac{\partial \mathcal{L}}{\partial g} &= Ev'(b)\pi p - \mu\left[E\pi p - \sigma E\frac{\partial(a+s)}{\partial g}\right] \end{aligned}$$

[16] This is a rather strong assumption particularly for low income households but that allows us to get intuitive results.

Combining the last two FOCs, we can write down the derivative of the Lagrangian with respect to the LTC benefit in compensated terms, that is, we calculate the effect of a marginal change of g on the Lagrangian when that change is compensated by a variation of ψ so as to maintain the government's budget balanced.

$$\frac{\partial \tilde{\mathcal{L}}}{\partial g} = \frac{\partial \mathcal{L}}{\partial g} + \frac{\partial \mathcal{L}}{\partial \psi}\frac{d\psi}{dg} = \quad E\pi \; cov(u'(d), \pi p) - E\pi p \; cov(u'(d), \pi) + \mu\sigma \left[E\frac{\partial (a+s)}{\partial g} - \frac{E\pi p}{\bar{\pi}} E\frac{\partial (a+s)}{\partial \psi} \right]$$

where $d\psi/dg = -E\pi p/\bar{\pi}$. To sign the above expression, we use our stylized facts pertaining to the relations between w, πp and π. They imply that the correlation between π and w is positive, and that the correlation between w and πp is negative, which in turn, imply that $cov(u'(d), \pi p) > 0$ and $cov(u'(d), \pi) < 0$.

Let assume, for simplicity, that $\sigma = 0$. In that situation, the above expression would be unambiguously positive. This means that, from a utilitarian perspective, a rise in LTC benefit g compensated by a fall in the pension allowance ψ contributes to increasing social welfare. So, if the government had to choose between LTC benefits and pension benefits, for a given level of government resources, it would have interest in giving priority to the financing of LTC benefits over pension benefits. The reason is that LTC benefits are an inferior good (which benefit first low-productivity / high-dependency individuals), whereas pensions are a superior good (which benefit first high-productivity / high-survival individuals). If σ is not too large, this result is preserved.

In the same way, one can compute the compensated Lagrangian of either ψ or g compensated by τ. The derivative of the compensated Lagrangian with respect to ψ is written as:

$$\frac{\partial \tilde{\mathcal{L}}^{\psi}}{\partial \psi} = \frac{\partial \mathcal{L}}{\partial \psi} + \frac{\partial \mathcal{L}}{\partial \tau}\frac{d\tau}{d\psi} = 0$$

with $d\tau/d\psi = \bar{\pi}/\left[Ew\ell + \tau Ew\frac{\partial \ell}{\partial \tau}\right]$. It yields, after simplifications,

$$\frac{\partial \tilde{\mathcal{L}}^{\psi}}{\partial \psi} = \frac{1}{\Delta}\left[\begin{array}{l} -\bar{\pi} cov(u'(c), wl) + cov(u'(d), \pi)Ew\ell + \sigma\bar{\pi}Ew\ell Eu'(c) \\ +\tau Ew\frac{\partial \ell}{\partial \tau}\left[Eu'(d)\pi\right] + \mu\Delta\sigma E\frac{\partial (a+s)}{\partial \psi} \end{array} \right] = 0$$

where $\Delta = \left[Ew\ell + \tau E\frac{\partial \ell}{\partial \tau}\right]$. Isolating τ, we obtain the level of τ that would give the optimal level of ψ. It is equal to:

$$\tau^{\psi} = \frac{cov(u'(d), \pi)Ew\ell - \bar{\pi} cov(u'(c), wl) + \mu\Delta\sigma E\frac{\partial (a+s)}{\partial \psi} + \sigma\bar{\pi}Ew\ell Eu'(c)}{-Ew\frac{\partial \ell}{\partial \tau}\left[Eu'(d)\pi\right]} \tag{12}$$

As usual in these types of models, the denominator accounts for the efficiency term: increasing taxation increases distortions on labour supply, resulting in lower resources for the government. The first two terms of the numerator account for the redistributive impact of taxation. The last two terms correspond to the total revenue effect of an increase in ψ. The tax rate τ^{ψ} can thus be positive or negative since $cov(u'(c), wl) < 0$, $cov(u'(d), \pi) < 0$ and $E\frac{\partial(a+s)}{\partial\psi} < 0$ so that $\tau^{\psi} \lesseqgtr 0$ depending on the size of the different effects.

Following the same approach for g, we compute the derivative of the compensated Lagrangian with respect to g. It is equal to

$$\frac{\partial\tilde{\mathcal{L}}^g}{\partial g} = \frac{\partial\mathcal{L}}{\partial g} + \frac{\partial\mathcal{L}}{\partial\tau}\frac{d\tau}{dg} = 0$$

where $d\tau/dg = E\pi p/\left[Ew\ell + \tau Ew\frac{\partial\ell}{\partial\tau}\right]$. It yields:

$$\frac{\partial\tilde{\mathcal{L}}^g}{\partial g} = \frac{1}{\Delta}\left[\begin{array}{l} -cov(u'(c), wl)E\pi p + cov(v'(b), \pi p)Ew\ell + \tau Ev'(b)\pi pEw\frac{\partial\ell}{\partial\tau} \\ +\mu\Delta\sigma E\frac{\partial(a+s)}{\partial g} \end{array}\right] = 0$$

so that again, isolating τ, we obtain the level of the tax rate that would be necessary to get the optimal level of LTC benefit, g:

$$\tau^g = \frac{cov(v'(b), \pi p)Ew\ell - cov(u'(c), wl)E\pi p + \sigma Ew\ell Eu'(c)E\pi p + \mu\Delta\sigma E\frac{\partial(a+s)}{\partial\psi}}{-Ew\frac{\partial\ell}{\partial\tau}Ev'(b)\pi p} \tag{13}$$

As a consequence of our stylized facts, we have that $cov(v'(b), \pi p) > 0$ (since high-productivity agents have a lower unconditional probability to be dependent than low-productivity ones) and $cov(u'(c), wl) < 0$, so that τ^g is positive if $\sigma E\frac{\partial(a+s)}{\partial\psi}$ is small enough.

Putting together these computations, one can therefore see that a utilitarian government would always prefer to favour the implementation of a LTC benefit g over a pension benefit ψ. This can be shown both from the arbitrage between g and ψ and from the fact that τ^g is likely to be positive, while nothing ensures that τ^{ψ} would be positive, in a second-best world. Proposition 4 summarizes our results.

Proposition 4 *Let us assume that the government has limited instruments and can only impose uniform taxes on labor and on aggregate savings to finance a uniform pension benefit and a uniform LTC benefit. At the second-best utilitarian optimum:*

- *For a given level of government resources, a uniform LTC benefit should be given priority over a uniform pension benefit.*
- *The tax on labor to finance a pension benefit is equal to equation (12). It can be positive or negative.*
- *The tax on labor to finance a LTC benefit is most likely to be positive and given by equation (13).*

Proof. See above. ■

For a given amount of governmental resources, a utilitarian government gives priority to the LTC benefit over the pension benefit. The intuition behind that result can be derived from the first-best utilitarian problem (Section 4). Utilitarianism leads to redistribute resources from high-productivity individuals to low-productivity individuals. In a second-best setting, that redistribution is achieved through giving priority to the LTC benefit, which is consumed by the dependent elderly (who include a higher proportion of low-productivity individuals in comparison to the elderly in general), over the pension benefit (which is consumed by all long-lived individuals). The priority given to the LTC benefit can thus be interpreted as a second-best way to reduce inequalities between high-productivity and low-productivity individuals.

If one considers now inequalities in lifetime well-being due to unequal lifetime or unequal old-age health status, we know from the first-best utilitarian problem that utilitarianism provides little support for reducing those inequalities. It should be stressed, however, that the priority of the LTC benefit over the pension benefit stated in Proposition 4 has the indirect effect of reducing *ex post* well-being inequalities between the healthy long-lived and the dependent long-lived. Thus the priority given to the LTC benefit tends to reduce inequalities in realized lifetime well-being among the long-lived.

Nevertheless, giving priority to the LTC benefit over the pension allowance does not allow to reduce inequalities in realized lifetime well-being between the long-lived and the short-lived. From that perspective, an important issue that arises concerns the level of the optimal second-best labor tax rate, which directly reduces the purchasing power of the young, and, thus affects first the short-lived.

Finally, let us notice that, while this section focused on linear and uniform instruments, another possibility would be to consider non-linear instruments in a setting wherein the government does not observe the individuals' types, namely their productivity and their survival and dependency probabilities. This alternative second-best utilitarian planning problem was solved by Nishimura and Pestieau (2016). In a two-type framework, they show that assuming a higher probability of survival and a lower unconditional probability of dependency for the more productive, the optimal policy is to subsidize LTC spending and to tax old-age consumption. This result is consistent with what is obtained in a linear setting.

In order to pursue our study on the robustness of optimal policy to the postulated social criterion, Section 6.2 derives a similar second-best problem, but under the *ex post* egalitarian criterion presented in Section 5.

6.2 The ex post egalitarian problem

We now turn to the second-best optimum under the *ex post* egalitarian criterion. The social planner chooses policy instruments τ, σ, ψ and g in order to maximize the lifetime well-being of the worst-off *ex post* (here, the short-lived), subject to the government's budget constraint, and subject to egalitarian constraints, which specify that the long-lived (healthy or dependent) is neither better off nor worse off, in realized terms, than the short-lived. That problem can be written as follows:

$$\begin{aligned}\max_{\tau,\sigma,\psi,g} \quad & u\left(\ell_i^* w_i(1-\tau) - (a_i^* + s_i^*)(1+\sigma) - e(\ell_i^*)\right) \\ & \text{s. to } \sum_i n_i \left[\tau \ell_i^* w_i + \sigma(a_i^* + s_i^*) - \pi_i \psi - \pi_i p_i g\right] \geq 0 \\ & \frac{s_i^*}{\pi_i} + \psi = \bar{c} \\ & \frac{s_i^*}{\pi_i} + \frac{a_i^*}{\pi_i p_i} + \psi + g - S = \tilde{c}\end{aligned}$$

where the first constraint is the government's budget constraint and the last two constraints are the egalitarian ones. Since both $s_i^* = s(\psi, g, \tau, \sigma; w_i)$ and $a_i^* = a(\psi, g, \tau, \sigma; w_i)$ depend on w_i, the only way to satisfy the above egalitarian constraints for both types H and L consists in taxing away both private savings and private LTC insurance benefits (i.e. setting $\sigma = 100\%$) and to give $\psi = \bar{c}$ to the healthy elderly and $g + \psi = \tilde{c} + S$ to the dependent elderly, independently of their type $\{H, L\}$. In that way, no inequality in realized lifetime well-being remains between agents of a given productivity type, w_i. As to the optimal level of τ, it is given by the budget constraint of the government and it is such that[17]

$$\tau E w \ell = \psi \bar{\pi} + g E \pi p.$$

Note, however, that because agents in the first-period of their life have $c_i = \ell_i^* w_i(1-\tau) - e(\ell_i^*)$, there remain *ex post* inequalities between short-lived and long-lived agents of different productivity types:

$$\begin{aligned}U^{i,SL} &= u(\ell_i^* w_i(1-\tau) - e(\ell_i^*)), \\ U^{i,LL,D} &= u(\ell_i^* w_i(1-\tau) - e(\ell_i^*)) + v(\tilde{c}), \\ U^{i,LL,A} &= u(\ell_i^* w_i(1-\tau) - e(\ell_i^*)) + u(\bar{c}).\end{aligned}$$

But, no inequality remains between agents with different length of life or health status at the old age, for a given productivity type i (since $u(\bar{c}) = v(\tilde{c}) = 0$).

Proposition 5 summarizes our results.

Proposition 5 *Let us assume that the government has limited instruments and can only impose a uniform tax on labor and on aggregate savings to finance a uniform pension benefit and a uniform LTC benefit. In the second-best ex post egalitarian optimum:*

- *It is optimal to set LTC and pension benefits such that* $g = \tilde{c} - \bar{c} + S$ *and* $\psi = \bar{c}$.

[17] Since savings and private LTC insurance are taxed away, agents choose neither to save not to invest in a private insurance so that taxation of aggregate savings does not appear in the government budget constraint.

- *No ex-post inequality remains between long-lived and short-lived as well as between healthy and unhealthy individuals of a given productivity type w_i.*
- *There remain ex-post inequalities across agents with different productivity.*

Proof. See above. ■

Proposition 5 can be interpreted by relying on the first-best *ex post* egalitarian optimum characterized in Section 5. From an *ex post* egalitarian perspective, the allocation of resources should be such that inequalities of outcomes caused by Nature - either in terms of life duration or health status at the old age - are neutralized. This motivation explains also the results obtained at the second-best. Indeed, the levels of the pension allowance and of the LTC benefit shown in Proposition 5 are exactly those allowing for the neutralization of the inequalities caused by unequal durations of life or unequal health status among the elderly.

Let us now briefly compare the results we obtain under the utilitarian and the egalitarian criteria. Under utilitarianism, priority is given to the dependent elderly at the expense of the healthy elderly, through a positive LTC benefit. Under *ex post* egalitarianism, priority is given to the prematurely dead, and both the healthy and the dependent elderly are given just enough resources to be exactly as well off as the unlucky short-lived. Also, it is very likely that the fiscal resources (obtained from the taxation of both aggregate savings and labour) necessary to finance $g + \psi$ at the egalitarian optimum are lower than at the utilitarian optimum. The reasons are twofold. First, at the *ex post* egalitarian optimum, consumptions in the old-age (whether the individual is autonomous or dependent) are set at their minimum, $\tilde{c}$ and $\bar{c}$ while at the utilitarian optimum, old-age consumptions are higher. This is a straightforward resource argument. Second, since survival and health conditions are uncertain, one way to compensate the short-lived for their early death is to increase welfare in the first-period, which is lived by all individuals (including those who will turn out to be short-lived). High taxation levels would go against compensation of the short-lived, since they decrease first-period consumption, and, hence, first-period well-being (equal to lifetime well-being for the short-lived).

Finally, in the Appendix, we derive the second-best non linear taxation problem, that is, when the government cannot distinguish between individuals' type. We still find that second-period consumptions when the individual is autonomous (resp. dependent) should be set to $\bar{c}$ (resp. to $\tilde{c}$). This ensures that no inequality remains between individuals of a same productivity type but with different lifetime or health condition. We also obtain the standard result that labour supply of the low type is distorted downward so as to avoid mimicking from the high-productivity type. In addition, c_L and c_H are likely to be different so that *ex post*, lifetime utilities cannot be equalized across productivity types. This is the consequence of asymmetric information.

7 Numerical illustration

Section 6 highlighted important qualitative differences between the optimal second-best policies under the utilitarian and the *ex post* egalitarian criteria. Those differences concerned mainly the levels of LTC and pension benefits, as well as the taxation of labor earnings at the young age. Moreover, Section 6 also provided interpretations of the key forces at work behind those differences. Having stressed this, one may wonder to what extent the different social criteria have strong *quantitative* effects on the design of optimal social insurance.

To explore that issue, this section proposes to impose some functional forms for preferences, and to calibrate our model, in such a way as to examine quantitatively the robustness of the optimal LTC social insurance to the underlying social criterion. For that purpose, we use French data on wages, on LTC expenditures, on survival probabilities and on probabilities to become dependent.

Using INSEE (2015) data on wages, we partition the French population into two categories with monthly wages, $w_H = 3176€$ and $w_L = 1681€$, with $n_H = 0.4$ and $n_L = 0.6$.[18] This leads to an average monthly wage of 2279€.

Regarding the calibration of LTC expenditures S, we use data on the monthly median price of a nursing home in France (CNSA, 2017), and we set $S = 1949€$.

In order to calibrate survival probabilities and (unconditional) probabilities to become dependent, we use the study by Cambois et al. (2011). Assuming that, in our 2-period model, young adulthood lasts from 25 years to 65 years, and old adulthood lasts from 65 years until 105 years, we use data on the life expectancy and on healthy life expectancy at age 65 to derive the probabilities:[19]

$$\begin{aligned} \pi_H &= 0.46 > \pi_L = 0.41 \\ p_H &= 0.60 < p_L = 0.67 \end{aligned}$$

Regarding preferences, we assume that temporal well-being takes a logarithmic form:

$$\begin{aligned} u(x) &= \log(x) + \alpha \\ v(x) &= \delta \log(x) + \beta \end{aligned}$$

To calibrate preference parameters $\{\alpha, \beta, \delta\}$, we proceed in three steps. First, following Becker at al (2005), we assume that $\bar{c} = 300/12€$, where 300€ is considered as the critical annual consumption that would make individuals indifferent between survival and death (less than 1€ a day). Normalizing the utility of being dead to zero, this allows us to derive α from the equation $\log(\bar{c}) + \alpha = 0$. This leads to $\alpha = -3.218$.

[18] See Tables 1 and 6, in INSEE (2015). We consider that high-qualification and intermediary occupations as type-H individuals and, clerks and manual workers as type-L individuals.

[19] See Table 2 (male column) of Cambois et al. (2011). Using the weights for the different socio-demographic categories used in the previous step, we aggregate life expectancy at 65 and life expectancy at 65 without daily-life restrictions for each type H and L, and obtain probabilities by dividing these by the length of the period (i.e. 40 years).

Second, we use Finkelstein et al. (2013) to calibrate the marginal utility of consumption under dependency. Finkelstein et al. (2013) shows that marginal utility of consumption under bad health (chronic diseases) is evaluated to be between 75% and 90% of the marginal utility under good health. Using this as a proxy for the marginal utility under dependency, we assume that $\delta = 0.9$.

Third, regarding the calibration of β, which captures the reduction of well-being due to loss of autonomy, we use the study by Ferrer-i-Carbonell and Van Praag (2002). That study shows that heart diseases is, from a welfare perspective, equivalent to a decrease in income equal to 47%. Using this as a proxy for the welfare loss due to becoming dependent, we obtain $\beta = -3.180$.[20]

Finally, we obtain the monthly minimum level of consumption under dependency, $\tilde{c}$ by solving the following equation: $\delta \log(\tilde{c}) + \beta = 0$.

Table 1 summarizes the values for our parameters.

Parameters	w_L	w_H	n_L	n_H	S	α	β	δ	$\bar{c}$	$\tilde{c}$
Value	1681	3176	0.6	0.4	1949	-3.218	-3.180	0.900	25	34.251

Table 1: Calibration of parameters

Our calibrated model is then used to derive the second-best optimal social insurance system under the utilitarian criterion and the *ex post* egalitarian criterion. More precisely, our computations allow us to derive the optimal levels for the four policy parameters studied in the previous section, $\{\tau, \sigma, \psi, g\}$, under the two alternative normative criteria. Our results are summarized in Table 2, which shows the optimal values of the four policy instruments, as well as the associated levels of realized lifetime well-being for the 6 types of individuals *ex post*.[21]

	SB utilitarian	SB ex post egalitarian
τ^*	42.43 %	24.12 %
σ^*	9.24 %	100.00 %
ψ^*	1060.82	25.00
g^*	1883.09	1958.25
total revenue	974.42	549.70
$U^{L,SL}$	3.66	3.93
$U^{H,SL}$	4.16	4.57
$U^{L,LL,D}$	6.69	3.93
$U^{H,LL,D}$	7.52	4.57
$U^{L,LL,A}$	7.40	3.93
$U^{H,LL,A}$	8.26	4.57

Table 2: Numerical results

[20] β solves: $\beta = \alpha - \delta \log(d) + \log(0.53d)$ with α and δ already obtained in previous steps and where $d = w_L/2$ (i.e. half of the income of the low wage individual).

[21] For simplicity, our calculations assume that individual labour supply is fixed. This implies some second-order changes on the government budget constraints and on the first-order condition with respect to τ at the second-best utilitarian optimum. Relaxing that assumption would not modify our results substantially.

Table 2 shows that relying on the utilitarian criterion or on the *ex post* egalitarian criterion has strong quantitative effects on the design of the optimal policy.[22] Some of those results could have been anticipated from our theoretical analysis of the second-best problem, but our calculations allow us to go beyond qualitative observations, to quantify the lack of robustness of the optimal policy to the postulated social criterion.

A first difference lies in the global level of taxation: fiscal revenues are significantly larger under the utilitarian second-best, since utilitarianism leads to spend more resources on the long-lived (both autonomous and dependent).

But besides the difference in the overall level of taxation, there is also a substantial difference in the structure of taxation. The *ex post* egalitarian second-best involves, in comparison to the utilitarian second-best, a much lower tax rate on the young adults's labor earnings (24 % against 42 %), as well as a higher tax rate on savings and private LTC insurance (100 % against 9 %).

As a consequence, consumption at the young age is higher under *ex post* egalitarianism than under utilitarianism (independently of types $i = \{H, L\}$). As explained above, favoring high consumption at the young age contributes to improve the situation of all the young, including those who will turn out to be short-lived. Higher consumption at the young age allows short-lived individuals to be better off at the *ex post* egalitarian optimum than at the utilitarian one.

Another important difference concerns the levels of old-age pensions and LTC benefits. Under utilitarianism, more resources are devoted to long-lived individuals (both autonomous and dependent), through the level of the pension benefit ψ: $\psi^E > \psi^U$ and $\psi^E + g^E > \psi^U + g^U$.[23] Overall, long-lived agents (either autonomous or dependent) have higher lifetime well-being at the utilitarian optimum than at the *ex post* egalitarian optimum.[24]

The reason why the *ex post* egalitarian solution involves fewer resources spent on the elderly follows logically from the underlying goal of compensating the unlucky short-lived. Clearly, dedicating fewer resources to the old makes these worse-off than under utilitarianism, but this allows to improve the situation of young individuals, and, hence, the situation of the unlucky individuals who turn out to be short-lived. Actually, Table 2 shows that the *ex post* egalitarian solution can achieve the full equalization of realized lifetime well-being levels for the short-lived and the long-lived with the same productivity level.

The utilitarian and the *ex post* egalitarian policies have also different effects concerning well-being inequalities among the long-lived. At the utilitarian optimum, individuals who are autonomous at the old age are better off than those who fall into dependency. On the contrary, at the egalitarian solution, the larger level of the LTC benefit g leads to equalize the well-being levels of the

[22] Note that, at the utilitarian second-best, we have corner solutions: $s_H^* > 0$ while $s_L^* = 0$ and $a_i^* = 0 \forall i$.

[23] Note also that, in line with Section 6.1, utilitarianism gives priority to the LTC benefit over the pension benefit.

[24] In unreported simulations,we show that, conditional on being long-lived, a utilitarian government chooses to favour more, in global terms, the autonomous agent than the dependent one (independently of his type $i = \{H, L\}$) since $d_i > b_i \forall i$.

autonomous and the dependent elderly (for a given productivity level). Thus the *ex post* egalitarian criterion leads to fully compensate the dependent elderly.

All in all, our numerical simulations confirm that the design of the optimal policy is not robust to the postulated social criterion. The *ex post* egalitarian second-best involves, in comparison to utilitarianism: (i) a lower tax rate on labor earnings; (ii) a higher tax rate on aggregate savings; (iii) lower pension benefits; (iv) higher LTC benefits. Moreover, the levels of the optimal fiscal instruments are shown to be strongly sensitive to the social criterion. Adopting one social criterion or another has sizeable quantitative effects on the design of the LTC social insurance.

8 Conclusions

What would be a fair LTC social insurance? That question is complex, since the construction of a fair LTC social insurance raises several difficulties. Among those difficulties, a crucial issue concerns the fact that individuals in situation of dependency do not have the same preferences as individuals who are autonomous and healthy. Moreover, another difficulty is that the fairness of a LTC social insurance should be valued not only by comparing the situations of healthy and dependent elderly individuals, but, also, by taking into account the situation of the unlucky prematurely dead individuals who do not face the risk of being dependent at the old age, simply because they do not reach the old age.

In order to address that issue, this paper developed a lifecycle model with risk about the duration of life and about the health status at the old age (autonomy or dependency), while allowing for state-dependent preferences at the old age. In order to examine the robustness of the optimal LTC social insurance to the underlying social welfare criterion, we contrasted two social optima: on the one hand, the standard utilitarian criterion, and, on the other hand, the *ex post* egalitarian criterion, which gives priority to the worst-off in realized terms.

Our motivation for considering that alternative social criterion lies in the fact that utilitarianism tends, under general conditions, to penalize short-lived individuals and the elderly dependent, on the grounds of their lower capacity to convert resources into well-being. We consider that feature of utilitarianism to lead to quite unfair social outcomes *ex post*. On the contrary, the *ex post* egalitarian social criterion tends, by construction, to do justice to the idea of compensating individuals for damages due to Nature, such as a premature death or old-age dependency.

Our analysis shows that the optimal LTC social insurance depends strongly on the adopted social welfare criterion. If one adopts a utilitarian objective, the dependent elderly should receive fewer resources than the healthy elderly, since the latter exhibits a higher capacity to convert resources into well-being. On the contrary, under the *ex post* egalitarian social criterion, this is the opposite, and more resources should be allocated to the dependent elderly than to the healthy elderly, so as to compensate him for his dependency.

The postulated normative criterion has also crucial consequences for the overall shape of consumption profiles. Whereas the optimal consumption age profile should be, under utilitarianism, flat in the absence of old-age dependency and decreasing under old-age dependency, it would be, under the *ex post* egalitarian criterion, decreasing with the age (with a stronger slope for the healthy elderly). The underlying intuition is that a consumption profile decreasing with the age pushes towards more resources being allocated to the young, who include those unlucky individuals who will turn out to be short-lived. Hence compensating individuals for arbitrarily short lives encourages more decreasing consumption profiles with the age.

We also carried out a second-best analysis with uniform policy instruments. The optimal policy was shown to vary significantly with the postulated social objective. In comparison with utilitarianism, the *ex post* egalitarian solution leads to a higher LTC benefit, a lower pension benefit, a lower tax on young's labor earnings, and a higher tax on aggregate savings. Those policies allow to neutralize inequalities in lifetime well-being between the long-lived and the short-lived, and, also, between the healthy elderly and the dependent elderly, unlike under utilitarianism. Hence, if neutralizing well-being inequalities due to the arbitrariness of Nature is an essential component of what a "fair" LTC insurance system is, then the design of social insurance should depart, on many dimensions, from what utilitarianism recommends.

All in all, this paper suggests that the construction of a fair LTC social insurance depends strongly on the postulated normative foundations. The relative size of the optimal LTC social benefit - as well as the overall burden of the funding of LTC - is not invariant at all to the social objective that is pursued. A corollary of this is that public debates on the future of LTC social insurance should not only focus on behavioral and technical issues, but should also discuss normative foundations. If one wants to treat the dependent elderly in a fair way, one can hardly leave these normative foundations aside. At the end of the day, how a society treats its elderly dependent members is a matter of social choice.

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Appendix

A Second best non linear taxation

Let us now consider the non linear taxation case. In the first best (see Section 5), we had $c_i = c^e$, $d_i = \bar{c}$, $b_i = \tilde{c}$ and $\ell_H > \ell_L$ so that if the planner were to propose the FB contracts, only type-H individual would be tempted to mimic type-L individuals. The second best problem then consists in adding to the first best problem (9) the following incentive constraint:

$$u\left(x_H - e\left(\frac{y_H}{w_H}\right)\right)+\pi_H[u(d_H)p_H+(1-p_H)v(b_H)] \geq u\left(x_L - e\left(\frac{y_L}{w_H}\right)\right)+\pi_H[u(d_L)p_H+(1-p_H)v(b_L)]$$

where $y_i = w_i \ell_i$. To insure perfect equality between agents with same productivity but different length of life and health status, one can still set $d_i^{**} = \bar{c}$ and $b_i^{**} = \tilde{c} + S \ \forall i$. The incentive constraint therefore simplifies to:

$$u\left(x_H - e\left(\frac{y_H}{w_H}\right)\right) \geq u\left(x_L - e\left(\frac{y_L}{w_H}\right)\right).$$

and the Egalitarian problem writes:

$$\begin{aligned} \max_{\ell_i, c_i, d_i, b_i} \quad & u\left(x_L - e\left(\frac{y_L}{w_L}\right)\right) \\ \text{s.t.} \quad & \sum n_i y_i = \sum n_i \left[x_i + \pi_i(1-p_i)\bar{c} + \pi_i p_i(\tilde{c} + S)\right] \\ \text{s.t.} \quad & u\left(x_H - e\left(\frac{y_H}{w_H}\right)\right) \geq u\left(x_L - e\left(\frac{y_L}{w_H}\right)\right). \end{aligned}$$

As usual, we find the result of no distortion at the top:

$$e'\left(\frac{y_H}{w_H}\right)\frac{1}{w_H} = 1$$

In contrast, we obtain that the labour supply of a type-L individuals should be distorted downward:

$$e'\left(\frac{y_L}{w_L}\right) = w_L \frac{1 - \mu\frac{u'(c_{L,H})}{u'(c_L)}}{1 - \mu\frac{u'(c_{L,H})}{u'(c_L)}\frac{e'(y_L/w_H)w_L}{e'(y_L/w_L)w_H}} < w_L$$

where $c_{L,H} = x_L - \frac{y_L}{w_H}$.