Premature deaths, accidental bequests and fairness*

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Abstract

While little agreement exists regarding the taxation of bequests in general, there is a widely held view that accidental bequests should be subject to a confiscatory tax. We reexamine the optimal taxation of accidental bequests in an economy where individuals care about what they leave to their offspring in case of premature death. We show that, whereas the conventional 100 % tax view holds under the utilitarian social criterion, it does not hold under the ex post egalitarian criterion (giving priority to the unlucky short-lived). From the perspective of compensating the short-lived, it is optimal not to tax, but to subsidize accidental bequests. We examine the robustness of those results in a dynamic OLG model of wealth accumulation, and show that, whereas the sign of the optimal tax depends on the form of the joy of giving motive, it remains true that the 100 % tax view does not hold under the ex post egalitarian social criterion. Finally, we provide a second-best egalitarian argument for taxing bequests at a rate that is increasing with the age of the deceased.

Keywords: mortality, accidental bequests, optimal taxation, egalitarianism, OLG models.

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1 Introduction

The taxation of bequests constitutes one of the most debated topics in public finance. The main reason why little agreement exists on this probably lies in the large number of issues raised by wealth transmission, which involve both the interests of the deceased, of his offspring, but also the whole distribution of wealth at the society level.

Those difficulties are illustrated by Mill's early works on inheritance taxation. In his *Principles of Political Economy* (1848), Mill argued that bequests go against the ideal of free competition, since these create an initial - and arbitrary - inequality among competitors. However, according to Mill, when there is a testimony of the deceased, one should respect the deceased's will to give.[1] To reconcile this with the ideal of free competition, Mill's solution consists in imposing an absolute limit on the amount of money that a man may inherit. Doing so, Mill respects the will of the deceased to give, under the constraint of limiting the induced inequalities thanks to the cap on received bequests.

When bequests are purely accidental (in case of early death), the second dimension mentioned by Mill is absent: no will was expressed by the deceased. Hence, in that case, there is a stronger support for a confiscatory tax on bequests. This may explain why the conventional view in public finance is that accidental bequests should be subject to a confiscatory tax (see Kaplow 2008).

To our knowledge, two objections were raised against that widely held view. First, Blumkin and Sadka (2003) argue that a non-confiscatory tax on accidental bequests has the desirable consequence of making the demogrant of an optimal linear income tax system effectively non-uniform. It will act as an additional instrument and increase the efficiency of the tax system. Second, Cremer et al. (2012) observe that bequests, when publicly observable, have informational content if they are correlated with relevant characteristics of taxable agents. This content must be incorporated in the design of optimal tax structures. This point challenges the 100 % tax view on accidental bequests.[2]

In this paper, we raise another objection against the 100 % tax view. That objection relies on the fact that risk about the timing of death generates inequalities not only among the descendants of the deceased (inequalities in bequests), but, also, among the deceased themselves (inequalities in consumption and longevity).[3] We reexamine the optimal taxation of accidental bequests in the context of risky and unequal lifetimes, and we show that, provided (i) individuals have preferences on how lost saving is distributed in case of premature death; (ii) governments care about the deceased's interests in giving; (iii) governments want to equalize realized lifetime well-being across individuals with unequal lifetime, then the 100 % tax view on accidental bequests does not hold.

[1] The underlying justification lies in Mill's adherence to the principle of free disposal of one's goods when being alive and even after one's death.

[2] Note that this point is general, and applies to all types of bequests.

[3] One could also mention inequalities in time spent with other generations in the family (a long life makes it possible for several generations to coexist longer), but we will focus here on the self-centered value of consumption and longevity.

Given that assumptions (i) to (iii) play a crucial role in our reassessment of the 100 % tax view on accidental bequests, it is worth explaining the reasons why our analysis departs from the existing literature on those dimensions.

One may argue against assumption (i) that preferences on how lost saving is distributed do not exist, precisely because those bequests are "accidental". However, it should be stressed that, from the perspective of the deceased, it is inaccurate to talk about "accidental bequest".[4] Death is the surest thing that happens to everyone, only its timing is unknown. Therefore any rational agent should have contingent plans about the use of her wealth after her death, at all times during life. The absence of an expressed will does not mean the absence of preferences about how the bequest is distributed. This paper assumes that, in case of (accidental) premature death, individuals benefit from the fact that their lost savings go to their children —following Mill's idea that the deceased have preferences about what happens to their wealth. The idea is not really different from the standard "joy of giving" (Andreoni 1990), even if the latter is generally understood as a joy of giving when being *alive*. In our model individuals enjoy giving to their offspring while being alive, but would, in case of premature death, enjoy giving their lost savings to their offspring.

Assuming a joy of giving makes preferences interdependent, which raises deep questions at the normative level. This leads us to point (ii). Should such interdependent preferences be taken into account in a social planning problem? Two opposite answers can be found in public economics (see Boadway 2012). On the one hand, some authors, such as Hammond (1987) and Mirrlees (2007, 2011), argue that joy of giving utility should not enter the social welfare function, because it would lead to double counting. On the other hand, other authors, such as Kaplow (1995, 2008), argue that, from a purely welfarist perspective, all aspects of individual preferences deserve to be taken into account when defining social welfare.[5] In this paper, we adopt the latter, purely welfarist, approach as a benchmark, and regard the deceased's interest in giving as one aspect of his preferences that should be taken into account in the social planning problem.

This paper departs also from the literature on accidental bequests taxation regarding the social welfare criterion used (assumption (iii)). Whereas the existing literature relies generally on the utilitarian criterion, we consider also another social welfare criterion, i.e. the ex post egalitarian criterion, which gives more weight to the unlucky short-lived individuals. The reason why we consider an alternative criterion is that the standard utilitarian criterion is inappropriate when considering inequalities in the duration of life, since it leads to counterintuitive redistribution from short-lived towards long-lived persons.[6] This is problematic, since a large part of longevity differentials is not under

[4] In this paper, we will follow the literature and use the term "accidental bequests", in the sense of a bequest that arises only because of the premature death of the donor, but that would not have occurred provided the donor had enjoyed a longer life.

[5] That perspective was also adopted in recent papers, such as Farhi and Werning (2013).

[6] Given that short-lived individuals have, due to the concavity of temporal utility, a lower capacity to convert resources into well-being than long-lived agents, the short-lived are usually penalized by a utilitarian planner, against any intuition of compensation (Fleurbaey et al 2014).

the control of individuals, but depends on external circumstances (e.g. genetic background), and, as such, deserves some compensation.[7] This leads us to consider here the ex post egalitarian criterion, which assigns a strong weight to the welfare of unlucky short-lived individuals (see Fleurbaey et al 2014).

The objective of this paper is to study the design of an optimal taxation of accidental bequests while paying a particular attention to the treatment of the deceased's interests in giving, and while adopting a social welfare criterion that is fair to the unlucky short-lived. For that purpose, we proceed in three stages. We first consider a static model composed of *ex ante* identical individuals who face risky lifetime, and whose initial endowments are equal. By doing so, we deliberately abstract from inequalities among the offspring, in order to focus only on inequalities between the long-lived and the short-lived. We compare the utilitarian optimum with the social optimum that equalizes lifetime utilities, and study the decentralization of those optima by means of a tax on bequests. Then, in a second stage, we extend this model to a dynamic overlapping generations (OLG) economy, where individuals enjoy unequal endowments due to longevity inequalities among their ancestors. Finally, in a third stage, we relax the assumption that the government can tax the unconditional and the accidental components of bequests at different rates, and consider a second-best setting where bequests are taxed at rates varying with the age of the deceased.

Anticipating our results, we show that, although the conventional 100 % tax view holds under the utilitarian social criterion, it does not hold any more under the ex post egalitarian criterion. It is also shown that the decentralization of the ex post egalitarian optimum requires not to tax, but to *subsidize* accidental bequests. Thus the 100 % tax view is not a good idea once it is acknowledged that prematurely dead individuals care about giving their lost savings to their children, and once one wants to be fair with respect to the unlucky short-lived.

The intuition behind our results goes as follows. If individuals have preferences on how lost saving is distributed in case of their premature death, accidental bequests do not only affect the distribution of well-being among inheritors: they also affect the distribution of well-being between short-lived and long-lived persons. When the use of lost savings matters for the prematurely dead person, accidental bequests can be regarded as a way to bring indirect compensation to the unlucky short-lived, beyond the consumption she can enjoy when being alive. Thus, when the social welfare criterion gives a strong weight to the interests of the unlucky short-lived, it is not optimal to tax accidental bequests at a 100 % rate. Such a tax would reinforce the extent of the deprivation caused by a premature death, and, hence, would raise well-being inequalities between the long-lived and the short-lived. On the contrary, since accidental bequests contribute to compensating the unlucky short-lived, it is optimal, from an egalitarian perspective, to subsidize these.

Note that, when our setting is extended to a dynamic OLG economy, whether it is optimal to tax or subsidize accidental bequests depends on the precise form

[7] Actually, genetic background was shown to explain between 1/4 and 1/3 of longevity inequalities within a given cohort (see Christensen et al 2006).

of the joy of giving motive (i.e. whether parents are interested in what they give to their children or, alternatively, in what their children receive net of all taxes and transfers). However, whatever the precise definition of the joy of giving motive is, it remains true that, under the ex post egalitarian criterion, the 100 % tax view does not hold. Thus adopting an ex post egalitarian social criterion definitely challenges the 100 % tax view on accidental bequests.

Whereas the above results assume that the government can tax the unconditional part and the accidental part of bequests at different rates, we relax that assumption in the last stage of our analysis, and consider a second-best setting where the government cannot tax unconditional and accidental bequests at different rates, but can only tax bequests at rates varying with the age of the deceased. We show that, whereas the utilitarian criterion recommends to tax bequests at a rate that is *decreasing* with the age of the deceased, the opposite prevails under the ex post egalitarian criterion, which recommends, on the grounds of the compensation of the unlucky short-lived, to tax bequests at a rate that is *increasing* with the age of the deceased.

Our paper contributes to the literature on several grounds. First, we supplement the recent literature on the optimal taxation of inheritance, such as Blumkin and Sadka (2004), Cremer et al (2012), Farhi and Werning (2013) and Piketty and Saez (2013). Whereas those papers focus on productivity differentials, our paper concentrates on another source of heterogeneity - the duration of life - and pays a particular attention to the treatment of the unlucky short-lived. In particular, this paper complements the seminal study of Vickrey (1945), by providing an alternative argument for a tax on bequests that is increasing with the age of the deceased. Second, our paper contributes also to the normative literature on the compensation of short-lived persons (Fleurbaey et al 2014, Fleurbaey et al 2016), by considering accidental bequests as an instrument for the compensation of the prematurely dead. Third, our paper, which assumes the absence of annuity markets, is also related to the literature on the annuity puzzle (Yaari 1965, Brown 2004, Davidoff et al 2005). We show that annuities have a double distributive role: they equalize the situation of all young individuals *ex ante* (by abolishing accidental bequests), but they exacerbate inequalities in lifetime well-being (i.e. *ex post*), by redistributing the savings of the unlucky short-lived to the lucky long-lived.

The paper is organized as follows. The baseline model is presented in Section 2, which studies the laissez-faire equilibrium with and without policy instruments. The utilitarian optimum and its decentralization are studied in Section 3. Section 4 examines the ex post egalitarian optimum and its decentralization. Section 5 extends our model to an OLG economy. Section 6 studies, in a second-best setting, the optimal taxation of bequests, while allowing for a tax rate on bequests that varies with the age of the deceased. Section 7 concludes.

2 The baseline model

Let us consider a two-period economy with risky lifetime. The duration of each period is normalized to 1. The population is a continuum of agents of size 1. The first period is the young age, during which individuals supply one unit of labour inelastically, have one child, consume and save for their old days. The second period, i.e. the old age, is a period during which agents consume their savings. Lifetime is risky: the old age is reached with a probability π $(0 < \pi < 1)$.

Throughout this paper, we distinguish between two kinds of transfers/bequests from parents to children.[8]

On the one hand, parents plan, at the beginning of their life, to give to their children a gift b. That transfer is given independently of the age at which the parent dies, that is, independently of whether the parent reaches the old age or not. This "non-accidental" bequest is the unconditional component of the parental bequest.

On the other hand, parents, in case of death before reaching the old age, transmit to their child the amount d that would have been consumed in case of survival. We call that transfer "accidental bequest". It is the component of parental bequest that is conditional on the parent's longevity.

Thus, in case of a long life, an individual transmits an amount b to his child, whereas, in case of premature death, an individual transmits an amount $b + d$, which includes an unconditional component b and an accidental component d.

2.1 Main assumptions

2.1.1 Preferences

Individual preferences are additive over time and satisfy the expected utility hypothesis. The utility of being dead (in the second period) without transmitting anything is normalized to 0. It is supposed that individuals derive some welfare from giving to their descendant. The utility function has the general form:

$$u(c) + \pi \left(u\left(d\right) + v(b)\right) + (1 - \pi)v\left(d + b\right), \tag{1}$$

where c and d denote first- and second-period consumptions. The period utility function $u(\cdot)$ is increasing and concave (i.e., $u'(\cdot) > 0$, $u''(\cdot) < 0$). We suppose that there exists a consumption level neutral for the continuation of existence $\bar{c} \geq 0$ such that $u(\bar{c}) = 0$. That consumption level $\bar{c}$ brings the same level of utility as being dead and giving nothing to the descendants.

The function $v\left(d\right)$ captures the joy of giving.[9] As stated above, when the individual reaches old age (with probability π), the bequest equals b. When the individual dies prematurely (with probability $1 - \pi$), the bequest is $d + b$, since the planned second-period consumption is also transferred to the offspring.

[8] Both transfers are received by the child in the first period of his life.

[9] Whereas assuming a joy of giving in case of premature death seems *a priori* to be a strong assumption, this can be regarded as a simple way to rationalize the low demand for annuitization, in line with the annuity insurance puzzle (Brown 2004, Davidoff et al 2005).

The joy of giving is distinct from either pure altruism or truncated altruism. Pure altruism would require parents to care about the total well-being of their children, which is not the case here. Truncated altruism would require parents to value the gifts they make to their children through their children's own utility of consumption, $u(c)$, which is not the case here.

We assume: $v(0) = 0 > u(0)$, $v(d) > 0$ under $d > 0$, $v'(\cdot) > 0$ and $v''(\cdot) < 0$. Following Hurd (1989), we assume that the marginal utility from giving is always lower than the marginal utility of consumption for a given amount, that is:

$$u'(d) > v'(d) \; \forall d \geq 0. \tag{2}$$

In this paper, we assume that there exists a level of $d > 0$, denoted $\tilde{d}$, such that $u\left(\tilde{d}\right) = v\left(\tilde{d}\right)$. $\tilde{d}$ is such that, in the absence of gift (i.e., $b = 0$), the utility from being alive at the old age and consuming $\tilde{d}$ is exactly equal to the utility from being dead at the old age and giving $\tilde{d}$ to the descendant.

When $d > \tilde{d}$, we have $u(d) > v(d)$, that is, in the absence of gift, a person would prefer surviving and consuming d at the old age rather than dying prematurely and giving accidentally d to his offspring. On the contrary, when $d < \tilde{d}$, we have $u(d) < v(d)$, that is, in the absence of gift, a person would prefer dying prematurely and giving accidentally d to his offspring rather than surviving and consuming d at the old age. The latter case, where a person prefers dying prematurely to surviving, concerns extremely poor economies (with very low productivity). Given that this latter case is extreme, we will, throughout this paper, assume that the economy is sufficiently affluent so that $d > \tilde{d}$.

An important corollary of assuming $d > \tilde{d}$ concerns the comparison of short-lived and long-lived individuals. Actually, under $d > \tilde{d}$, it is also the case, under our assumptions on $u(\cdot)$ and $v(\cdot)$, that for any $b > 0$, we have $v(b + d) < v(b) + u(d)$, that is, the prematurely dead are worse-off than the long-lived, which is a reasonable assumption in affluent economies.[10]

2.1.2 Markets

Throughout this paper, it is assumed that the labor market is perfectly competitive, and that workers are paid at a wage rate $w > 0$. Their labor supply is inelastic and their earnings equal w. Regarding the capital market, we suppose that savings bring a return R equal to 1 plus the interest rate. For simplicity, we assume, in the baseline model, that $R = 1$, i.e., a zero interest rate.

When considering economies with risky lifetime, it is common, after Yaari (1965), to assume that the economy includes perfectly competitive annuity markets, yielding an actuarially fair return. The function of those markets is to insure individuals against the risk of a long life. Given that we would like here to consider the issue of accidental bequests, we will suppose that annuity markets

[10] To see this, let us assume that $d = \tilde{d}$. The utility of the long-lived in period 2 is $u(\tilde{d}) + v(b) = v(\tilde{d}) + v(b)$, whereas the utility of the short-lived in the second period is $v(b + \tilde{d})$. But since, by concavity, $v(\tilde{d}) + v(b) > v(b + \tilde{d})$, it follows that $u(\tilde{d}) + v(b) = v(\tilde{d}) + v(b) > v(b + \tilde{d})$. A fortiori, when $d > \tilde{d}$, we have $u(d) + v(b) > v(d) + v(b) > v(b + d)$.

do not exist in the laissez-faire.[11] This assumption is in line with the empirical literature on the so-called annuity insurance puzzle (Brown 2004, Davidoff et al 2005). In the absence of annuities, individuals will then, in case of premature death, transmit the proceeds of their savings to their offsprings. As stated above, this transmission is valued by the donors through the function $v(\cdot)$.

2.1.3 Budget constraints

The first-period and second-period budget constraints are, respectively:

$$c = w - s - b + b_0 \tag{3}$$
$$d = s \tag{4}$$

where s denotes savings for future consumption, b denotes the gift the agent is willing to give to his own child, while b_0 denotes the bequest received from the parents when being young.

As a consequence of the absence of annuities, a young generation is composed of individuals with quite different endowments, depending on how large the received bequest b_0 is. However, for the sake of presentation, the baseline model examined in the first part of this paper deliberately abstracts from inequalities in initial endowments and sets b_0 to 0, in order to focus first on inequalities in lifetime well-being between short-lived and long-lived individuals, and on the impact of accidental bequests on those inequalities. Section 5 will then reintroduce inequalities in initial endowments due to unequal received bequests.

2.2 The laissez-faire

The problem of a representative individual consists in allocating his resources along his life, so as to maximize his expected lifetime utility subject to the resource constraints and the market factor prices.

That problem can be written as:

$$\max_{s,b} u(w - s - b) + \pi (u(s) + v(b)) + (1 - \pi) v(s + b)$$

The first-order condition (FOC) for optimal savings, s is given by:

$$u'(w - s - b) = \pi u'(s) + (1 - \pi) v'(s + b) \tag{5}$$

and the FOC for the gift, b is:

$$u'(w - s - b) = \pi v'(b) + (1 - \pi) v'(s + b) \tag{6}$$

The first FOC equalizes, at the margin, the welfare loss from savings due to a decrease in consumption at the young age (LHS) and the welfare gain from

[11] Similarly, we assume that there is no private life insurance market at the laissez-faire. Note that our results would still hold with annuities or life insurance characterized by high loading costs.

savings due either to a larger consumption at the old age in case of survival (first term of the RHS), or to a larger bequest left to the descendant in case of premature death (second term of the RHS). The second FOC equalizes the marginal loss of utility induced by a lower consumption due to a larger gift (LHS) and the marginal joy of giving (RHS).

Obviously, if individuals do not care about giving to the descendant ($v \equiv 0$), then $b = 0$ and one obtains the standard Euler equation:

$$u'(w - s) = \pi u'(s)$$

Note that, even in this case, consumption is not smoothed along the lifecycle, because of the absence of annuities. Alternatively, in the presence of actuarially fair annuities, the return on savings would be $\frac{1}{\pi}$ and the FOC would be reduced to $u'(w - s - b) = u'(s/\pi)$ implying $c = d$. This is not the case in the absence of annuities, where we have, given $\pi < 1$, $u'(s) > u'(w - s - b)$, implying $c > d$.

When individuals care about giving to their descendants in case of premature death ($v \neq 0$), the consumption profile is, under our assumptions, necessarily decreasing with age. Proposition 1 summarizes the key facts.

Proposition 1 *At the pure laissez-faire, consumption decreases over the life cycle ($c > d$), and the gift is smaller than old-age consumption ($b < d$). The long-lived is better off than the short-lived.*

Proof. Regarding the consumption profile, note that $u'(w - s - b)$ is a linear combination of $u'(s)$ and $v'(s + b)$ with $u'(s) > v'(s + b)$ from our assumptions on the forms of $u(\cdot)$ and $v(\cdot)$. Therefore, $u'(w-s-b) < u'(s)$ and $c = w-s-b > d = s$. Comparing the FOCs for s and for b , we obtain that $u'(s) = v'(b)$, which under our assumptions on the forms of $u(.)$ and $v(.)$, implies that $b < s = d$.

Regarding well-being inequalities, we know that, if $d > \tilde{d}$, the short-lived is worse off than the long-lived,

$$U^{LL} = u(c) + u(d) + v(b) > U^{SL} = u(c) + v(d + b)$$

since $v(b + d) < v(b) + u(d)$. ■

Proposition 1 states that, at the laissez-faire equilibrium, the long-lived are better off than the short-lived. This inequality arises despite the fact that the prematurely dead derive some well-being from having the proceeds of their lost savings being redistributed towards their offsprings. The reason is that, in affluent economies, we have $u(d) + v(b) > v(d + b)$, which implies that the well-being of long-lived persons living at the old age is larger than the well-being derived by the short-lived from transmitting their lost savings to their offspring.

Thus, although our individuals are all identical *ex ante* (i.e. before the duration of life is revealed), and make similar decisions, those persons turn out, *ex post* (i.e. after the duration of life is revealed), to enjoy quite different lifetime well-being levels, because of unequal lifetimes.

2.3 Introducing policy instruments

Having presented the laissez-faire, let us now introduce some standard policy instruments, in order to examine their redistributive impact, that is, their effect on lifetime well-being inequalities between the long-lived and the short-lived. For that purpose, this subsection considers three instruments: first, a collective annuitization of savings through public actuarially fair annuities to a fixed degree $\alpha \in [0, 1]$; second, an actuarially fair public life insurance scheme a; third, a tax θ on accidental legs (tax revenues being redistributed at the young age).

Under those three instruments, the agent chooses savings (net of gifts) s and gifts b so as to maximize expected lifetime well-being:

$$\max_{s,b} u(c) + \pi [u(d) + v(b)] + (1 - \pi)v(e + b)$$

with

$$\begin{aligned} c &= w - b - s - a + T \\ d &= \alpha\frac{s}{\pi} + (1 - \alpha)s \\ e &= (1 - \alpha)s(1 - \theta) + \frac{a}{1 - \pi} \end{aligned}$$

where $T = s\theta(1 - \alpha)(1 - \pi)$ is a lump sum transfer equal to the taxation of accidental bequests. Note that the "joy of giving" term includes what the parent leaves to his child net of the inheritance tax.

FOCs for s and b are now:

$$u'(c) = \pi u'(d)\left(\frac{\alpha}{\pi} + (1 - \alpha)\right) + (1 - \pi)v'(e + b)(1 - \alpha)(1 - \theta) \tag{7}$$

$$u'(c) = \pi v'(b) + (1 - \pi)v'(e + b) \tag{8}$$

When there is full annuitization, no life insurance and no tax on accidental bequest (i.e., $\alpha = 1, a = \theta = 0$), we obtain that $e = 0$ and after some rearrangements, that $u'(c) = u'(d) = v'(b)$. Hence we have that $d = \frac{s}{\pi} > b$, which implies, in our affluent economy, that the long-lived are better off than the short-lived.

The above FOCs also show that, in the absence of life insurance, a full annuitization of savings (i.e., $\alpha = 1$) is equivalent to a 100 % tax on accidental bequests (i.e., $\theta = 1$), since assuming either $\alpha = 1$ or $\theta = 1$ leads in both cases to $u'(c) = u'\left(\frac{s}{\pi}\right) = v'(b)$.

In order to study the redistributive impact of our instruments, note that the difference in lifetime utility between the long-lived and the short-lived is:

$$U^{LL} - U^{SL} = u\left(\alpha\frac{s}{\pi} + (1 - \alpha)s\right) + v(b) - v\left((1 - \alpha)s(1 - \theta) + b + \frac{a}{1 - \pi}\right) \tag{9}$$

The marginal impact of collective annuitization on inequalities is:

$$\begin{aligned}\frac{d\left[U^{LL}-U^{SL}\right]}{d\alpha} &= u'(d)(\frac{s}{\pi}-s)+v'(b+e)\,s(1-\theta) \\ &+ \frac{ds}{d\alpha}\left[u'(d)\left(\frac{\alpha}{\pi}+(1-\alpha)\right)-v'(b+e)(1-\alpha)(1-\theta)\right] \\ & +[v'(b)-v'(b+e)]\frac{db}{d\alpha}\end{aligned}$$

The direct impact (in the first line) of a variation of α is positive, while the indirect impacts (in the second and third lines), through the responses of s and b, are ambiguous. However, using numerical simulations with CES utility, we find that, for any values of the parameters (a,θ) and for reasonable preference parameters, the direct effect dominates the indirect effects, so that the overall sign of the above expression is positive. Hence a higher degree of annuitization raises welfare inequalities between the long-lived and the short-lived.

Concerning the impact of life insurance, we have:

$$\begin{aligned}\frac{d\left[U^{LL}-U^{SL}\right]}{da} &= -v'(b+e)\left[\frac{1}{1-\pi}\right] \\ & +[u'(d)\left(\frac{\alpha}{\pi}+(1-\alpha)\right)-v'(b+e)(1-\alpha)(1-\theta)]\frac{ds}{da} \\ & +[v'(b)-v'(b+e)]\frac{db}{da}\end{aligned}$$

As before, whether welfare inequalities between the long-lived and the short-lived increase or decrease with the size of life insurance depends on both the direct effect (the first line) and indirect ones (the last two lines). Numerical simulations show that, for any values of (α,θ) and for reasonable preference parameters, the overall sign of the above expression is negative, so that higher levels of life insurance decreases welfare inequalities.

Regarding the tax on accidental bequests, we have:

$$\begin{aligned}\frac{d\left[U^{LL}-U^{SL}\right]}{d\theta} &= v'(b+e)(1-\alpha)s \\ &+ \left[u'(d)\left(\frac{\alpha}{\pi}+(1-\alpha)\right)+v'(b+e)(1-\alpha)(1-\theta)\right]\frac{ds}{d\theta} \\ &+ [v'(b)-v'(b+e)]\frac{db}{d\theta}\end{aligned}$$

Hence, whether a tax on accidental bequests raises welfare inequalities between the short-lived and the long-lived depends both on a positive direct effect and on indirect effects through behavioral responses, which are a priori indeterminate. In the same way, our simulations show that, for any values of (α,a) and for reasonable preference parameters, the overall sign of the above expression is positive, so that higher levels of tax always increase welfare inequalities.

Thus, the size of inequalities in lifetime well-being between the long-lived and the short-lived is, under mild conditions, increasing with the degree of annuitization and with the tax on accidental bequests, but decreasing with the life insurance. Taxing accidental bequests tends thus to reinforce inequalities in lifetime well-being between the long-lived and the short-lived.

Finally, note that reducing the degree of annuitization α to zero contributes to reducing inequalities in lifetime well-being, but is not sufficient to achieve the equality $U^{LL} = U^{SL}$. Indeed, under $\alpha = 0$, we have:

$$U^{LL} - U^{SL} = u(s) + v(b) - v\left(s(1-\theta) + b + \frac{a}{1-\pi}\right)$$

That expression reveals that, with no annuitization possible, the life insurance and the subsidy on accidental bequests are two equivalent ways to equalize lifetime well-being between short-lived and long-lived agents.[12]

Proposition 2 summarizes our results, which will prove to be useful for the decentralization of social optima in the following sections.

Proposition 2 *Inequalities in lifetime well-being between the long-lived and the short-lived are, in general, increasing in the degree of annuitization, decreasing in life insurance, and increasing in the tax on accidental bequests. Introducing a life insurance and subsidizing accidental bequests are two equivalent ways to achieve equality in lifetime well-being.*

Thus, once we consider accidental bequests from the perspective of the donors, taxing accidental bequests is unfair as it increases inequalities in lifetime well-being between the long-lived and the short-lived, in the same way as annuities increase those inequalities.

3 The utilitarian optimum

As a benchmark case for normative analysis, let us first briefly review the problem faced by a utilitarian social planner, whose goal is to maximize the average lifetime well-being of the population (which is also equal here to the expected lifetime well-being of a representative individual).

As recalled in the introduction, there is a debate about whether the joy of giving term should be taken into account in the social welfare function. We will here adopt a comprehensive approach to well-being, and incorporate the joy of giving in the social welfare function. We will also briefly mention what happens when this term is omitted in the social welfare function.

Another degree of freedom lies in how accidental bequests are distributed. It may be possible to transfer only a portion e of d to the descendant in case of premature death and redistribute the remainder equally, for instance, among all individuals. We can therefore distinguish between two social optima. The first

[12] To see this, suppose $\theta = 0$. Then, under our assumptions, there exists always, by continuity, a value of life insurance $a^* > 0$ such that $U^{LL} - U^{SL} = 0$. Alternatively, if we now assume no life insurance ($a = 0$), it is also the case that there exists a value of the subsidy on accidental bequests $\theta^* < 0$ such that $U^{LL} - U^{SL} = 0$.

social optimum is the *optimum optimorum*: in that case, we do not impose, in the social planning problem, that $e = d$. The second social optimum is a kind of constrained first-best optimum, in the sense that this assumes, as in the laissez-faire, that the constraint $e = d$ is necessarily satisfied.

3.1 The utilitarian optimum optimorum

Let us first characterize the optimum optimorum, that is, the social optimum when both savings consumed in old age d and the accidental bequest e can be controlled (without the constraint $e = d$). The problem of the utilitarian social planner can be written as:

$$\begin{aligned}&\max_{c,d,b,e} u(c) + \pi\left(u(d) + v\left(b\right)\right) + (1-\pi)v(e+b)\\&\text{s.t. } c + \pi d + (1-\pi)e + b = w.\end{aligned}$$

The FOCs yield:

$$u'(c) = u'(d) = v'(e+b) = v'(b) \tag{10}$$

Hence, consumption is smoothed along the life cycle ($c = d$) and $e = 0$, meaning that accidental bequests are totally eliminated. In addition, one deduces from $u'(d) = v'(b)$ that $b < d$.

A short-lived person, with utility $u(c) + v(b)$, is worse off than a long-lived person whose utility is $u(c) + u(d) + v(b)$, if and only if consumption d is above $\bar{c}$, which is a weak assumption in an affluent economy.

Proposition 3 *At the utilitarian optimum optimorum, consumption is smoothed, accidental bequests e are zero, $b < d$, and the short-lived are worse off than the long-lived if and only if $d > \bar{c}$. The utilitarian optimum optimorum can be decentralized either by introducing full annuitization of savings (i.e., $\alpha = 1$) or by introducing a full taxation of accidental bequests (i.e., $\theta = 1$).*

Proof. The first part of Proposition 3 follows from the FOCs. Regarding the decentralization, it is achieved by comparing the FOCs of the agent's problem in Section 2.3 with the FOCs of the social planning problem in Section 3.1. ■

The utilitarian optimum optimorum involves a flat consumption profile as well as some positive gift, but no accidental bequest.[13] Its decentralization involves either the full annuitization of savings, or a 100 % tax on accidental bequests. Quite interestingly, both annuitization and taxation of bequests contribute, in the presence of joy of giving, to increasing inequalities in lifetime well-being between the long-lived and the short-lived with respect to our laissez-faire equilibrium (see Section 2.3). By eliminating accidental bequests, Bentham makes the short-lived worse-off than at the laissez-faire.

[13] Note that, when the joy of giving is not counted in the social welfare function, the optimal e remains null, and the well-being gap between short-lived and long-lived remains the same.

3.2 The first-best constrained optimum

Let us now consider the constrained utilitarian optimum, where the planner cannot control both d and e, but faces the constraint $e = d$. The problem is:

$$\begin{aligned} &\max_{c,d} u(c) + \pi \left(u(d) + v\left(b\right)\right) + (1-\pi) v\left(d+b\right) \\ &\text{s.t. } c + d + b = w. \end{aligned}$$

The FOCs yield:

$$\begin{aligned} u'(c) &= \pi u'(d) + (1-\pi) v'(d+b) \\ u'(d) &= v'(b). \end{aligned}$$

One recognizes the laissez-faire program and its FOCs. Thus the constrained utilitarian optimum coincides with the pure laissez-faire equilibrium.

Proposition 4 *At the constrained utilitarian optimum, which coincides with the laissez-faire, consumption declines over the life cycle, accidental bequests are positive, $b < d$, and the short-lived are worse off than the long-lived.*

Proof. See above. ■

Since the first-best constrained optimum coincides with the laissez-faire, there is here no need for any type of public intervention ($\theta = a = \alpha = 0$). When comparing the constrained utilitarian optimum with the utilitarian optimum optimorum, it is crucial to notice that inequalities in lifetime well-being between the long-lived and the short-lived are stronger at the utilitarian optimum optimorum than at the utilitarian constrained optimum, since the latter involves accidental bequests which decrease welfare inequalities.

4 The ex post egalitarian optimum

Up to now, we examined the design of an optimal tax on accidental bequests from a utilitarian perspective. In the optimum optimorum, accidental bequests are taxed away fully, comforting the widespread view on this issue. One may wonder whether this result is robust to the social welfare criterion. Clearly, the use of a utilitarian social welfare function in a context of unequal lifetimes can be questioned, on the grounds of the little attention it pays to inequalities in realized lifetime utilities. As we showed above, at the utilitarian optimum, the short-lived is, in an affluent economy, worse off than the long-lived.

Those inequalities in realized lifetime well-being are difficult to justify, since all individuals are here exactly identical ex ante, and behave in exactly the same way. In our framework, it is a pure matter of luck whether someone reaches the old age or not. This section aims at exploring the question of the compensation of the short-lived, and its impact on the optimal taxation of accidental bequests. For that purpose, we consider here a social welfare criterion of the ex post egalitarian type, as suggested in Fleurbaey et al (2014, 2016). This social objective selects the allocation that maximizes the realized —and not expected— lifetime well-being of the worst-off individual.

4.1 The egalitarian optimum optimorum

The problem of the ex post egalitarian social planner is:

$$\max_{c,d,e,b} \min \left\{U^{SL}, U^{LL}\right\}$$
$$\text{s.t. } c + \pi d + (1-\pi)e + b = w$$

where $U^{LL} \equiv u(c) + u(d) + v(b)$ and $U^{SL} \equiv u(c) + v(e+b)$.

The social objective is continuous but not differentiable. However, it is possible to rewrite the above problem in a more tractable way. We can rewrite it as the maximization of the lifetime well-being of the long-lived conditionally on the resource constraint of the economy and conditionally on the constraint that short-lived individuals are not worse-off than long-lived individuals. That constraint can be called an egalitarian constraint. This was violated at the utilitarian optimum for an affluent economy. The egalitarian constraint is:

$$u(c) + u(d) + v(b) = u(c) + v(e+b) \tag{11}$$

The Lagrangian can be written as

$$\begin{aligned} L &= u(c) + u(d) + v(b) - \mu\left[c + \pi d + (1-\pi)e + b - w\right] \\ & \quad -\lambda\left[u(d) + v(b) - v(e+b)\right] \end{aligned}$$

where μ is the Lagrange multiplier associated with the resource constraint of the economy, while λ is the Lagrange multiplier associated with the egalitarian constraint. The FOCs yield:

$$u'(c) = \mu = \frac{1-\lambda}{\pi} u'(d) \tag{12}$$

$$(1-\lambda)v'(b) + \lambda v'(e+b) = \mu = \frac{\lambda}{1-\pi} v'(e+b) \tag{13}$$

Regarding the consumption profile (first expression), two cases can arise. If $\pi > 1-\lambda$, we have $c > d$, while if $\pi < 1-\lambda$, we have $c < d$. Given that the latter case corresponds to extremely poor economies, we focus here on the first one. From the last expression, we obtain:

$$v'(b) = \frac{\lambda\pi}{1-\lambda-\pi+\lambda\pi} v'(e+b) \tag{14}$$

Given $\pi > 1-\lambda$, we have $v'(b) > v'(e+b)$, which implies $e > 0$. Moreover, combining this with the FOC for d, we have:

$$u'(d) = \frac{\lambda\pi}{1-\lambda-\pi+\lambda\pi} v'(e+b) = v'(b) \tag{15}$$

leading to $d > b$. Thus we have $c > d > b$.

Finally, let us study the ranking between e and d. Given the egalitarian constraint $u(d) + v(b) = v(e+b)$, and the concavity of $v(\cdot)$, we have $v(e+b) <$

$v(e) + v(b)$. Hence, the egalitarian optimum optimorum is characterized by $0 \leq u(d) < v(e)$. Under our assumption of an affluent economy (i.e. $d > \tilde{d}$), $u(\cdot)$ is always above $v(\cdot)$, so that e must be much larger than d to obtain that $u(d) < v(e)$. Accidental bequests of the short-lived must therefore be augmented ($e > d$) in order to provide them with sufficient compensation for the loss of an affluent retirement period.

Proposition 5 *At the ex post egalitarian optimum optimorum, consumption decreases along the life cycle, accidental bequests are augmented with respect to the laissez-faire, and the short-lived and long-lived are equally well off. That optimum can be decentralized by imposing no collective annuitization (i.e., $\alpha = 0$), and by introducing either life insurance $a > 0$ or a subsidy on accidental bequests $\theta < 0$.*

Proof. The first part of Proposition 5 follows from the above FOCs. The decentralization is achieved by recognizing that the difference in lifetime utilities between short-lived and long lived (eq. 9) is increasing in α and θ and decreasing in a. ■

Comparing this result with the utilitarian optimum optimorum, one sees that compensating the short-lived involves not only allowing them to give accidental bequests, but also imposing a low old-age consumption for the long-lived.[14]

Concerning the decentralization of the ex post egalitarian optimum, it should be stressed that this involves no collective annuitization of savings, since, as discussed in Section 2.3, collective annuitization, by eliminating accidental bequests, raises well-being inequalities between the long-lived and the short-lived.

Note, however, that the absence of annuitization is a necessary, but not a sufficient condition to achieve equality of well-being between the long-lived and the short-lived. The decentralization requires also either the introduction of life insurance, or a subsidy on accidental bequests in order to have $e > d$. Subsidizing accidental bequests seems counterintuitive, but it should be reminded that the goal of policies is here to compensate short-lived persons. From that perspective, taxing accidental bequests would be unfair, as it would prevent the short-lived from giving to their descendant, and, as such, would make them worse off. On the contrary, accidental bequests should be subsidized, to achieve equality of well-being between the short-lived and the long-lived.

4.2 The constrained egalitarian optimum

Let us now consider the constrained first-best problem, where the social planner cannot control for both d and e, but faces the constraint $e = d$. In this case, the egalitarian constraint becomes:

$$u(c) + u(d) + v(b) = u(c) + v(d + b). \tag{16}$$

[14] In absence of joy of giving, bequests would disappear altogether and the equality between short-lived and long-lived would boil down to $u(c) + u(d) = u(c)$, requiring $d = \bar{c}$, i.e., making old-age consumption low enough to eliminate the advantage of living long (see Fleurbaey et al. 2014 for further discussion of this case).

It is easy to show that, under our assumption of a sufficiently rich economy (i.e. $d > \tilde{d}$), that egalitarian constraint cannot be satisfied. Indeed, since $v(d+b) < v(d) + v(b)$, the equality would require $u(d) < v(d)$, implying $d < \tilde{d}$. Thus equality cannot be achieved in a sufficiently affluent economy: the short-lived will remain the worse off. Thus the problem amounts to maximizing the well-being of the short-lived. The Lagrangian is:

$$\mathcal{L} = u(c) + v(d+b) - \mu [c + d + b - w].$$

The FOCs require $u'(c) = v'(d+b)$, meaning that $c > d+b$ since $v'(d+b) < u'(d+b)$. In this program, the division of $d+b$ between d and b does not matter, because b and d affect the short-lived in the same way. Given the constraint of doing the maximum for the short-lived, one can still maximize the utility of the long-lived, which then implies $v'(b) = u'(d)$ and so $d > b$.

Proposition 6 *At the ex post egalitarian constrained optimum, equality is no longer achieved and the long-lived remain better off, but consumption is still decreasing over the life cycle and $b < d$. The decentralization of that optimum can be achieved by imposing no collective annuitization ($\alpha = 0$), and by introducing either life insurance ($a > 0$) or by subsidizing accidental bequests ($\theta < 0$).*

Proof. The first part of Proposition 6 follows from the above FOCs. The proof of the decentralization follows the same lines as the one of Proposition 5. ■

In comparison to Proposition 5, a major difference is that the egalitarian constrained optimum does not equalize the lifetime well-being of the long-lived and the short-lived.[15] Given the inability to distinguish between old-age consumption and accidental bequests, it is not possible to make the lifetime well-being of the short-lived as high as the one of the long-lived. Increasing the realized lifetime well-being of the short-lived would require to increasing also the consumption of the long-lived at the old age, which prevents equalization of lifetime well-being levels, unlike what can be achieved under the ex post egalitarian optimum optimorum (where the two variables are distinct).

Regarding the decentralization of the constrained optimum, it is also the case, as in Proposition 5, that there should be no collective annuitization but a life insurance or a subsidy on accidental bequests, in order to limit welfare inequalities between the short-lived and the long-lived.

5 Extension: the OLG economy

In the previous sections, we deliberately ignored inequalities in endowments at the young age, in order to focus on inequalities between the short-lived and the long-lived. We showed that, by doing so, it may be optimal, when the prematurely dead have some joy of giving, not to tax accidental bequests, but

[15] Note, however, that, in absence of joy of giving in the social welfare function (or in the agents' preferences), it would always be possible to achieve equality between short-lived and long-lived, as in the previous subsection.

to subsidize them. It should be stressed, however, that ignoring inequalities in endowments simplifies the picture significantly. Indeed, as Mill (1848) emphasized, a major argument for taxing accidental bequests lies precisely in the fact that those bequests lead to large inequalities among descendants.

In order to take into account the impact of bequests on next generations, we need to consider now a dynamic OLG model, where agents are heterogeneous in terms of the bequests they received from their parents. This section develops the dynamic extension of the baseline model, and examines the robustness of our results concerning the optimal taxation of accidental bequests.

5.1 The model

We now consider a two-period OLG model with risky lifetime. As in the baseline model, each cohort is a continuum of agents of size 1. The duration of each period is normalized to 1. Period 1 is young adulthood, during which individuals supply inelastically 1 unit of labor, consume, have one child, and save for their old days.[16] The old age - period 2 - is reached with a probability π $(0 < \pi < 1)$. During the old age, individuals consume their savings and do not work.

The shift from a static to a dynamic intergenerational model raises some complexity in terms of intracohort heterogeneity. In our static model, the unique source of intracohort heterogeneity was the duration of life. This source of heterogeneity remains present here, but a second source of heterogeneity lies in individual initial endowments.[17] The individual's endowment depends on the bequest received from his parent, and, hence, depends on his parent's duration of life, as well as on the duration of life of all his ancestors (which affect the parent's endowment). This leads, after a small number of generations, to a large heterogeneity in endowments. In order to keep the analysis tractable, we will, throughout this section, assume a particular structure on preferences, which makes the intergenerational dynamics of wealth accumulation Markovian, in the sense that the endowment of an agent born at t only depends on the longevity of his parent born at $t-1$, and not on the longevity of previous ancestors.

5.1.1 Preferences

To make the wealth dynamics across generations Markovian, we assume that all agents have the same preferences, which take the quasi-linear form:

$$c_t + \pi\big[u\left(d_{t+1}\right) + v(b_{t+1})\big] + (1-\pi)v\left(d_{t+1} + b_{t+1}\right) \tag{17}$$

where c_t is the consumption of a young adult at time t, b_{t+1} denotes gifts (i.e. the unconditional component of parental bequest), while d_{t+1} is either the old-age consumption for a young adult at time t who survives to the old age, or the accidental bequest that he leaves to his child in case of premature death.

[16] We abstract here from the cost of childbearing (the same for all), and thus neglect a potential period 0 (childhood) in which people consume and make no decision whatsoever.

[17] Note that this section only considers inequalities in endowments due to unequal bequests received from the parents, but does not consider the case of inequalities in earnings w.

Under those preferences, the initial endowment received affects only the level of first-period consumption, but has no impact on the levels of savings s_t and on the gift b_{t+1}. Put it differently, under quasi-linear preferences, whether an agent has a long-lived or a short-lived parent will only affect the level of his initial endowment, but will not impact the endowment of his child (which only depends on the agent's duration of life, independently from ancestors).

5.1.2 Budget constraints

Given our assumption of quasi-linear preferences, there are only two groups of young adults at time t, depending on whether the parent died early or late:

- Type-E_t agents: young adults at time t whose parents die early;
- Type-L_t agents: young adults at time t whose parents die late.

Given that, at any period, a proportion π of individuals reach the old age, while a proportion $1-\pi$ dies before reaching the old age, by the law of large numbers, the proportion of agents of type L_t in the cohort is equal to π, while the proportion of individuals of type E_t is equal to $1-\pi$.

The budget constraints faced by an agent of type i_t in the first and second periods are:

$$c_t^{i_t} + s_t^{i_t} = w_t + b_t^{i_t} + B_t^{i_t} \quad (18)$$

$$d_{t+1}^{i_t} + b_{t+1}^{i_t} = R_{t+1} s_t^{i_t} \quad (19)$$

where $b_t^{i_t}$ is the gift, i.e. the unconditional component of bequests received from the parents (which is the same for all agents, whatever the realized longevity of parents is). On the contrary, $B_t^{i_t}$ is the accidental component of bequest received at the death of his parent. This is conditional on the duration of life of the parent. If the parent dies early, it is equal to the savings of his parent, multiplied by the interest factor, R_t, so that $B_t^{E_t} = R_t s_{t-1}$. If the parent dies late, then there is no accidental bequest and $B_t^{L_t} = 0$.

Hence the associated intertemporal budget constraint is:

$$w_t + b_t^{i_t} + B_t^{i_t} = c_t^{i_t} + \frac{b_{t+1}^{i_t} + d_{t+1}^{i_t}}{R_{t+1}} \quad (20)$$

Obviously, individuals of type E_t, for whom $B_t^{i_t} > 0$, face better budget conditions than agents of type L_t, for whom $B_t^{i_t} = 0$. All agents face the same prices, but the initial endowment is higher for individuals who received an accidental bequest due to the premature death of their parent.

5.1.3 Production

Production takes place with labour ℓ_t and capital k_t, according to a production function with constant returns to scale:

$$Y_t = F(k_t, \ell_t) \quad (21)$$

Under our assumptions, i.e. each young individual supplies 1 unit of labour inelastically, and each cohort being a continuum of size 1, we have $\ell_t = \ell = 1$. Hence, throughout this section, we will write the production process as:

$$Y_t = f(k_t) \tag{22}$$

where $f(k_t) \equiv F(k_t, 1)$.

Capital fully depreciates after one period of use. Hence the capital accumulation equation is:

$$k_{t+1} = \pi s_t^{L_t} + (1-\pi) s_t^{E_t} \tag{23}$$

Factors are paid at their marginal productivity:

$$w_t = f(k_t) - k_t f'(k_t) \tag{24}$$

$$R_t = f'(k_t) \tag{25}$$

5.2 The laissez-faire

The problem of a type i_t agent is:

$$\max_{c^{i_t}, d_{t+1}^{i_t}, b_{t+1}^{i_t}} c_t^{i_t} + \pi \left[u(d_{t+1}^{i_t}) + v(b_{t+1}^{i_t}) \right] + (1-\pi) \left[v(d_{t+1}^{i_t} + b_{t+1}^{i_t}) \right]$$

$$\text{s.t.} \quad w_t + b_t^{i_t} + B_t^{i_t} = c_t^{i_t} + \frac{d_{t+1}^{i_t} + b_{t+1}^{i_t}}{R_{t+1}}$$

The FOCs of this problem yield $u'(d_{t+1}^{i_t}) = v'(b_{t+1}^{i_t})$, and:

$$R_{t+1} \left[\pi u'(d_{t+1}^{i_t}) + (1-\pi) \left[v' \left(d_{t+1}^{i_t} + v'^{-1}\big(u'(d_{t+1}^{i_t})\big) \right) \right] \right] = 1. \tag{26}$$

This implies that $d_{t+1}^{i_t}$ is independent from the wealth initially inherited (i.e. the same for $i_t = E_t, L_t$). Let us denote the second-period consumption level satisfying the above expression as $\breve{d}_{t+1}$. Given that $u'(d_{t+1}^{i_t}) = v'(b_{t+1}^{i_t})$, it is straightforward to see that the level of unconditional bequest is:

$$b_{t+1}^{i_t} = v'^{-1}\big(u'(d_{t+1}^{i_t})\big) = v'^{-1}\big(u'(\breve{d}_{t+1})\big) = \breve{b}_{t+1}. \tag{27}$$

Thus, whatever the inheritance was, all individuals choose the same level of second-period consumption and gifts, which only depend on time through the interest rate. The only difference between dynasties concerns what is consumed in the first period, which varies depending on what was received from the parent, which can be either $\breve{b}_t$ (for type L_t) or $\breve{b}_t + \breve{d}_t$ (for type E_t).

Hence, ex ante, before the realization of lifetime duration, the population is divided, in the long-run, in two groups: those whose parent died late (type L in proportion π) and those whose parent died early (type E in proportion $(1-\pi)$).

Throughout the rest of this section, we assume that a stationary equilibrium exists, that it is unique and stable.[18] At the stationary equilibrium, optimal consumption profiles and gifts are:

[18] Note that, since the heterogeneity is reduced to two groups with time-invariant proportions, this assumption is close to what is usually assumed when considering simple OLG models of capital accumulation with a representative agent.

$$c^E = w + \left(\breve{d} + \breve{b}\right) - \left(\frac{\breve{d}+\breve{b}}{R}\right) > c^L = w + \breve{b} - \left(\frac{\breve{d}+\breve{b}}{R}\right);$$

$d^E = d^L = \breve{d}$, where $\breve{d}$ is such that $\pi u'(\breve{d}) + (1-\pi)\left[v'\left(\breve{d} + v'^{-1}\left(u'(\breve{d})\right)\right)\right] = 1/R$;

$b^E = b^L = \breve{b}$, where $\breve{b} = v'^{-1}\left(u'(\breve{d})\right)$.

Proposition 7 summarizes our results regarding the stationary equilibrium.

Proposition 7 *At a stationary laissez-faire equilibrium with quasi-linear utility, savings and gifts are independent from initial wealth, and individuals with short-lived parents have higher first-period consumption than individuals with long-lived parents. For a given longevity, individuals of type E are better off than individuals of type L. Within a given type $i = E, L$, the long-lived is better off than the short-lived.*

Proof. See above. ■

In comparison with the baseline model, a major difference lies in the fact that inequalities in the parent's duration of life lead here to inequalities in endowments at the next generation, and, hence, to well-being inequalities for a given longevity. However, it remains true, as in the baseline model, that, within a given type, short-lived individuals are worse off than long-lived individuals.

It should be stressed that survival conditions, summarized here by the parameter π, have a crucial impact on the distribution of wealth and well-being in the economy. To see this, let us note that, when life expectancy is low, there are lots of young individuals who benefit from accidental bequests. Also, they are likely to inherit small amounts of accidental bequest, because people facing a large probability of dying save less. On the contrary, when life expectancy is high, there are fewer young individuals who receive accidental bequests, implying that wealth is much more concentrated in the hands of the few ones who could benefit from those bequests. Moreover, since life expectancy is high, savings are likely to be higher so that each of the few inheritants receives a higher accidental bequest. Therefore, our framework provides a purely demographic explanation for the widely documented rise in wealth inequalities: inequalities would be reinforced by the improvement of survival conditions, which both raises the average size of accidental bequests and reduces the proportion of heirs in the population. That explanation is in line with Miyazawa (2006).

5.3 The utilitarian optimum

5.3.1 The utilitarian optimum optimorum

Like before, we first derive the utilitarian optimum optimorum, where the planner can control for both savings consumed in old age d^i and the accidental bequest e^i. We consider here the problem of a social planner who selects the levels of consumptions, gifts, accidental bequests and capital so as to maximize the average lifetime welfare prevailing at the stationary equilibrium, while satisfying the resource constraint of the economy prevailing at the steady-state:

$$\begin{aligned}
\max_{c^i,d^i,b^i,e^i,k} \quad & (1-\pi)c^E + \pi(1-\pi)\left[u(d^E)+v(b^E)\right] + (1-\pi)^2 v(b^E+e^E) \\
& +\pi c^L + \pi^2\left[u(d^L)+v(b^L)\right] + \pi(1-\pi)v(b^L+e^L) \\
\text{s.t.} \quad & f(k) = \pi c^L + (1-\pi)c^E + (1-\pi)b^E + \pi(1-\pi)d^E \\
& +(1-\pi)^2 e^E + \pi b^L + \pi^2 d^L + \pi(1-\pi)e^L + k
\end{aligned}$$

The solution to this problem is not unique because of the linearity in c^E and c^L, which makes any allocation of first-period consumption between groups E and L indifferent for a given level of aggregate first-period consumption. Without loss of generality, we will focus here on the case where $c^E = c^L$.[19]

The FOCs yield $u'(d^E) = u'(d^L)$, so that $d^E = d^L$. Also, for any $i = E, L$ we obtain $\pi v'(b^i) + (1-\pi)v'(b^i + e^i) = v'(b^i + e^i) = 1$. Hence, we necessarily have $e^i = 0$ and $b^E = b^L = v'^{-1}(1)$. Similarly, $u'(d^E) = u'(d^L) = 1$, so that $b^E = b^L = u'^{-1}(1)$. By assumption, $u'^{-1}(1) > v'^{-1}(1)$, so that savings are larger than gifts. Finally, the optimal intertemporal allocation of resources implies that $f'(k) = 1$. Proposition 8 summarizes these results.

Proposition 8 *At any long-run utilitarian optimum optimorum with quasi-linear utility, there is no accidental bequest ($e^i = 0$) and individuals of types E and L benefit from the same gifts, which are smaller than savings. The capital stock satisfies the Golden Rule $f'(k) = 1$. There exists a long-run utilitarian optimum optimorum such that first-period consumption is the same for individuals of types E and L. Short-lived individuals are worse off than long-lived ones if and only if $u(d) = u \circ u'^{-1}(1) > 0$.*

The decentralization of that optimum requires a system of intergenerational lump-sum transfers leading to the Golden Rule. The decentralization requires also either introducing full collective annuitization (i.e., $\alpha = 1$) or introducing full taxation of accidental bequests (i.e., $\theta = 1$).

Proof. The first part of Proposition 8 follows from the above FOCs of the planner's problem. Regarding the decentralization, it is achieved by comparing the FOCs of the agent's problem in Section 2.3 rewritten at time t for agents of types E and L with the FOCs of the utilitarian planner's problem. ■

Proposition 8 states that, as in the baseline static model, the long-run utilitarian optimum optimorum involves zero accidental bequests. Hence, without surprise, the decentralization of the long-run utilitarian optimum optimorum requires, as in the baseline model, a full taxation of accidental bequests, or, alternatively, a full annuitization of savings. Thus the conventional 100 % tax view still holds here, as in the baseline static model.

The only difference with respect to the baseline model is that annuitization or accidental bequests taxation does not suffice here to decentralize the social optimum, since individual savings decisions may lead to under- or over-accumulation of capital. Hence intergenerational lump-sum transfers are needed

[19] We consider this specific solution as it simplifies comparisons with the egalitarian optima below.

here. Note, however, that since accidental bequest are null at the optimum optimorum, there is no heterogeneity in initial endowments received by the agent, $B_i = e_i = 0$, and thus the decentralization of the long-run utilitarian optimum does not require to have intra-generational lump sum transfers.[20]

5.3.2 The utilitarian constrained optimum

Let us now consider the constrained utilitarian optimum, where the social planner cannot control for both d^i and e^i, but faces the constraint $e^i = d^i$. The problem of the utilitarian social planner is:

$$\begin{aligned}
\max_{c^i,d^i,b^i,k} \quad & (1-\pi)c^E + \pi(1-\pi)\left[u(d^E) + v(b^E)\right] + (1-\pi)^2 v(b^E + d^E) \\
& + \pi c^L + \pi^2 \left[u(d^L) + v(b^L)\right] + \pi(1-\pi)v(b^L + d^L) \\
\text{s.t.} \quad & f(k) = \pi c^L + (1-\pi)c^E + (1-\pi)b^E + (1-\pi)d^E \\
& + \pi b^L + \pi d^L + k
\end{aligned}$$

Like before, we focus, without loss of generality, on the solution where $c^E = c^L$. The FOCs yield, for each $i = E, L$, $\pi u'(d^i) + (1-\pi)v'(b^i + d^i) = \pi v'(b^i) + (1-\pi)v'(b^i + d^i)$, so that $u'(d^i) = v'(b^i)$, and $d^i > b^i$. Therefore, $b^i = v'^{-1}\big(u'(d^i)\big)$ Also, for any $i = E, L$ we obtain $\pi u'(d^i) + (1-\pi)v'(b^i + d^i) = \pi u'(d^i) + (1-\pi)v'\big(d^i + v'^{-1}\big(u'(d^i)\big)\big) = 1$. Hence, $d^E = d^L$ and $b^E = b^L$. Furthermore, the optimal intertemporal allocation of resources implies that $f'(k) = 1$, so that $1/R = 1$. The optimal levels of savings and gifts are the same as in a laissez-faire equilibrium provided the capital level takes its socially optimal level. To achieve this optimum, we thus only need to equalize initial wealths and modify capital accumulation to reach the Golden Rule level.

Proposition 9 *At the long-run constrained utilitarian social optimum with quasi-linear utility and equal first-period consumption, old-age consumption and gifts are identical for all individuals, the former is larger than the latter ($d > b$), and accidental bequests are positive. The capital stock satisfies the Golden Rule. Within a given type $i = E, L$, the long-lived is better off than the short-lived.*

Decentralization of that optimum requires a system of intergenerational lump-sum transfers leading to the Golden Rule, as well as a system of intragenerational lump-sum transfers equalizing endowments across types E and L.

Proof. The proof follows the same lines as the one of Proposition 8. ■

The constrained utilitarian optimum involves the same capital stock as the optimum optimorum. Moreover, like at the optimum optimorum, consumptions are equalized across types, and gifts as well. However, an important difference with respect to the utilitarian optimum optimorum is here that the individuals

[20] Note that, in case of inequalities in endowments across dynasties at time 0, the equality between c^E and c^L is not achieved at $t = 0$. But given the full taxation of accidental bequests, we know that, at the stationary equilibrium, the equality $c^E = c^L$ will necessarily prevail. Thus the decentralization of the long-run utilitarian optimum does not require intra-generational lump-sum transfers.

leave more to their children when they die early than when they die late, since $d+b > b$, unlike what prevailed at the unconstrained utilitarian optimum where there were no accidental bequests. But the additional bequest left by short-lived parents will not be enjoyed by his child, since the system of lump-sum transfers equalizes, by construction, the initial endowments across types E and L. Thus, although short-lived parents are allowed here to give an accidental bequest, their children are not allowed to consume it.

5.4 The ex post egalitarian optimum

5.4.1 The ex post egalitarian optimum optimorum

At the stationary equilibrium, and allowing for accidental bequest e different from old-age consumption of the surviving people d, there are four groups of individuals ex post:

- Individuals who die early and whose parents died early: they have utility $U^{ESL} = c^E + v(b^E + e^E)$;
- Individuals who die late and whose parents died early: they have utility $U^{ELL} = c^E + u(d^E) + v(b^E)$;
- Individuals who die early and whose parents died late: they have utility $U^{LSL} = c^L + v(b^L + e^L)$;
- Individuals who die late and whose parents died late: they have utility $U^{LLL} = c^L + u(d^L) + v(b^L)$.

Let us consider the problem of a social planner who maximizes the realized lifetime well-being of the worst off individuals living at the stationary equilibrium.

The social planner's problem can be written as follows:

$$\max_{\substack{c^E,d^E,b^E,e^E\\ c^L,d^L,b^L,e^L,k}} \min\{U^{ELL}, U^{ESL}, U^{LLL}, U^{LSL}\}$$

$$\text{s.t. } f(k) = \pi c^L + (1-\pi)c^E + \pi(1-\pi)\left(d^E + b^E\right) + (1-\pi)^2(e^E + b^E)$$
$$+\pi^2(d^L + b^L) + \pi(1-\pi)(e^L + b^L) + k$$

From the egalitarian point of view, there should be an equality of all variables across types E and L, that is, $c^E = c^L = c$, $b^E = b^L = b$, $d^E = d^L = d$ and $e^E = e^L = e$. Thus the planning problem can be written as the one of maximizing lifetime utility at the stationary equilibrium under the egalitarian constraint:

$$c + u(d) + v(b) = c + v(b + e).$$

which guarantees that long-lived and short-lived agents are equally well-off.

Hence the social planner's problem can be written as follows:

$$\begin{aligned}
&\max_{c,b,d,e,k} c + u(d) + v(b) \\
\text{s.t.} \quad & f(k) = c + b + \pi d + (1-\pi)e + k \\
& u(d) + v(b) = v(b+e)
\end{aligned}$$

The Lagrangian for this problem is

$$\mathcal{L} = c+u(d)+v(b)+\mu\big[f(k)-c-b-\pi d-(1-\pi)e-k\big]-\lambda\big[u(d)+v(b)-v(b+e)\big].$$

The FOCs are similar to the ones in Section 4.1. The unique difference is that utility is linear in first-period consumption, so that $\mu = 1$. We still have that $v'(b) = u'(d)$, so that $b < d$. Assuming, here again, $\pi > 1 - \lambda$, the FOCs imply that $u'(d) = \frac{\pi}{1-\lambda} > 1$ and $v'(b)(1-\lambda) = 1 - \lambda v'(b+e)$. This can be rewritten as $1 - v'(b) = -\lambda [v'(b) - v'(b+e)]$. Given that $u'(d) = v'(b) > 1$ and $\lambda > 0$, we obtain $v'(b) > v'(b+e)$ and therefore $e > 0$. Thus the conventional 100 % tax view does not hold here.

Proposition 10 *At the long-run ex post egalitarian optimum optimorum with quasi-linear utility, consumptions, gifts and accidental bequests are equal across types E and L, while the capital stock satisfies the Golden Rule. Accidental bequests are augmented with respect to the laissez-faire, and the short-lived and long-lived are equally well off.*

The decentralization of that optimum requires a system of intergenerational lump-sum transfers leading to the Golden Rule, as well as a system of intra-generational lump-sum transfers equalizing endowments across types E and L. The decentralization requires also imposing no annuitization (i.e., $\alpha = 0$), and introducing either life insurance $a > 0$ or a subsidy on accidental bequests $\theta < 0$.

Proof. The first part of Proposition 10 follows from the above FOCs of the planner's problem. Regarding the decentralization, it follows the same lines as the one of Proposition 5. ■

The ex post egalitarian optimum involves the Golden Rule capital level, as well as a full equalization of lifetime welfare across all individuals (long-lived and short-lived) and across dynasties (i.e. types E and L).

Proposition 10 informs us on the robustness of the results obtained in the baseline model. It appears that, as in the static model, the decentralization of the egalitarian optimum still requires to subsidize accidental bequests (even though this is not sufficient here to fully decentralize the egalitarian optimum).

The subsidization of accidental bequests is not, in the above setting, incompatible with the equalization of lifetime utilities across children of short-lived and long-lived parents, since, in our framework, parents are interested in what they *give* to their child (net of inheritance tax), and not in what their child actually *receives* (including lump sum transfers aimed at equalizing endowments across types).[21] This explains why the decentralization of the egalitarian optimum

[21] Thus, in that setting, the short-lived parent's well-being is not affected by the fact that their children will have to pay some lumpsum taxes, and, hence, will not enjoy the whole bequest left by their parent.

involves, in a first-best setting, both subsidizing accidental bequests (so as to increase the well-being of the prematurely dead through a higher joy of giving), while, at the same time, redistributing the transmitted wealth (in a lump-sum way) in such a way as to equalize the endowments of the young across dynasties. That policy mix allows the full equalization of the well-being of the long-lived and the short-lived without preventing the equalization of endowments (and, thus, of lifetime well-being) among the children of the next generation.[22]

But then what would happen if, alternatively, parents would be interested in what they give to their child *net of all taxes and transfers*, that is, in what their child actually *receives*? Would accidental bequests still be subsidized in that alternative setting? Proposition 11 provides the answer.[23]

Proposition 11 *Suppose now that parents are interested in what they give to their children net of all taxes and transfers. Then the decentralization of the long-run ex post egalitarian optimum requires a system of intergenerational lump-sum transfers leading to the Golden Rule. In the absence of annuitization (i.e.,* $\alpha = 0$*) and life insurance (i.e.,* $a = 0$*), the decentralization requires also imposing a tax on accidental bequests* $0 < \theta < 1$*, a tax* τ *on second-period consumption and a lump-sum transfer compensating individuals of type* L*.*
Proof. *See the Appendix.* ■

Proposition 11 states that, once parents are interested in what their child receive net of all taxes and transfers, it is no longer the case that the decentralization of the ex post egalitarian optimum involves a subsidy on accidental bequests. Actually, the decentralization requires here not to subsidize, but to tax accidental bequests. Moreover, the decentralization of the ex post egalitarian optimum requires also to tax old-age consumption.

The underlying intuition goes as follows. Once parents are interested in what their children actually receive (i.e. in their initial endowment net of all taxes and transfers), then, given the equalization of lifetime utilities across children of types E and L, the joy of giving terms of all parents become equal, whatever these have a long life or a short life (since parents care about children's endowments, which are equalized). Hence, one can no longer rely on accidental bequests to reduce welfare inequalities between the long-lived and the short-lived. This tends to annihilate the redistributive motive for subsidizing accidental bequests.

Moreover, given that short-lived and long-lived individuals share the same first-period utility, and the same joy of giving term, the equalization of lifetime welfare across them requires here $u(d) = 0$, that is, old-age consumption d fixed to the monetary equivalent to being dead $\bar{c}$ (i.e. such that $u(\bar{c}) = 0$). This explains why a tax on old-age consumption is here required (unlike above).

[22] Note, however, that, in a second-best world with limited available policy instruments, such a policy mix could not be implemented, and hence there could be a tension between equalizing the well-being across durations of life and across dynasties.

[23] Obviously, that alternative setting presupposes a strong form of transparency, in the sense that parents are here able to anticipate, for any level of saving, what their children will exactly receive in case of their premature death or in case of them reaching the old age.

The comparison of Propositions 10 and 11 reveals that whether one should tax or subsidize accidental bequests depends crucially on the precise formulation of the joy of giving motive. But it should be stressed, however, that Proposition 11 does not recommend the full taxation of accidental bequests (since we have $\theta < 1$). Thus it still remains true, under that alternative formulation, that the standard 100 % tax view on accidental bequests is challenged.

5.4.2 The ex post egalitarian constrained optimum

Let us now turn back to our baseline assumption on joy of giving, and consider the constrained egalitarian optimum. This amounts to considering a stationary equilibrium where the accidental bequest is necessarily equal to the old-age consumption of the surviving people (i.e. $e^i = d^i$). In that case, the social planner's problem becomes:

$$\max_{\substack{c^E, d^E, b^E \\ c^L, d^L, b^L, k}} \min\{U^{ELL}, U^{ESL}, U^{LLL}, U^{LSL}\}$$
$$\text{s.t. } f(k) = \pi\left[c^L + d^L + b^L\right] + (1-\pi)\left[c^E + d^E + b^E\right] + k$$

where $U^{ELL} = c^E + u(d^E) + v(b^E)$, $U^{ESL} = c^E + v(b^E + d^E)$, $U^{LLL} = c^L + u(d^L) + v(b^L)$ and $U^{LSL} = c^L + v(b^L + d^L)$.

From the egalitarian point of view, we should have $c^E = c^L = c$, $b^E = b^L = b$ and $d^E = d^L = d$. As argued before, imposing the egalitarian outcome, $U^{iSL} = U^{iLL} \forall i$, implies that $d < \tilde{d}$, which goes against our assumption. Thus, under $d > \tilde{d}$, we have $u(d) + v(b) > v(d) + v(b) > v(d + b)$: the long-lived is better-off than the short-lived. Therefore, the problem can be written as:

$$\max_{c,b,d,e,k} c + v(d + b)$$
$$\text{s.t.} \quad f(k) = c + b + d + k$$

The Lagrangian for this problem is:

$$\mathcal{L} = c + v(d + b) + \mu\left[f(k) - c - b - d - k\right].$$

The FOCs require $v'(d + b) = 1$. Note that the division of $d + b$ between d and b does not matter here, because it affects the short-lived in the same way. Given the constraint of doing the maximum for the short-lived, one can still maximize the utility of the long-lived, which then implies the usual $v'(b) = u'(d)$.

Proposition 12 *At the long-run ex post egalitarian constrained optimum with quasi-linear utility, consumptions, gifts and accidental bequests are equal across types E and L, while the capital stock satisfies the Golden Rule. Old-age consumption must be larger than gifts ($d > b$). Within a given type $i = E, L$, the long-lived remains better off than the short-lived.*

The decentralization of that optimum requires a system of intergenerational lump-sum transfers leading to the Golden Rule, as well as a system of intragenerational lump-sum transfers equalizing endowments across types E and L. The *decentralization requires also imposing no annuitization ($\alpha = 0$), and introducing either life insurance ($a > 0$) or a subsidy on accidental bequests ($\theta < 0$).*

Proof. The proof follows the same lines as the one of Proposition 10. ■

As in Proposition 10, there is here an equalization of lifetime well-being across dynasties for an equal duration of life. However, the main difference is that the ex post egalitarian constrained optimum cannot achieve the equalization of lifetime well-being between the long-lived and the short-lived, unlike what can be done under the ex post egalitarian optimum optimorum. Thus the short-lived has to remain here worse off than the long-lived.

It should be stressed, however, that the main result obtained in the baseline static model still holds in the dynamic OLG economy: it remains optimal to encourage accidental bequests, in such a way as to reduce inequalities in lifetime well-being between the short-lived and the long-lived. Note also that the robustness of that result depends, here again, on the precise specification of the joy of giving term. If parents were interested in what their children receive, then the decentralization would no longer involve a subsidy on accidental bequests, but a tax. However, that tax would be, as above, less than 100 %.

6 Bequest tax and the age of the deceased

Up to now, we carried out our analysis in a first-best framework, that is, while assuming that all fiscal instruments are available. In that setting, the government could tax unconditional and accidental bequests at different rates. To conclude our explorations, this section considers a second-best setting with a limited number of policy instruments. For that purpose, we remain in our OLG setting with quasi linear preferences, but assume now a small open economy at its stationary equilibrium (that is, factor prices w and R are fixed).[24] Moreover, as in Sections 2 to 4, we also deliberately abstracts from inequalities in initial endowments, and consider only two types of agents, depending on the duration of their own life.

Our second-best analysis assumes here that only three instruments are available: a first-period demogrant T, a tax θ_E on bequests left in case of early death, and a tax θ_L on bequests left in case of late death.[25] The underlying motivation for considering those instruments is twofold.

First, although it is difficult, for a government, to impose different tax rates on the unconditional part of the bequest and the accidental part of the bequest, the government can potentially use the age of the deceased as an indirect way to tax these two components at different rates. The underlying intuition is that the relative part of the accidental component in the total bequest is decreasing with the age of the deceased.

Second, the public finance literature already considered taxing bequests differently depending on the age of the deceased. Vickrey (1945) argued that the

[24] Moreover, we also assume in this section that $R = 1$ (i.e. a zero interest rate).

[25] Thus we abstract here from the possibility of life insurance and collective annuitization.

tax on bequests should be increasing with the age gap between the donator and the receiver, in order to prevent fiscal arbitrages.[26] As we will see, this section will provide also an argument for a tax on bequests that is increasing with the age gap between the donator and the receiver, but on egalitarian grounds.

6.1 The individual problem

Individuals choose saving s and unconditional bequest b to maximize utility:

$$\max_{s,x} w - s - b + T + \pi \left[u(s) + v(\underbrace{(1-\theta_L)\,b}_{x_L}) \right] + (1-\pi)v(\underbrace{(1-\theta_E)\,(b+s)}_{x_E})$$

where x_L and x_E denote the bequest (net of tax) in case of, respectively, late death and early death.

FOCs for s and b yield, respectively:

$$-1 + \pi u'(s) + (1-\pi)v'((1-\theta_E)\,(b+s))\,(1-\theta_E) = 0 \quad (28)$$

$$-1 + \pi v'((1-\theta_L)\,b)\,(1-\theta_L) + (1-\pi)v'((1-\theta_E)\,(b+s))\,(1-\theta_E) = 0 \quad (29)$$

Combining those FOCs, we obtain:

$$u'(s) = v'((1-\theta_L)\,b)\,(1-\theta_L) \quad (30)$$

Both the unconditional bequest $b \equiv b\,(T, \theta_E, \theta_L)$ and saving $s \equiv s\,(T, \theta_E, \theta_L)$ are functions of the three tax instruments.

6.2 The second-best utilitarian problem

The problem of the utilitarian social planner is to find taxes T, θ_E and θ_L that maximize average social welfare subject to the revenue constraint. This problem can be written by means of the following Lagrangian:

$$\max_{T,\theta_L,\theta_E} \quad \pounds = w - s - b + T + \pi\,[u(s) + v((1-\theta_L)\,b)] + (1-\pi)v((1-\theta_E)\,(b+s)) + \mu\,[\pi\theta_L b + (1-\pi)\theta_E(b+s) - T]$$

Using the Envelope Theorem, we obtain the following FOCs:

$$\frac{\partial \pounds}{\partial T} = 1 + \mu \left[\pi\theta_L \frac{\partial b}{\partial T} + (1-\pi)\theta_E (\frac{\partial b}{\partial T} + \frac{\partial s}{\partial T}) - 1 \right] = 0 \quad (31)$$

$$\frac{\partial \pounds}{\partial \theta_L} = -\pi v'(x_L)b + \mu \left[\pi b + \pi\theta_L \frac{\partial b}{\partial \theta_L} + (1-\pi)\theta_E \left(\frac{\partial b}{\partial \theta_L} + \frac{\partial s}{\partial \theta_L} \right) \right] = 0 \quad (32)$$

[26] Vickrey's argument states that, under a fixed tax rate on bequests, grand-parents transmitting their wealth directly to their grand children would lead to less taxation than if these were first transmitting their wealth to their children, and, then, if their children were transmitting the wealth to their own children. The tax schedule proposed by Vickrey prevents this kind of fiscal arbitrage.

$$\frac{\partial \pounds}{\partial \theta_E} = -(1-\pi)v'(x_E)(b+s)+\mu\left[\pi\theta_L\frac{\partial b}{\partial \theta_E} + (1-\pi)(b+s) + (1-\pi)\theta_E\left(\frac{\partial b}{\partial \theta_E}+\frac{\partial s}{\partial \theta_E}\right)\right] = 0 \tag{33}$$

Those FOCs can then be used to examine the design of optimal taxes on bequests. For that purpose, let us first combine the last two FOCs to write the effect of a compensated change of the tax rate on bequests left by a person who died early (i.e. "early bequests"), that is, the effect, on the objective function of the social planner, of a marginal change in the tax rate on early bequests when this change is compensated by a change in the tax rate on late bequests, so as to maintain the government's budget constraint balanced:

$$\begin{aligned}\frac{\partial \tilde{\pounds}}{\partial \theta_E} \equiv \frac{\partial \pounds}{\partial \theta_E} + \frac{\partial \pounds}{\partial \theta_L}\frac{d\theta_L}{d\theta_E} = \quad & -(1-\pi)(b+s)\left[v'(x_E) - v'(x_L)\right] \\ & +\mu\left[\pi\theta_L\frac{\partial \tilde{b}}{\partial \theta_E} + (1-\pi)\theta_E\left(\frac{\partial \tilde{b}}{\partial \theta_E}+\frac{\partial \tilde{s}}{\partial \theta_E}\right)\right]\end{aligned}$$

where $\frac{\partial \tilde{b}}{\partial \theta_E} \equiv \frac{\partial b}{\partial \theta_E} + \frac{\partial b}{\partial \theta_L}\frac{d\theta_L}{d\theta_E}$ and $\frac{\partial \tilde{s}}{\partial \theta_E} \equiv \frac{\partial s}{\partial \theta_E} + \frac{\partial s}{\partial \theta_L}\frac{d\theta_L}{d\theta_E}$ denote the effects on, respectively, b and s, of compensated changes in the tax rate θ_E.[27]

In the absence of any tax on bequests, this expression collapses to:

$$\left.\frac{\partial \tilde{\pounds}}{\partial \theta_E}\right|_{\theta_E=\theta_L=0} = -(1-\pi)(b+s)\left[v'(x_E) - v'(x_L)\right] > 0$$

The above expression states that, starting from a zero tax on bequests, a marginal increase in the tax on bequests left by individuals who die earlier, when compensated by a change in the tax on bequests of those who die later on so as to maintain the government's budget balanced, increases the welfare of the society. As such, this formula provides a utilitarian argument for taxing the bequests of individuals who die earlier at a higher rate than the bequests of those who die later on: $\theta_E > \theta_L$.

6.3 The second-best ex post egalitarian problem

Let us now consider the problem of an ex post egalitarian social planner, who would like to maximize the realized lifetime well-being of the worst-off while equalizing the well-being of the short-lived and the long-lived.

That problem can be written as:

$$\begin{aligned}\max_{T,\theta_L,\theta_E}\ \min \quad & \{w - s - b + T + u(s) + v((1-\theta_L)\,b), w - s - b + T + v((1-\theta_E)\,(b+s))\} \\ \text{s.t.} \quad & \pi\theta_L b + (1-\pi)\theta_E(b+s) = T\end{aligned}$$

[27] Note that $\frac{d\theta_L}{d\theta_E} = \frac{-(1-\pi)}{\pi}\left(\frac{s+b}{b}\right)$.

As usual, we rewrite this problem as the maximization of the well-being of the long-lived subject to the egalitarian constraint, according to which the short-lived is not worse off than the long-lived:

$$\begin{array}{ll} \max\limits_{T,\theta_L,\theta_E} & w - s - b + T + u(s) + v((1-\theta_L)\, b) \\ \text{s.t.} & \pi\theta_L b + (1-\pi)\theta_E(b+s) = T \\ \text{s.t.} & v((1-\theta_E)\,(b+s)) \geq u(s) + v((1-\theta_L)\, b) \end{array}$$

It is straightforward to see that, in the absence of taxation, the egalitarian constraint would not be satisfied, since:

$$v\,(b+s) < u(s) + v(b)$$

that is, short-lived individuals are worse off than long-lived individuals. Hence, in order to achieve equality, it is necessary that the tax on the bequests left by those who die earlier is strictly lower than the tax on the bequests left by those who enjoy a long life, that is: $\theta_E < \theta_L$.

We obtain here a result that is opposite to what is optimal under the utilitarian social welfare function. It is here optimal to tax more the bequests left by individuals who die at higher ages, on the grounds that lower tax rates on the bequests of those who die earlier favor the compensation of the unlucky short-lived with respect to the lucky long-lived.

Proposition 13 summarizes our results.

Proposition 13 *Consider a second-best setting with, as available instruments, a demogrant T, and tax rates on bequests θ_E and θ_L, which vary with the age of the deceased. Abstracting from inequalities in endowments across children, we obtain that:*

- *under the utilitarian objective, the tax on bequests should be decreasing with the age of the deceased ($\theta_E > \theta_L$);*
- *under the ex post egalitarian objective, the tax on bequests should be increasing with the age of the deceased ($\theta_E < \theta_L$).*

Proof. See above. ■

Proposition 13 suggests that how governments should differentiate bequests taxation depending on the age of the deceased is not robust to the underlying social welfare objective. Assigning a large weight to the unlucky short-lived leads here to a lower tax on the bequests left by those who die earlier, and to a higher tax on bequests left by those who die later on. This compensation argument for a differentiated treatment of bequests depending on the age of the deceased is quite different, in nature, from the argument of Vickrey (1945), even though both arguments recommend a tax rate increasing with the age of the deceased.[28]

[28] Strictly speaking, Vickrey (1945) recommends a taxation increasing with the age gap between the donator and the receiver. This is equivalent, under a fixed age of the receiver, to a tax rate increasing with the age of the donator (i.e. the deceased).

7 Conclusions

The goal of this paper consists in revisiting the optimal taxation of accidental bequests by departing from the existing literature in two main ways. First, we paid a particular attention to the fact that individuals may care about what they would, in case of premature death, leave to their offsprings. Second, we also departed from the standard utilitarian social optimum, and compared it with an ex post egalitarian optimum, in order to give more weight to the interests of the unlucky prematurely dead persons. The underlying motivation for considering that alternative social criterion is that taxing accidental bequests raises not only an issue in terms of fairness among offsprings, but, also, an issue of fairness in the treatment of short-lived *versus* long-lived persons.

Our main result is that, once we depart from the literature in those two ways, the conventional view according to which accidental bequests should be taxed at a confiscatory rate is not true anymore. The underlying intuition goes as follows. If individuals care about what they leave to their children in case of premature death, then accidental bequests become a way to allow for an improvement of the situation of the unlucky prematurely dead, and, hence, for a reduction of inequalities in lifetime well-being between the long-lived and the short-lived. Hence, if one pays attention to the compensation of unlucky short-lived persons, this tends to question the conventional 100 % tax view on accidental bequests. From the perspective of inequalities along the lifecycle between long-lived and short-lived individuals, a tax on accidental bequests is unfair to the short-lived. This motivates subsidizing - rather than taxing - accidental bequests.

When examining the robustness of those results to a dynamic OLG model (which allows us to take into account inequalities in endowments across children at the next generation), it is shown that, although the sign of the optimal tax on accidental bequests depends on the form of the joy of giving motive (i.e., on whether parents are interested in what they give or in what their children receive net of all taxes and transfers), it remains true, in that broader setting, that the 100 % tax view does not hold under the ex post egalitarian criterion.

What are the implications of our results for real economies? To answer that question, it should be stressed that the above results are obtained while assuming that the government can tax the unconditional and the accidental components of bequests at different rates. However, in reality, such fiscal instruments are not available. One possibility to overcome this difficulty, which was explored in Section 6, may be to rely, as a proxy, on the age of the deceased. The relative part of the accidental component in total bequests is decreasing with the age of the deceased. Governments could thus use the age of the deceased as a basis for indirectly taxing the two components of bequests differently. Our analysis shows that, whereas the utilitarian criterion recommends a tax on bequests that is decreasing with the age of the deceased, the opposite is true under the ex post egalitarian criterion, which recommends a tax on bequests that is increasing with the age of the deceased. This confirms that the optimal tax on bequests is highly sensitive to the underlying social welfare criterion, and, in particular, to the ethical treatment of longevity inequalities.

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9 Appendix: Proof of Proposition 11

Suppose now that parents are interested in what they give to their children net of all government taxes and transfers. In addition to the egalitarian constraint, $u(d)+v(b) = v(b+e)$ of the original problem (see beginning of Section 5.4.1), the social optimum now also includes an additional constraint, $v(b) = v(b+e)$ which accounts for the fact that children would obtain the same amount of bequest independently of whether their parents die late or early. The combination of these two equations yields that necessarily at the optimum, $e^* = 0$ and $u(d) = 0$ so that $d^* = \bar{c}$. Hence, the social optimum can be rewritten in the following simplified way:

$$\begin{aligned} &\max_{c,b,k} c + u(\bar{d}) + v(b) \\ &\text{s. to } f(k) = c + b + \pi\bar{d} + k \end{aligned}$$

Rearranged FOCs are

$$\begin{aligned} v'(b^*) &= 1 \\ f'(k^*) &= 1 \end{aligned}$$

Let us now consider the decentralization of this problem. Let us assume, for simplicity, that there is no life insurance ($a = 0$) and no annuitization ($\alpha = 0$).[29] Let us suppose that savings (equivalently old-age consumption) are taxed at a rate τ, accidental bequests at a rate θ and that in case of late death of the parent, children are compensated through a lump sum transfer T.[30]

The problem of any agent therefore writes:[31]

$$\begin{aligned} \max_{s,b} \quad & w - b - s + B + \pi\left[u(s(1-\tau)) + v(b+T)\right] \\ & +(1-\pi)v\left(b + s(1-\theta)\right) \end{aligned}$$

where B is the amount received by children net of all government taxes and transfers and which is independent of whether their parents died early or late.

FOCs for s and b are now:

$$1 = \pi u'\left(s(1-\tau)\right)(1-\tau) + (1-\pi)v'\left(b + s(1-\theta)\right)(1-\theta) \tag{34}$$

$$1 = \pi v'(b+T) + (1-\pi)v'(s(1-\theta)+b) \tag{35}$$

The government budget constraint is such that, in equilibrium, it is balanced:

$$\pi T = \pi s\tau + (1-\pi)\theta s \tag{36}$$

[29] Introducing the possibility of annuitization and life insurance would not change the results.
[30] Note that we could have as well assumed a lump sum tax on savings.
[31] We can remove indexes $i = E, L$ from this problem since at the decentralized problem we ensure that no inequality remains between children whose parents were long-lived or short-lived.

We also need to have $T = s(1-\theta)$ so as to ensure that the amounts children receive from their parents net of all taxes and transfers are independent of whether these died early or late. Replacing for this expression into the above government budget constraint, we obtain that the tax on savings (equivalently old-age consumption) should be such that:

$$\tau = 1 - \frac{\theta}{\pi} \tag{37}$$

At the optimum, τ is chosen so as to ensure that $s(1-\tau) = \bar{c}$ so that

$$\tau^D = 1 - \frac{\bar{c}}{s^D} < 1$$

and thus, using (37), we obtain

$$\theta^D = \pi \frac{\bar{c}}{s^D} < 1$$

where s^D is obtained from the FOC on saving of the individual:

$$1 = \pi u'(\bar{d})(1-\tau^D) + (1-\pi)v'\left(b^D + s^D(1-\theta^D)\right)(1-\theta^D)$$

and is a function of policy parameters (τ^D, θ^D). The level of voluntary bequest b^D is such that $1 = v'(b^D + T^D) = v'(b^D + s^D(1-\theta^D))$ with

$$T^D = s^D\tau^D + \frac{(1-\pi)}{\pi}\theta s^D$$

obtained from (36).

This tax and transfers scheme ensures that at the decentralized equilibrium, we obtain optimal values $s^D(1-\tau^D) = d^* = \bar{c}$ and $b^D + T^D = b^*$ since $v'(b^D + T^D) = 1$.